Statistics 120
Perception II

Graphical Perception

- Some visual processing takes place without any concious effort on our part.
- Psychologists call this *preattentive vision*.
- In the context of extracting information from graphs we will call it *graphical perception*.
- Graphs which convey their information at this unconcious level allow us to extract the information without any concious effort on our part.
- Such graphs are said to provide *inter-occular traumatic impact* (this is a joke).

Graphical Cognition

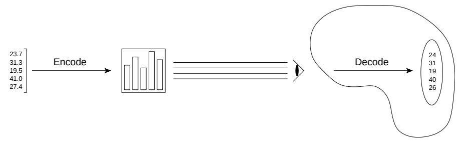
- Some visual processing requires that we conciously inspect the things that we are looking at.
- Psychologists refer to this kind of activity as *cognitive*.
- It is *graphical cognition* which allows us to make statements like
  - The largest person is third from the right.
  - The steepest slope in the graph is near $x = 4.5$.

Graphical Design

- Graphs can affect us at either the perceptual or cognitive level (or both).
- The most effective (and eye-catching) graphs always operate to some extent at the perceptual level.

Graph Drawing

- When we draw a graph we *encode* a numerical value as a graphical attribute.
- When we look at a graph the aim is to *decode* the graphical attributes and extract information about the numbers which were encoded.

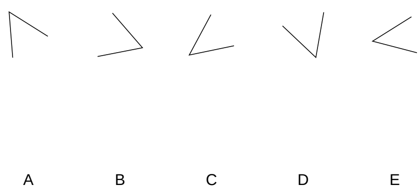


- If possible we should arrange for the decoding to happen at the perceptual level.

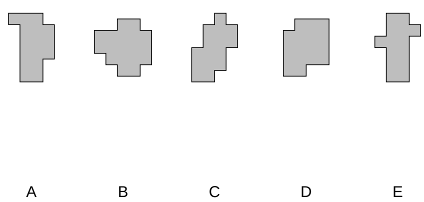
Designing Effective Graphs

- To design effective graphs we must know which graphical attributes are most easily decoded.
- We need a selection of possible graphical attributes and an ordering of their ease of decoding.
- There are a number of possibilities.

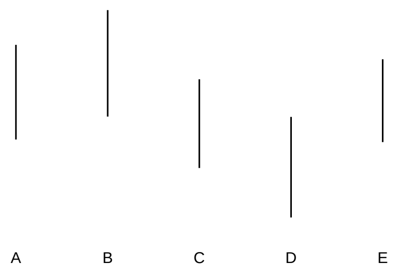
Encoding Using Angles



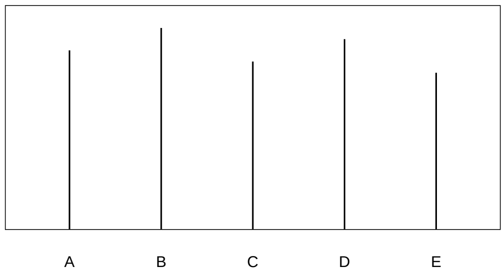
Encoding Using Areas



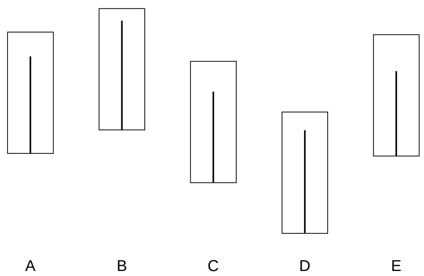
Encoding Using Lengths



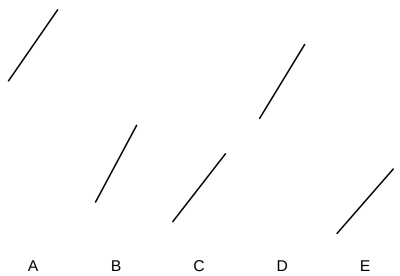
Position on a Common Scale



Position on Identical but Unaligned Scales



Encoding Using Slopes



Perception "Laws"

- Perceptual psychologists have established a number of empirical laws for perception.
- The two most important laws hold very generally and apply to many graphical encodings.

Weber's Law

Weber's law applies to a variety of perceptual encodings, but we will apply it to length.

- Consider two lines with lengths x and $x+w$.
- If w is very small, there is only a very small chance that we will notice that the lines have different lengths.
- As w gets larger, the chance of detecting a difference increases.
- Weber's law says that the chance of detecting a difference depends on the value of w/x.

Weber's Law

- For a given individual the difference between x and $x+w$ will be detected with probability $p_x(w)$.
- Now turn this around. For a fixed p, let the value of w which is detected with this probability be

$$w_p(x)$$

- Weber's law states that

$$w_p(x) = k_p x$$

- Since $w_p(x) \propto x$, we detect *relative* differences in values.

Weber's Law

- We detect a 1cm change in a 1m length as easily as we detect a 10m change in a 1km length.
- Weber's law appears to hold for many different graphical encodings.

Stephens' Law

- Suppose that we encode a value x and then decode it to obtain a perceived value $p(x)$.
- Stevens' law says

$$p(x) = Cx^{\beta}$$

- The values of C and β depend on both the encoding method used and the observer.
- This law holds for many encodings.

Perception of Area and Volume

- Our perception of areas and volumes is conservative.
- When values are presented as areas or volumes, we underestimate the large values relative to the small ones and overestimate the small ones relative to the large ones.

β and Nonlinearity

- If the relationship $p(x)$ and x is nonlinear, we can be mislead in what we infer about the x values.
- The linearity of the relationship between $p(x)$ and x depends only on the value of β, with linearity if and only if $\beta = 1$.
- Here are some typically observed ranges for β.

Length:	0.9 – 1.1
Area:	0.6 – 0.9
Volume:	0.5 – 0.8

Combining Weber's and Stevens' Laws

Consider encoding using length with $\beta = 1$ and area with $\beta = 0.7$. When we compare the values x and $x+d$ we perceive the relative value

$$\frac{x+d}{x} = 1 + \frac{d}{x}$$

for length and the relative value

$$\frac{(x+d)^{0.7}}{x^{0.7}} = \left(1 + \frac{d}{x}\right)^{0.7} \approx 1 + \frac{0.7d}{x}$$

for area.

We are more likely to detect small differences when a length encoding is used than when area encoding is used.

Stevens' Law – Perception of Large Values

- Consider area with $\beta = 0.7$.
- Suppose that we compare an area of size 2 with an area of size 1. The perceived ratio of areas is:

$$\frac{p(2)}{p(1)} = \frac{2^{0.7}}{1^{0.7}} = 1.62$$

- We don't see the bigger area as twice as large.

Ranking Graphical Encodings

- Cleveland and McGill carried out an extensive study of graphical encodings to obtain a best to worst ranking.
- The rankings they examined were

  angle, area, colour hue, length,
  colour brightness,
  position (common scale),
  position (identical unaligned scales),
  colour purity,
  slope, volume

Stevens' Law – Perception of Small Values

- Consider area with $\beta = 0.7$.
- Suppose that we compare an area of size $1/2$ with an area of size 1. The perceived ratio of areas is:

$$\frac{p(1/2)}{p(1)} = \frac{0.5^{0.7}}{1^{0.7}} = 0.62$$

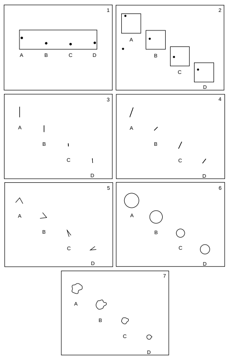
- We don't see the smaller area as half the size.

One Cleveland and McGill Experiment

- 7 graphical encodings
- 3 judgements per encoding
- 10 replications per subject
- 127 experimental subjects
- Assessment criterion:

$$\text{error} = |\text{judged\%} - \text{true\%}|$$

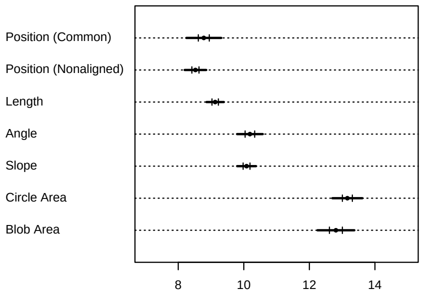
One Cleveland and McGill Experiment



Perception In Maps

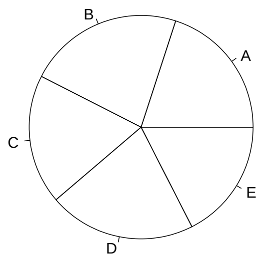
Our perception of the size of areas is affected by the shape of those areas. Here Georgia is bigger than Florida, but doesn't look it.

Results



Angular Encoding in a Pie Chart

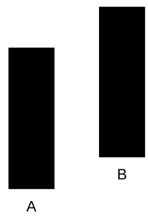
Pie charts have a very weak perceptual basis.



The Encoding Ranking

1. Position on a common scale
2. Position along identical, non-aligned scales
3. Length
4. Angle / Slope
5. Area
6. Volume
7. Colour properties

Encoding Using Length



Recommendations

- Use the highest possible encoding on the Cleveland-McGill scale.
- The preferred encodings are:
  - Position on a common scale.
  - Position on identical, unaligned scales.
  - Length.
- Be careful when using angles and slopes when encoding numerical values.
- Don't use area or volume to encode numerical values.
- Don't use colour to encode numerical values.

Encoding Using Identical Unaligned Scales