

# Teaching Statistics Using Forensic Examples

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Statistics plays an important role in forensic science. For example, the conditional probability can be easily demonstrated by the so called prosecutor fallacy. In this talk, ideas of mean, standard deviation, coefficient of variation, prior probability, Bayes theorem and graphical methods are illustrated using forensic examples. Students find the examples interesting and the applications of statistics useful in forensic science.

## 1. DNA Profiling: Test of Independence

DNA profiling/fingerprinting is a very powerful method for human identification (Jeffreys, et al., 1985). It is regarded as one of the most important discoveries in forensic science since the introduction of dermal fingerprinting. In theory, no two persons, except for identical twins, have the same DNA sequence. However, with the DNA markers/loci that are currently in use, a unique identification cannot be assured.

Consider a case where a crime has been committed, a blood stain (from perpetrator) is found in the crime scene and a suspect has been identified. Suppose that the DNA found at the crime scene is typed with alleles (distinct types or lengths of DNA) {8, 9} (a person inherits two alleles from his or her parents) at the DNA locus TPOX and that the DNA of the suspect has the same alleles. Under the current legal system, the frequency or the match probability of the alleles has to be calculated and reported in the courtroom. This can be obtained as  $2p_8p_9$ , where  $p_8$  and  $p_9$  are the frequencies of alleles 8 and 9 respectively. In the Hong Kong Chinese population, this frequency is evaluated as  $2 \times 0.545 \times 0.1 = 0.109$ ; see Table 1 (Wong et al., 2001).

**Table 1. Allele frequencies for locus TPOX in Hong Kong Chinese**

Allele	8	9	10	11	12
Frequency	0.545	0.100	0.022	0.313	0.020

The assessment of frequency uses the product rule in which the alleles are taken to be independent statistically, or in Hardy-Weinberg equilibrium genetically. The frequencies of locus TPOX for the Hong Kong Chinese were determined from a database of  $n = 275$  persons, with a total of  $2n = 550$  alleles. The locus TPOX takes the allele value  $A_i = i + 7$ ,  $i = 1, \Delta, 5$ , and the 275 pairs of alleles can be grouped into a special contingency table with the row and column factors being  $A_i$  and  $A_j$ ,  $i \leq j$ ,  $i, j = 1, \Delta, 5$ . A particular  $\chi^2$  test of independence of alleles within the locus TPOX (independence of row and column factors or in Hardy-Weinberg) can be constructed, and it is shown to be chi-squared distributed with  $5(5-1)/2 = 10$  degrees of freedom (Geisser and Johnson, 1992). The  $\chi^2$  value in this case is 5.97 with a  $p$ -value 0.818. This idea can be generalized to test for independence of alleles between loci (Fung, 1997), since the product rule is also used to evaluate the overall frequency across all loci.

## 2. Conditional Probability and Prosecutor Fallacy

Many countries use 7 or more loci for forensic DNA analysis. Therefore, it is not uncommon to find a frequency as small as 1 in 1 million or even 1 in 1 billion (this happens in many places such as Hong Kong in which 9 loci is used). This shows the powerfulness of the product rule. Students find it amusing that such a small frequency can be arrived based on a small number of loci, and difficult to understand why the frequency can be 1 in 1 billion for a city with only 6.5 million people. The confusion may be explained as follows.

Consider two possible explanations hypotheses for the blood stain evidence,  $H_p$ : the perpetrator left the blood stain, and  $H_d$ : some other person left the blood stain. Based on the product rule, we may have arrived that “the chance/probability observing this blood type if the blood came from someone other than the suspect is 1 in 1 billion”. The match probability idea is of course different from the frequency idea and it could be as small as 1 in 1 billion for a population size of only 6.5 million.

In the following discussion, we slightly modify the above statement to “the chance  $\Lambda$  is 1 in 1000” for easy understanding. In the presentation of DNA evidence or forensic evidence in general, it is not too uncommon, at least in the early court cases with DNA evidence, to find a (incorrect) statement in the courtroom in such a form as “the chance that the blood came from someone else is 1 in 1000”. What is the difference between these two statements?

Let  $G_c$  be the DNA typing results for the crime sample. The first quoted sentence means

$$P(G_c | H_d) = 0.001,$$

while the second says

$$P(H_d | G_c) = 0.001.$$

The second sentence is equivalent to say that “the chance that the blood came from the suspect is 99.9%”. This is a very strong statement suggesting a high chance that the suspect is the contributor of the blood stain. In fact, from the probability statement given above, it is clear that the order of  $H_d$  and  $G_c$  is mixed up in the second sentence in the assessment of the conditional probability. This is often called the error of the transposed conditional or termed the “prosecutor's fallacy” (Thompson and Schumann, 1987; Evett and Weir, 1998) even though defense lawyers may also make such a mistake.

This error has been committed by members of the legal profession. The following lists some examples (Evett and Weir, 1998): “*you testify that there is  $\mathbf{L}$  a 1 in 71 chance that a pair of contributors at random could have left the stain*” (people v. Simpson); “*Q: So the likelihood of this being any other man than Andrew Deen is one in three million? A: In three million, yes*” (R. v. Deen, U.K., 1994); “*Q: What is the combination, taking all those [alleles] into account? A: Taking them all into account, I calculated the chance of finding all those bands [alleles] and the conventional blood groups to be about one in 40 million. Q: The likelihood of it being anybody other than Alan Doheny? A: Is about one in 40 million.*” (R. v. Doheny, R. v. G. Adams, UK, 1997).

## 3. Defender's Fallacy

The defender's fallacy or the defense attorney's fallacy (Thompson and Schumann, 1987) which assigns prior probabilities of guilt from transfer evidence, is more likely to be made by defense lawyers. Consider the following illustrative statement from a defense lawyer (Aitken, 1995).

“The evidence for blood types has very little relevance for this case. Only 1 in 10,000 of the population has the rare blood type found at the crime scene and in the suspect. However, in a city, like this one in which the crime occurred, with a population of 2,000,000 people who may have committed the crime this blood type would be found in approximately 200 people. The evidence merely shows that the suspect is one of 200 people in the city who might have

committed the crime. The blood test evidence has provided a probability of guilt of 1 in 200.

Such a small probability has little relevance for proving the suspect is guilty.”

Statistically, part of the argument of the defense lawyer is correct. The number of people having the same profile as the crime stain is approximately 200, but it is very doubtful that all these 200 people have an equal chance of guilt. The 1/10,000 frequency suggests a strong evidence and it is expected that only one of 10,000 people selected randomly will have the DNA match the evidence. The 1/200 suggests a “weak” DNA evidence and it is not correct because all 200 people are evaluated equally, a priori, in committing the crime.

#### 4. Blood Alcohol Concentration Measurements

The blood alcohol concentration (BAC) were measured from blood specimens collected from drivers in drink driving cases in Hong Kong (Fung et al., 2000). Four measurements,  $x_i, i = 1, \Lambda, 4$ , were obtained for each specimen by 2 analysts using 2 instruments. One basic assumption in many statistical models is the constant variance assumption, whose validity can be established by a graphical presentation of the data. Fig. 1 plots the variance against the mean BAC values obtained by each analyst for every specimen. Obviously, the scattering of the data was not structureless, rather they scattered in a half-cone shape. The variance increases as the mean of BAC value increases. Instead, the coefficients of variation of the BAC are rather similar.

One way to deal with the non-constant variance problem is to transform the data. Let  $y_i = x_i / \bar{x}, i = 1, \Lambda, 4$ , and  $y_i$  is therefore a measure of the proportional deviation of the BAC values from their mean. After removing some “outlying observations”, Fig 2 plots the variance of  $y$ 's against the mean of  $x$ 's. The scattering of the data is structureless and the transformation solves the non-constant variance problem. The subsequent analyses based on the transformed data would be more sound and valid statistically.

#### 5. Conclusion

The use of forensic examples in the teaching of statistics is promising. Students are interested in learning the background of some real cases such as the Simpson case. They appreciate the usefulness of statistics in tackling the prosecutor's and defender's fallacies, and obtaining a very small match probability for the DNA evidence.

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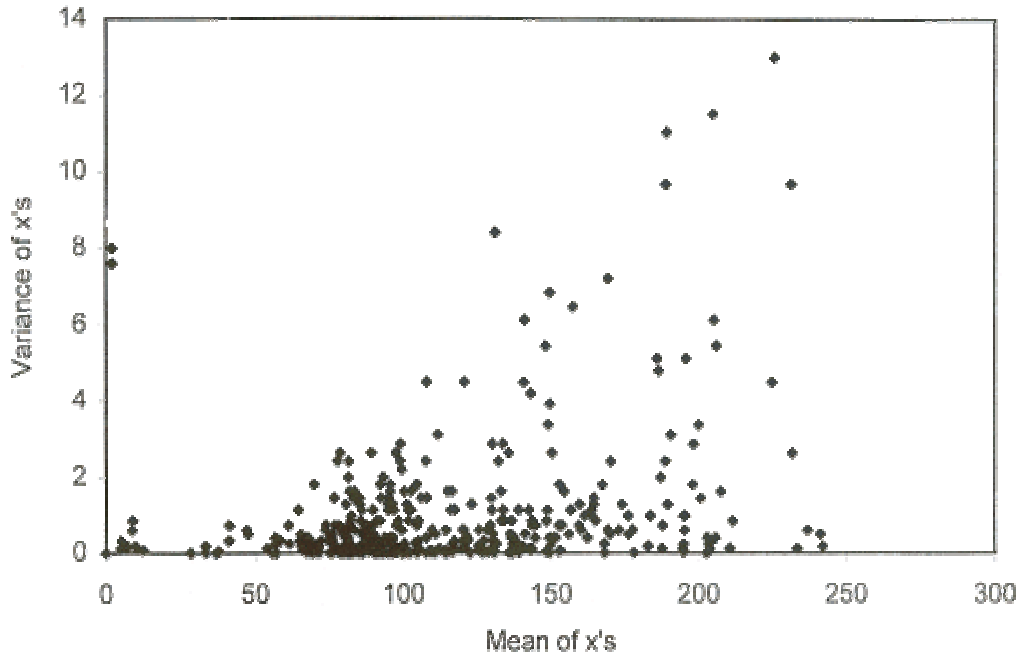


Fig. 1. Plot of variance versus mean BAC value of  $x$ 's.

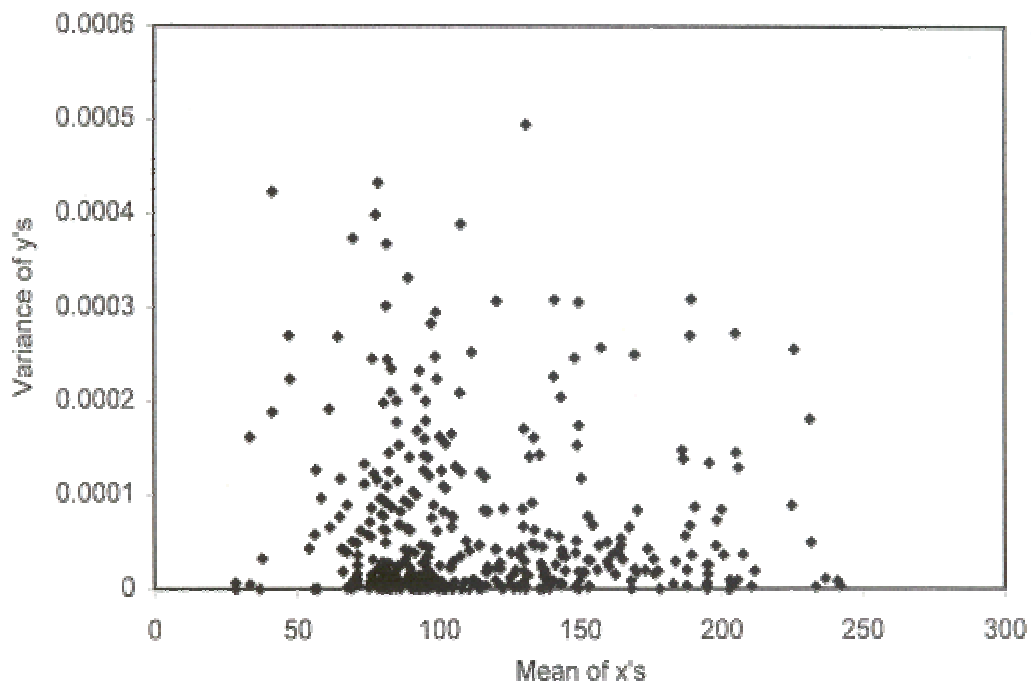


Fig. 2. Plot of variance of  $y$ 's against mean of  $x$ 's.