SHARING IDEAS AND STATISTICS LEARNING: THE ROLE OF PEER INTERACTIONS IN SCHOOL CONTEXT

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The role of Statistics is becoming increasingly important in today's society. According to several authors, collaborative work has shown to be one of the most adapted forms of facilitating knowledge appropriation and the mobilisation of competencies. The project Interaction and Knowledge has studied and encouraged peer interactions in the Mathematics class, in association with a new didactic or experimental contract, as a way of promoting pupils' performances, allowing them to reach relational knowledge. By analysing excerpts of these interactions we can understand the facilitating character of this working method.

INTRODUCTION

The past decades have brought teachers new challenges: the term instruction was replaced by education; educational goals include contents but also attitudes and values; compulsory schooling was increased to a greater number of years; pupils are more heterogeneous, and literacy was considered to be the competency that Everyone should desire.

In Portugal, as in most western countries, the 70s and 80s witness a concern in finding solutions for the problematic that the Mathematics subject comprises. From this moment on Statistics starts belonging to the set of contents that are taught in the compulsory Maths curricula. Contributions to the search for solutions came not only from authors close to these two fields of knowledge, but also from Social Genetic Psychology, which has developed research that stresses the importance of social interactions in the classroom, namely the role of peer interactions in knowledge appropriation and in the mobilisation of mathematical competencies.

THEORETICAL FRAMEWORK

In present societies a great deal of numerical information is produced that needs to be handled. Statistics has become imperious for the challenge of transforming raw data into organised data that allows us to read and understand reality. Many of the decisions in which individuals are asked to have a say have consequences. As modern societies regulate the lives of their citizens through numerical indicators, they force them to appropriate knowledge that helps them understand their meaning and how this process comes about, while at the same time mobilising them when they have to make choices. To have some statistical knowledge has become inevitable in order to practice a critical, reflexive and participant citizenship, since our decisions are based on critical data analyses, whether individually or collectively. Consequently, Statistics has become a part of Mathematics curricula in basic education, integrated in the form of a curricular unit (Gal & Garfield, 1997). However, it is not only the adults who have to be critical as far as statistical information is concerned (Lajoie, Jacobs, & Lavigne, 1993). As Lopes reminds us (1998), "young people are already citizens; we need to help them develop their critical capacity and sense of autonomy so that they have a better chance of elaborating reflections, giving opinions and/or making decisions" (p. 114). The process by which children and youths build attitudes and create values regarding many of the choices they will make in personal and social terms is a gradual one. They must therefore become critical and reflective towards statistical information, even when they used it inappropriately or in a naïve manner.

The emphasis of teaching must not be placed in a so-called knowledge acquisition, but on the mobilisation of competencies (Abrantes, Serrazina, & Oliveira, 1999), that is, on knowing in action. In the case of Statistics, that means knowing what is present in a statistical study, how to interpret it, how to pose questions, how to select a sample, or which measures are used according to a certain context and a given situation. The nature of Statistics has led several authors to prefer small groups as the ideal way of working it, as this allows for the exploration of themes that pupils feel attached to, making it more motivating and producing new discoveries concerning the situations that are studied (Cobb, 1999; Shaughnessy, 1992). But for this to happen we must change classroom practices. Collaborative work has shown to have great potential in the classroom (César, 1998, 2000; Cobb, 1994; Schubauer-Leoni & Perret-Clermont, 1997; van der Linden et al., 2000), for it allows pupils to work in their Zone of Proximal Development (ZPD), to

establish conditions where socio-cognitive conflicts occur, didactic contracts to take on novel characteristics, thus allowing pupils to develop their competencies to learn to communicate mathematically. In other words, this form of work with pupils in the classroom promotes the conditions to improve their mathematical and statistical power.

Starting with Vygotsky's (1962, 1978) work, we began to understand how co-operation with others can produce the tools [mental and physical artefacts] that allow us to act upon the atmosphere, at which time the social dimension becomes vital in the understanding of human development. Social interactions, or better yet, their quality, are now regarded as a crucial element for the progress of the human being. The notion of ZPD becomes one of the most explored concepts in Education for allowing the (re)discovery of the importance and the need of the other in learning. As César (2000) states working in the ZPD of each pupil is one of the greatest challenges that are currently put to teachers.

To do so, the subject must interact with others, diversifying the forms of interaction according to the social context where he/she lives and where this activity takes place (Moll and Greenberg, 1996). This way, "the inter-psychological functioning found in the ZPD may vary according to the social and institutional contexts where this functioning takes place" (Wertsch, 1988, p. 91). But it also seems that the partner with whom the subject interacts has a decisive role in the inter-psychological functioning. Tudge (1990) feels that when Vygotsky states that more competent peers help children's development, he introduces the possibility that the more competent peer is another child or youth. As he declares "in many classroom situations children help their less competent colleagues to learn" (p. 151).

In Portugal, César and collaborators (Carvalho, 2001; Carvalho & César 2001; César, 1998, 2000; César & Torres, 1998) found that it is not only the less competent peer who shows progress: the more competent peer also progresses when interacting in a dyad with the less competent one. However, different reasoning processes must take place, with several forms of resolution in confrontation. These resolutions all involve finding a common intersubjectivity, such that a portion of representations is shared by both (Wertsch, 1988).

A task that is novel for students in terms of either its contents or its working instructions facilitates an exchange of resolutions and reasonings and brings about situations of socio-cognitive conflict. In a social interaction, students face a socio-cognitive conflict when they have to confront themselves with a resolution that differs from that which they originally considered. This is the cognitive feature of the conflict; but at the same time they have to deal with the social aspect that is expressed in the other's behaviour.

The confrontation of responses is beneficial when both engage in discussion. From here a double imbalance emerges: between the subjects' answers, therefore inter-individual; and also intra-individual when, upon debating a different answer from his/hers, the subject interrogates himself about it. Solving a socio-cognitive conflict implies overcoming a situation of cognitive conflict and managing a social relationship with a partner with whom it is necessary to co-ordinate points of view in order to reach a consensus and solve the task. Through this attempt to overcome the inter-individual cognitive imbalance, the child can solve his/her own intra-individual cognitive imbalance (Gilly, Fraisse, & Roux, 1988). It is also in situations of socio-cognitive conflict that we find the features that bring the theories of Piaget and Vygotsky closer, turning them into a fundamental conceptual tool when trying to understand the construction of knowledge (Tryphon & Vonèche, 1996).

When pupils have the chance to experience forms of collaborative work in the classroom and confront themselves with tasks and situations where they are encouraged to work in a non-routine manner, we find a great wealth of solving strategies, some resulting from socio-cognitive conflicts. It has also been found that pupils shift from an instrumental approach to a relational approach (Skemp, 1978). In the first type of approach , pupils just master an isolated set of rules and algorithms, learnt through repetition and routine. Relational approach allows for the construction of a scheme of a concept that can be updated whenever this is demanded by new situations, that is, knowledge that can be mobilised in face of new situations and which satisfy the current educational suggestions for the Statistics unit.

METHOD

This study is part of a broader project called *Interaction and Knowledge* whose main goal is to study and implement peer interactions in Mathematics classes as a way of promoting pupils' academic self-esteem, socio-cognitive development and school achievement. The project *Interaction and Knowledge* is divided into two different levels: 1) - A micro-analysis level, in which we studied different types of peers,

their interactions, the tasks we propose, the mistakes they make and the progress that peer interactions are able to generate in statistical contents (Carvalho, 2001; César, 1994); 2) - An action research level, in which some mathematics teachers implemented peer interactions as a daily practice during at least one school year (César, 1998, 2000; César e Torres, 1998). The data we are going to present are from level 1, related to the content of Statistics.

The sample had 533 subjects (25 classes) and data were gathered in two consecutive school years. All subjects were attending the 7th grade. The subjects were divided into an experimental group and a control group and the ones from the experimental group worked in peers (136 dyads) during three sessions, solving challenging problems. After that each class participated in a general discussion that was videotaped and in which pupils discussed some of the tasks they had solved in peers.

Task

In a factory, five of the workers were randomly chosen to do a study about their salaries. The salaries were:

Worker	A	B	C	D	E
Salaries (escudos/month)	54 000	42 000	60 000	48 000	180 000

1 - Do you think that the five workers would agree if we say that the great majority of them have a salary equal to the mean? Why?
2 - Do you choose the mean or the median to represent the salaries of the workers? Why?

RESULTS

An example of how interacting can enrich what we already know. The excerpt we are about to discuss is from a much longer interaction and it is representative of a larger data corpus. It aims to illustrate how pupils, by negotiating a resolution through co-elaboration, can increase their knowledge on the concepts of mean and median, going from a knowledge that merely encompassed the simple application of the algorithm of mean or the application of a set of procedures concerning the median to a richer statistical knowledge. It also stresses how the task and the instructions given are reflected in the interactions that are established between both elements of the dyad. The pupils of this dyad are considered to be fair by their Maths teacher and define themselves as average pupils in this subject.

[They start by reading the question individually]

43. *D – We've got to do the mean.*
44. *A – The mean is summing everything up.*
45. *D – Then dividing by the number of parts.*
46. *A – Which are five.*
47. *D – 54 thousand, 42 thousand, 60 thousand, 48 thousand, 180 thousand. Now divide by five.*
48. *A – The mean is 76 contos 800* [After using the calculator].
49. *A – Put the sum there.*
50. *D –* [Starts writing] *Do I divide 384 thousand 400 or do I put the total straight away?*
51. *A – You'd better put everything.*
52. *D –* [Writes what A said] *And now we have to put why.*
53. *A –* [Looks at the sheet] *The why is because that's how you do the mean.*
54. *D – Do you think so? I think it's to know if the mean is what represents the salaries best. They want to know the employees' opinion when we say they make the mean.*
55. *A – Then I disagree, because the pay should be more equal.*
56. *D – Yes, if the salary were equivalent to the mean, if each employee had this* [Pointing to the mean value].
57. *A – So we put no, because the salaries are different from what the mean is.*
58. *D – Different how? Because they don't all make the same?*
59. *A – Yes, they mustn't all make the same.*
60. *D – That's why they don't make the same. E must be the one who's in charge.*
61. *A – Then we say no, because the salaries are all different. Is that OK?*
62. *D – Yeah. A firm always has to have bosses and employees.*
63. *A – That's why the salaries are so different.*
64. *D – And they can't agree with the mean because it would look like they all have the same...*
65. *A – Money and that's wrong because they're making less.*

66. *D – It's not wrong, it's deceiving them... because the mean is right.*
67. *A – What's wrong is they put the boss and the cleaning staff and it looks like the cleaning staff are making good money...*
68. *D – And they're making little...I think.*

[They read the question 2.3 individually]

73. *D – Now we have to do the median.*
74. *A – Do you know what the median is?*
75. *D – Yes. It's the one in the middle.*
76. *D – The median is putting them all in order and seeing which one's in the middle.*
77. *A – The one in the middle is...*
78. *D – It's 54 thousand.*
79. *A – Would you choose the mean or the median to represent the employees' salaries in this firm? I'd choose the mean because it's a higher salary. The mean is 76 800.*
80. *D – I wouldn't. Because the mean isn't what they're making. It's the calculation, don't you see the salaries?*
81. *A – Yes, but the calculation is made with the salaries they have, man.*
82. *D – Yeah, you're right. But then after finding the mean the salaries are lower than the mean. That's why I think it's the median.*
83. *A – The mean, because it's a higher salary. They make more.*
84. *D – It's more money, but the mean isn't what they make, it's the sum of what they make...*
85. *A – That's why they want the mean to make more.*
86. *D – I know what. With the mean it looks like they make more because E raises this all because he's the boss and makes much more than the others. If they remove the boss from the mean they're all much closer to each other and then they might want the mean.*
87. *A – So they choose the median because they picked the wrong salaries to do this mean.*

In this interaction we are looking at a co-elaboration through co-construction of the resolution: both pupils complete and continue the other's reasoning and neither of them stands out as the leader. A joint search for a solution is verified, in which each one contributes to the progress of the resolution, hearing and clarifying the other's interventions and enlarging them (Gilly, Fraisse and Roux, 1988). Both agree on the resolution strategy and the inherent computational procedures. From a social point of view (who leads, how he does it, when is there agreement, when are concessions made), the management of the interactive process is done naturally, with no need to solve strong differences. However, as the authors we quoted declare, this apparent harmony does not exclude the possibility of one's interventions disturbing the other or provoke a new conjecture, which would never emerge without this dynamic. In fact, in line 53 we observe a socio-cognitive conflict emerging, that is responsible for going from a solving strategy just based on the calculation of the mean algorithm to a richer strategy where both pupils interrogate themselves about both the representativeness of the data and the reasons that might underlie the choice of the sample upon which data were gathered.

The excerpt of the interaction that starts with line 53 shows us how the fact that the pupils debated their arguments helped A. enrich his resolution, which probably would not have happened if he were working individually. In this situation, which began with a socio-cognitive conflict, we did not manage to discover whether this conflict resulted from a difficulty regarding the statistical concept of representativity, from one regarding the actual lexical meaning of the word, or from both. What remains clear is that during an interactive situation, when pupils are solving a task, linguistic, semantic and lexical competencies are all at stake, as well as those concerning the actual concept that is under study (Trognon, de Almeida & Grossen, 1999).

The literature regards the notion of the representativity of central tendency measures is a more difficult notion for pupils than that of localisation (Batanero, 2000; Goodchild, 1988; Leon & Zawojewski, 1991), for pupils have to make use of knowledge other than that of calculation. In this attempt to clarify what may be causing the difficulty of the colleague there is a negotiation of the meaning of the task solution, in the game between the inter-individual and the intra-individual, when one of the partners interrogates the other about his reasoning.

The fact that the pupils worked in a familiar context, as that of the social problematic around salaries, also facilitated the mobilisation of other necessary information to fulfil their original strategy. In this case, we have a task that has a social mark (Doise & Mugny, 1981), which is a facilitating aspect for pupils to develop more complex and flexible forms of knowledge.

From line 73 onwards we find that, due to both their collaborative interactions and to specific characteristics of the task itself, the pupils manage to find a statistical property of the mean: the mean cannot be equal to the values of the observations. This empirical evidence allows them to construct a relational meaning of the concepts of mean and median, for they go beyond the simple application of an algorithm or the use of a procedure. However, as the calculation of the algorithm is present right in the first lines of A., without being asked for explicitly in the question, it makes us think that in the usual didactic contract these pupils have with their teacher, it is more common to start by calculating it before analysing which is the best parameter for the situation.

During the first part of the interaction we see how both pupils question each other, forcing the other to express his arguments more precisely. Each one picks up the other's solving strategy again and enriches it. Without this dynamic of co-construction of a common strategy, based on each element's interventions, it would be very hard for both subjects to have reached such a complete answer, that is, an understanding of the concepts of mean and median that is so close to its meaning.

FINAL REMARKS

The interaction we presented reveals a harmony in terms of the interactive dynamics between both pupils, confirming the following claim made by Gilly, Fraisse and Roux (1988).

> Observing solving behaviours in pairs makes us think that co-elaboration benefits the partners of the relationship in different ways and that it is not absolutely necessary to establish this within a typical socio-cognitive conflict (...). With or without a manifest socio-cognitive conflict, co-elaboration is likely to disturb or destabilise individual functioning. But this destabilisation is more efficient when it is within an interactive structure where it is combined with others such as: stimulation, reinforcement, control, expansion of the field of possibilities (p. 91).

The task and instructions related to it are fundamental elements for subjects' cognitive functioning. These allow the establishment of an interactive dynamic that can support subjects' type of action or another, thus originating an improvement in their competencies.

Collaborative work has proved to facilitate better performances on behalf of the pupils, regardless of these having difficulties in knowledge appropriation or not. Therefore, this kind of work reveals countless potentialities for attaining some of the most ambitious objectives of current educational politics documents.

NOTES

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