Teaching Probability to Pre-service Primary School Teachers through Simulation

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1. Introduction

Over the past 20 years probability has been added to the mathematics curricula for primary schools in many countries (e.g. Holmes, 1980; NCTM, 2000), due to the presence of randomness in daily life, the need for basic stochastic knowledge in many professions and its role in developing critical reasoning. A problem to succeed in introducing the teaching of probability at this level is that teachers frequently lack specific preparation. For example, in Spain primary school teachers do not receive any specific training in probability or statistics and this situation is common to many countries.

In this paper we first present results of an initial assessment that show that students entering the Faculty of Education frequently present various probabilistic misconceptions. We then analyse some activities that were used to make student teachers realise their own probability misconceptions at the University of Granada. We conclude that a better prior training for teachers as well as permanent support for these teachers from University departments and research groups is an urgent necessity.

2. Trainee teachers' probabilistic conceptions. Results from an initial assessment

Spanish trainee teachers only study probability with a formal, mathematical approach in the first year of secondary school (14 years-old) for four weeks, 3 hours per week. That is why we expect them to present some of the probabilistic misconceptions that have been widely reported in adults.

In order to assess the extent of these misconceptions amongst trainee teachers, we carried out an assessment study with a group of 132 students at the Faculty of Education, University of Granada. We gave the subjects in the sample the questionnaire by Garfield (1991) which was developed to measure a variety of components in statistics and probabilistic reasoning. In Table 1 we reproduce 5 items, which refer to probabilistic misconceptions and the percentage of responses for each category. These misconceptions are the following:

- *Neglect of sample size: law of small numbers.* Item 2 is adapted from Kahneman et al (1992) to assess whether the trainee teachers appear to be neglecting the sample size in judging probabilities. This is a special case of the representativeness heuristic, referred to as the "law of small numbers", because people tend to judge small samples as being equally representative of a population as large samples.
- *Equiprobability:* Items 3 and 4 are adapted from Lecoutre and Durand (1988) to assess whether the trainee teachers tended to reason using the equiprobability bias.

Outcome approach: Konold et al (1993) suggested that some of the students' incorrect answers in item 3 might be explained by the outcome approach. He then suggested using a different version of item 5 asking for the least likely event. Konold suggests that students choosing answer c in item 5 might be reasoning according to the outcome approach.

Table 1. Percentages of responses to probabilistic items

	Percentage
Item 1. Five faces of a fair die are painted black, and one face is painted white. The die is rolled six times. Which of the following results is more likely?	
a. Black side up on five of the rolls; white side up on the other roll	31.1
b. Black side up on all six rolls	64.4
c. a and b are equally likely	5.5
Item 2. Half of all new born babies are girls and half are boys. Hospital A records an average of 50 births a day. Hospital B records an average of 10 births a day. On a particular day, which hospital is more likely to record 80% or more female births?	
a. Hospital A (with 50 births a day)	24.4
b. Hospital B (with 10 births a day)	13.7
c. The two hospitals are equally likely to record such an event.	61.9
Item 3. When two dice are thrown simultaneously it is possible that one of the following two results occurs: Result 1: A 5 and a 6 are obtained. Result 2: A 5 is obtained twice. Select the response that you agree with the most:	
a. The chances of obtaining each of these results is equal	66.2
b. There is more chance of obtaining result 1.	6.2
c. There is more chance of obtaining result 2.	2.3
d. It is impossible to give an answer.	25.3
Item 4. When three dice are thrown simultaneously, which of the following results is MOST LIKELY to be obtained?	
a. Result 1: "A 5, a 3 and a 6"	14.6
b. Result 2: "A 5 three times"	0.8
c. Result 3: A 5 twice and a 3"	3.1
d. All three results are equally likely	81.5
Item 5. When three dice are thrown simultaneously, which of these three results is LEAST LIKELY to be obtained?	
a. Result 1: "A 5, a 3 and a 6"	3.1
b. Result 2: "A 5 three times"	23.1
c. Result 3: A 5 twice and a 3"	73.1
d. All three results are equally unlikely	0.8

The results in Table 1 suggest that representativeness (item 2, option c), equiprobability (item 3, option a; item 4, option d) and the outcome approach (item 5 option d) are widespread amongst the trainee teachers in this sample. Hence, it is foreseeable that these teachers might feel unconfident when they teach probability to their students and also that they might contribute to spread their misconceptions amongst them.

One related problem is how best we should use the brief time available to teach probability to these future teachers. Since we cannot introduce a large number of concepts or carry out a complete formal study of probability in this time, the best solution might be using the teaching time to make the student teachers conscious of their probabilistic misconceptions, help them to overcome some of them and increase their interest in probability and its teaching. This change in attitudes might lead teachers to try to complete their probability knowledge by themselves, by reading some probability books or by attending some probability courses for in-service teachers. Below we suggest how simulation might be a useful didactic tool for achieving this aim.

3. Simulation Experiments based on Random Number Tables

The experiment described was carried out within a compulsory course of Mathematics and Didactics (90 hours long), at the Faculty of Education, University of Granada. All the students were trainee teachers and 10 hours were available to teach statistics and probability. Before the experiment was carried out the students had been introduced to the notions of random events, sample space, a priori and frequential probability (Godino and Batanero, 2002).

The aim of the experiment was to introduce the students to the use of random table numbers and the idea of simulation; making them carry out random simulations with manipulatives and random number tables, and use simulation to solve and reflect on counterintuitive probability problems.

The simulation experiment was organised into six steps:

1. Solving the two dice problem. Students were asked to solve a problem similar to item 3. The results matched the assessment described in section 1, since most of the students considered the three results were equally likely.

2. Simulation with small samples. Each student was asked to simulate using random table numbers 10 throwing of two dice and to record his/her results in a table. Then they were asked to compare results with other classmates and with their answers to the problem in step 1. They had to explain why different students obtained different results in the simulations.

3. Simulation with large samples. Each student was asked to simulate using random table numbers 100 throwing of two dice and to record his/her results in a table. Then they were asked to compare again their results with other classmates and with their answers to the problem in step 1.They had to explain the differences in the simulations carried out in steps 2 and 3.

4. Solving the hospital problem. Students were asked to solve a problem which is a variation of item 2 and to record their answers in their report. Most of them reasoned according to representativeness heuristics and considered the event to be equally likely in both hospitals.

5. Simulations with small samples. The students were asked to carry out 10 simulations using random table numbers for hospitals A and B and record for each of them the number of days for which more than 70% of the babies were female. Again the results with small samples were too variable and did not allow the students to overcome their incorrect intuitions.

6. Simulations with large samples. The results for all the students were collected to produce 200 simulations for both hospitals (20 groups of students x 10 simulations) and a debate was held about the results.

4. Some precautions

With the previous description we aimed to show the interest of using simulation for teaching probability that is mentioned by Biehler (1997). As suggested by Biehler the temporal and spatial features of many random phenomena make them difficult to observe. Simulations create new dialogue contexts between teachers and students that help them to confront their mutual interpretations, to progress in fitting together the institutional and personal meanings (Godino y Batanero, 1998).

Nowadays, many complex problems are solved by simulation so that showing the students some simple examples of this technique may serve to show them their applicability to real problems. In teacher training, simulation may also help students to recognise the differences between theoretical and experimental probability.

We should, however not be too optimistic as regards the difficulties in simulation. In our experiments we observed the following difficulties that were also described by Countiñho (2001):

a. Distinguishing the estimation of probability given through simulation from the real theoretical value of probability which is also accessible by formal calculus, in case it is possible.
b. Since simulation and the frequentist approach to probability only provide an estimate for the problem solution and not the reason why this solution is valid it has no explanatory power. Consequently, it lacks the validative value that only the classical approach and formal probability calculus can provide. The teacher's role is also crucial in the institutionalisation phase of didactical interactions (Brousseau, 1997) and for solving semiotic conflicts when interpreting tasks and language (Godino, 2002).

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REFERENCES

Batanero, C. y Godino, J. D. (2002). Estocástica para maestros. Granada: Departamento de Didáctica de la Matemática. URL: http://www.ugr.es/local/jgodino/edumat-maestros/

Bieler, R. (1997).Software for learning and for doing statistics. International Statistical Review, 65 (2), 167-190.

Brousseau, G. (1997). *Theory of didactical situations in mathematics*. Dordrecht: Kluwer A. P.

Countinho, C. (2001): *Introduction aus situations aléatoires dés le Collége: de la modélisation à la simulation d'experiences de Bernoulli dans l'environment informatique Cabri-géomètre-II,* Tesis Doctoral, Universidad de Grénoble.

Garfield, J. (1991). Evaluating Students' Understanding of Statistics: Development of the Statistical Reasoning Assessment. In *Proceedings of the Thirteenth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Volume 2* (pp. 1-7). Blacksburg, VA.

Godino, J. D. (2002). Un enfoque ontológico y semiótico de la cognición matemática. *Recherches en Didactique de la Mathématique*, *22 (2/3)..*

Godino, J. D., & Batanero, C. (1998). Clarifying the meaning of mathematical objects as a priority area of research in mathematics education. In J. Kilpatrick, & A. Sierpinska (Eds.), *Mathematics education as a research domain: A search for identity* (pp. 177-196). Dordrecht: Kluwer.

Holmes, P. (1980). Teaching statistics 11-16. Sloug: Foulsham Educational.

Lecoutre, M. P. y Durand, J. L. (1988). Judgements probabilistes et modèles cognitifs: Étude d'une situation aleatoire. *Educational Studies in Mathematics*, 19, 357- 368.

Kahneman, D., Slovic, P. y Tversky, A. (Eds.) (1982). *Judgement under uncertainty: Heuristics and biases.* Cambridge: University Press.

Konold, C., Pollatsek, A., Well, A., Lohmeier, J. y Lipson, A, (1993). Inconsistencies in students' reasoning about probability. *Journal for Research in Mathematics Education*, 24(5), 392-414.

NCTM (2000). *Principles and standards for school mathematics*. Reston: VA: National Council of Teachers of Mathematics.

RÉSUMÉ

Cet article analyse les raisons pour lesquelles l'enseignement des probabilités est difficile pour les professeurs des écoles primaires. Nous présentons les résultats d'une évaluation initiale sur un échantillon de professeurs stagiaires montrant des conceptions erronées très profondes. Puis nous décrivons une expérience didactique basée sur une simulation utilisant des tables de nombres aléatoires. Les résultats montrent que les professeurs pourraient bénéficier de cette expérience pour acquérir des connaissances probabilistes et pour surmonter leurs conceptions erronées initiales, même dans le temps limité disponible pour leur formation.