

AN EXPERIMENTAL STUDY OF PROBABILISTIC LANGUAGE IN SECONDARY SCHOOL TEXTBOOKS

Juan J. Ortiz, M. J. Cañizares, C. Batanero, and Luis Serrano
University of Granada
Spain

The teaching of probability is currently being reinforced in many countries, as it is visible from new curricular documents such as the NCTM Standards (2000), where the acquisition of a precise language in connection to chance and probability is considered to be a main learning goal. On the other hand, textbooks are an important resource for teachers who can find in these books ideas and activities to facilitate students' learning. In recent Spanish curricular documents (M.E.C., 1992) the teaching of probability is introduced at earlier ages with a teaching methodology based on simulation and experimentation. A main concern is children's progressive acquisition of a precise language in connection to chance and probability. This curriculum is not an exception, since we find similar concerns in curricular documents from other countries, such as the United Kingdom or the United States.

On the other hand, when children are first taught probability, they have frequently used terms and expressions to refer to randomness, sometimes with a meaning different to what is usual in the mathematics classroom. All these reasons suggest the interest to carry out an empirical study to determine the specific language that about chance and probability is presented in the textbooks.

In this paper, we summarize an empirical study of the terms and expressions included in three different Spanish secondary school textbooks, which is part of a wider study on the treatment of probability in the textbooks (Ortiz, 1999) and continues other related research in our department (Sánchez Cobo, 1987). On main concern for mathematics teachers is to facilitate students' learning with a variety of didactic resources and materials, the main one being textbooks. As suggested in the Cockcroft report (1985) "textbooks constitute an invaluable help for the teachers' daily work in the classroom" (p. 114). We suggest that the linguistic differences observed in these textbooks might have an influence on the students' learning and that, therefore this analysis may provide information that will guide the teachers when preparing and carrying out the teaching of probability.

The Study

To carry out this study, we selected three textbooks that will be referred as [A], [B] and [C] in the following, and which appear in the Appendix. These books were selected from those used in our previous study (Ortiz, 1999), because they were widely used by the teachers and presented the most comprehensive account of probability in the whole sample of books. They were published by three different very popular Spanish publishers. Book [B] was published in 1975, book [A] in 1991 and book [C] in 1998. These years correspond to the beginning of the so-called "modern mathematics" period, the year in which the last Spanish school reform (LOGSE) was started, and finally we took a recent book, where the LOGSE has been consolidated in Spain. The aim is to assess the impact of the LOGSE over the period.

LANGUAGE AND MATHEMATICS

Many authors have reflected about the relevance of language in the study of mathematics. For example Orton (1990) suggests that many aspects of language can influence the learning of mathematics, since many children do not understand the terms we use in the classrooms, as a part of the mathematical vocabulary. Pimm (1987) draws our attention to the use of everyday words, with particular mathematical sense, which arise very frequently in the case of probability. He also considers that analogies and metaphors, where everyday words can be given particular meanings are very important for the construction of mathematical meaning. Dickson and cols. (1991) summarise diverse theories that point to the relevance of language in the development of mathematical thinking.

Rothery (1980) differentiates three categories of words used in the teaching of the mathematics: a) words that are specific to mathematics and that, usually, are not part of the daily language; b) words that appear both in mathematics and in ordinary language, although they are given the same meaning in both contexts; c) words that have the same or very close meanings in both

contexts. In the teaching of probability most of the words used belong categories b) and c). However for young children or for children not familiar with probability many words in category c) may, in fact, become terms of category b). This created communication difficulties in the classroom.

Hartwell et al (1996) suggest the lack of literature-based guidance for conducting evaluations of statistics textbooks and that this is an important activity that might provide criteria for guiding new evaluation and use of textbooks. Moreover, most research in this area is related to University level and therefore our research may provide these criteria in reference to secondary school textbooks. In the following we describe our study with two main objectives:

1. Showing the wealth and variety of probabilistic expressions and words in the textbooks.
2. Remarking possible differences of expressive means in textbooks at the same educational level.

Our results are presented in two sections: the language of randomness and the language of probability. Tables with all the different words and expressions in each language category for each book will be presented and compared throughout the presentation of the paper.

THE LANGUAGE OF RANDOMNESS

We first analysed words and terms used in reference to randomness and random experiment, their properties and examples of random situations. These books in general provided scant opportunities for children to carry out random experiments and to observe their results. The introduction of the idea of randomness is just made through the description of examples of random and non-random situations. For this reason, language plays a primordial role in children's understanding of this idea. The words and expressions related to randomness were classified in the following differentiated groups:

- a) *References to randomness*: Words and expressions that evoke the idea of randomness, that are used to define randomness or to describe its features: "*Chance is what is opposed to order, to any law, to prediction. How can we draw a law for something which is unpredictable*"
- b) *References to random experiment or random results*. We found adjectives, such as *uncertain*, and also the words "random" "chance" is used as an adjective, as in *random number, chance game*. Some verbs are given a precise meaning in the context of random experiments: *throwing* (a die), *drawing* (a ball; a card). They evoke a series of action familiar to children's experience with popular games, and, at the same time they call some implicit agreements: we throw the die, but we only take into account the number shown on the upper side and all the numbers are equally likely. We assume children understand this; however, some research show that for young children these agreements might not be natural. For example some of them think not all the numbers in a die are equally likely.
- c) *References to random generators*. Two components can be isolated from a theoretical point of view in analysing the concept of randomness: the outcome or outcomes sequence, and the experiment itself - experimental device producing random outcomes-. We found a great variety of words and expressions related to random generators, that. According to Truran (1994) might be considered to be non-equivalent, they have the same mathematical structure. These words refer to the random generators themselves (die, cards, urns, spinners), random situations in everyday life (birth, traffic, rain) and computer devices (random key). They also made reference to their properties, such as fairness of symmetry.

Our analysis showed a richer randomness vocabulary in textbooks [A] and [C]. The variety of adjectives and expressions used to describe this concept was greater. These books also included examples of random experiments for which the Laplace cannot be applied and that are, therefore, associated to a frequentist conception of probability (e.g., throwing a thumbtack). References to random numbers and the different forms to generate them, with calculators, computers, random numbers tables were found in book [C]. In book [A], the computers' random function is also described. All of this suggests that a bigger emphasis is given to the concept and phenomenology of randomness in these two books, as compared to book [B] and also a gradual widening of randomness vocabulary in textbooks used for the same teaching level over time.

THE LANGUAGE OF PROBABILITY

There is also a great variety in the vocabulary used to refer to the idea of probability, assigning and comparing probabilities and different interpretations of probabilities. We will

present a summary of this vocabulary in the presentation of the paper. We have classified these expressions in the following categories:

- a) *Concept, interpretation, types of probability, conceptions.* A first group of words and expressions refer to the concept of probability, their various interpretations and suggest different underlying conceptions of probability, such as laplacian, frequentist, subjective or formal. An example is the use of the word *confidence* in relation to subjective assignment of probability. In the same way the expressions *degree of belief*, *degree of uncertainty* are used: "*probability is the part of mathematics which tries to put numbers to uncertainty (degree of uncertainty)*". Book [A], with a formal axiomatic introduction of probability uses the word *number* and symbolic notations to define probability "It is possible to assign a non negative number $p(S)$ which is called the probability of event S to any $S \in B$ ".
- b) *Probability as a function and assignment of their values.* Different words are used to suggest the way in which probabilities are assigned: *assign*, *compute*, *estimate*, *measure*, which are not equivalent and sometimes might confuse the students: "*we will try to measure the probability for these events*". The only book where the idea of *function* is related to probability is text [B] where a formal approach is included.
- c) *Graduation of probabilities.* The textbooks use different words to express in a qualitative way a graduation of the probability of different events: *Certain*, *confirming*, *almost certain*, *probable (highly, very few, few, half)*, *equiprobable*, *equally probable*, *rare (very)*, *impossible (almost impossible)*.

Again the vocabulary is much richer in textbooks [A] and [C], confirming the tendency to find a greater variety in the most recent textbooks. Qualitative expressions for the probabilities of events do not appear in book [B], which only includes numeric values of probabilities. Therefore the study of probability which is carried out in the classroom is isolated from the applications of probability in daily life where we often make probabilistic evaluations using colloquial expressions such as "very likely", "not expected", etc. to refer to our perceived subjective probability for a random event. There is no reference to the different ways of obtaining the initial values for probability, since subjective or frequentist conceptions of probability are not mentioned in this book. Consequently, there is not reference to "estimation" or to subjective assignment of probabilities and the distinction between theoretical and experimental probability is not mentioned. Finally, we found no connection to combinatorics, even when most of the problems proposed should be solved using combinatorial concepts.

CONCLUSIONS

In this research, we tried to highlight the usefulness of textbooks as a didactical resource in the teaching of probability, as well as possible limitations in the textbooks. We have also shown the relevance of language in the teaching and learning of mathematics. In particular the language used in relation to randomness and probability suggest these are very complex concepts and also the variability of expressions that appear in books that have been written for the same teaching level in relation to randomness and probability. Our study also shows the significant role of textbook authors, who set a new level in the didactical transposition of a theme, when interpreting and developing what is established in the official curricular designs. Finally, the teachers decide what textbook they will use in their classroom, how they will be using the books, and how they will supplement their content. We hope this paper can contribute to increase the interest towards the language of randomness and probability amongst teachers and textbook authors and thus improve the teaching of randomness and probability, at the secondary school level.

ACKNOWLEDGEMENTS:

This paper is part of Project BSO-2000-1507. MEC. Madrid.

REFERENCES

- Cockcroft, W.H. (1985). *Las matemáticas sí cuentan. Informe Cockcroft* (Mathematics count. Cockcroft report). Madrid: Ministerio de Educación y Ciencia.
- Dickson, L., Brown, M., & Gibson, O. (1991). *El aprendizaje de las matemáticas* (Learning of mathematics). Madrid: Labor.

- Hartwell, M.R., Herrick, M.L., Curtis, D., Mundfrom, D., & Gold, K. (1996). Evaluating statistics texts used in education. *Journal of Educational and Behavioral Statistics*, 21, 3-34.
- N.C.T.M. (2000). *Principles and standards for school mathematics*. Reston, VA; N.C.T.M. <http://standards.nctm.org/>
- M.E.C. (1992). *Matemática secundaria obligatoria* (Secondary school mathematics). Madrid: Ministerio de Educación y Ciencia.
- Ortiz, J.J. (1999). *Significado de conceptos probabilísticos en los textos de Bachillerato* (Meaning of probabilistic concepts in textbooks). Doctoral Thesis. University of Granada.
- Orton, A. (1990). *Didáctica de las matemáticas* (Mathematics education). Madrid: M.E.C. and Morata.
- Pimm, D. (1987). *Speaking mathematically*. New York: Routledge and Kegan Paul.
- Truran, J. (1994). Children's understanding of random generators. In J. Garfield (Eds.), *Research papers from the 4th International Conference on Teaching Statistics (ICOTS IV)*. The International Study Group for Research on Learning Probability and Statistics. University of Minnesota.
- Sánchez-Cobo, F.T. (1996). *Análisis de la exposición teórica y de los ejercicios de correlación y regresión en los textos de Bachillerato* (Analysis of regression and correlation theoretical content and exercises in secondary school textbooks). Master's Thesis. University of Granada.

APPENDIX: TEXTBOOKS ANALYSED

- [A]: Guzmán, M., Colera, J., & Salvador, A. (1988). *Matemáticas, Bachillerato 1º*. Ed. Anaya. Madrid.
- [B]: Valdés, J., & Marsinyach, S. (1975). *Matemáticas Bachillerato 1º*. Ed. Bruño. Madrid.
- [C]: Berenguer, L. et al. (1998). *Construir las matemáticas. 3º E.S.O.* Proyecto Sur de Ediciones. Granada.