STATISTICAL THINKING MODELS

Maxine Pfannkuch and Chris Wild
The University of Auckland
NewZealand

Models for statistical modes of thinking and problem solving have been developed, and continue to be developed, by teachers and researchers. The purpose of these models range from helping to understand how individual students solve problems to developing instruments for educational research. These models have arisen with particular perspectives and primary uses in mind. In this paper we compare and contrast some statistical thinking models originating from statistics education research (Ben-Zvi & Friedlander, 1997; Jones, Thornton, Langrall, Mooney, Perry & Putt, 2000) with some models arising from the discipline of statistics and sub-disciplines (Wild & Pfannkuch, 1999; Hoerl & Snee, 2001). Drawing upon models from both these areas we discuss issues that include their development and use, how they might illuminate one another and what we can learn from them.

INTRODUCTION

> "All models are wrong, but some are useful" (George Box, 1979).
> "Most of our tacit, intuitive models are imperfect mediators, leading often to incorrect or incomplete interpretations" (Efraim Fischbein, 1987).

Statisticians and statistics education researchers recognize the part models play in constructing and interpreting the world around us. Whether models are constructed mentally for cognitive purposes or externally for communication purposes one function of a model is to provide an organized, global account of a process including abstract relationships. Fischbein (1987) believes that a good model not only has to have the function of an autonomous entity but also simultaneously has to be "a trusty mediator between the original situation and the solver's intellectual activity" (p. 126). Taking this into account, a second function of a model is as a reasoning tool that gives control over one's mental processes and helps in thinking productively.

According to Brown (1998) the benefits of such models in education are that they provide a vocabulary and a classification system that help people to think in a more focussed way about an area of mathematics. This can lead to a different way of conceptualizing a content area of mathematics. It is also a means through which thought, discussion, further theory building and action can be operationalized. This is similar to the way statisticians think about their models as they also state that models provide a standard vocabulary as well as tools which are useful for sharing, discussing and learning from one another (Hoerl & Snee, 2001). Both educators and statisticians recognize that while models may simplify complexity, they nevertheless can be useful. Thus models should allow us to think and communicate more effectively about phenomena which are complex.

Statisticians Hoerl and Snee (2001), believe that "the most effective way to create process knowledge is to develop a model that describes the behavior of the process" (p. 230). This means that the actual construction of a model assists in deepening knowledge and understanding about a process. Brown (1998) an educator, however, warns that the model constructors should acknowledge that the models are a product of their own perspective which might be based on intuitive and/or particular theories. For example a model would be based on their understandings of statistical thinking, statistical, pedagogical, and cognitive knowledge, and on the type of data collected, the interpretation of the data, and the framework constructed. Given these boundaries it would appear that actively constructing a model of a situation aids thinking and stimulates insight about that situation.

Statistics is regarded by some statisticians and statistics educators as ways of thinking as well as a specific body of knowledge. These ways of thinking, which were once largely implicit, are now being explicitly formulated as frameworks or models. Both the statistician and educator are modeling the intellectual activity in the statistical domain through observing and attempting to understand either patterns of thoughts in students in the statistics classroom or patterns of thought

from their collective experiences in the statistics discipline. Therefore we have statistical thinking models being developed by statisticians and educators. Drawing upon models developed in both these areas we will compare and contrast these models, discuss how they might illuminate one another, and consider what we can learn from them.

In this paper we will compare four statistical thinking models, two originating from statistics education research, those of Jones et al. (2000) (JT model) and Ben-Zvi and Friedlander (1997) (BF model), and two arising from the discipline of statistics and sub-disciplines, those of Wild and Pfannkuch (1999) (WP model) and Hoerl and Snee (2001) (HS model). Apart from our own WP model, we have not discussed our interpretation of the other models with the respective authors and therefore acknowledge that the ensuing discussion is subject to our own biases, understandings, tacit models and imperfect reasoning.

DESCRIPTION OF THE MODELS

The JT Model

The purpose of the JT model is to provide a coherent picture of young children's thinking and their cognitive knowledge. It is a four by four matrix model that has four levels of thinking (idiosyncratic, transitional, quantitative, analytical) based on the SOLO taxonomy theory of Biggs and Collis (1982) and four key constructs (describing data displays, organizing and reducing data, representing data, analyzing and interpreting data) for statistical analysis. It is a sequential model in that there is an assumption that students will proceed through each of the four levels of thinking for each construct. It is also an hierarchical model as the thinking levels are ordered from low to high. Jones et al. (2000) use the model to demonstrate the growth in children's thinking from Grades 1 to 5. Students were given specific questions for each construct and their response level was assessed. The model is also a measurement instrument for categorizing student responses and as such gives detailed descriptions of the types of thinking expected for each cell of the matrix. Jones et al. (2000) state that the JT model can be used to inform instruction and to predict the kind of thinking that will occur in students. It should enable curriculum designers and teachers to build instructional sequences and learning tasks within students' capabilities.

The BF Model

The purpose of the BF model is to define and characterize the levels of thinking that were observed for 13 to 15 year olds in a teaching experiment. It is a one by four matrix model (Mode 0: Uncritical thinking, Mode 1: Meaningful use of a representation, Mode 2: Meaningful handling of multiple representations: developing metacognitive abilities, Mode 3: Creative thinking) that documents possible developmental stages of thinking and it provides examples of student responses that were classified for each level. These stages were observable through the way students analyzed their data and reached their final conclusions when conducting a statistical investigation of their choice. The model is assumed to be a working model that requires further refinement. The researchers intended to test the model with more data with respect to whether the developmental stages identified were hierarchical and whether or not the students were going through each stage linearly.

The WP Model

The WP model is built upon the empirical enquiry cycle, historical and statistical literature. Its purpose is to describe how people think within the statistics discipline. It has four separate components or dimensions (the investigative cycle, types of thinking, the interrogative cycle, dispositions) and claims that people think within each dimension simultaneously. It is non-hierarchical with two dimensions being sequential and two non-sequential. These patterns of thought were observable in the way statisticians and statistics project students described their approach to problem solving. The beginnings of such thought processes have since been observed in middle-school students conducting statistical investigations (Rubick, 2000; Yoon, 2001). The model is an initial attempt at giving a global overview of statistical thinking from which refinements or new models can emerge. In each dimension categories are given but the meaning of each category needs further clarification and amplification.

The HS Model

The HS model documents the processes and procedures used in solving problems and improving processes. It promotes a statistical way of thinking for everyone involved in improving some aspect of their business or system and promotes a way of viewing the world. Their model comprises four main models, an overview model (the statistical thinking model), two models to use and work with (the problem-solving strategy, the process improvement strategy) and an evaluation model (key elements of statistical thinking). The overview model illustrates the cycle of enquiry in a linear form, while the two working models are flowcharts giving a sequence of identifiable steps with each step including suggestions for thinking tools to use. The evaluation model is a checklist on the key elements of process, variation, and data to determine whether there is evidence of statistical thinking being utilized within an organization. These models are based on experience in the field, the philosophies and theories of such people as W. Edwards Deming (e.g., 1986) and Peter Senge (1990), and have been developed over a number of years.

COMPARISON OF MODELS

All these models could be considered as tentative explanations of the intellectual activity within the statistics domain. When comparing these models we are mindful of Box's (1979) statement about models: "The relevant question is not 'Which is the best?' but 'Do these different entities have a role and if so, what is it?' "(p. 2).

The different roles of the models can be exemplified by these multi-model and single model entities and the assumptions on which they are based. The WP and HS models acknowledge that several different models are operationalized for statistical thinking as they are illustrating the different types of thinking involved in statistical thinking whereas the other two are based on a single model notion as they are illustrating growth or developmental stages of statistical thinking. A single model may convey clarity but at the same time may consider only one main aspect of thinking whereas the multi-models, although difficult to grasp, indicate the possibility that statistical thinking may be a complex activity. The JT model assumes learning growth is according to age and that all students will proceed through the stages of their model. The HS model assumes that not everyone is able to operate at the same level of capability. It is used by leaders who involve people at different levels, depending on their position in the work hierarchy, but everyone in the system participates and understands why they are using such a model. Both models, however, appear to assert that learning growth comes from being engaged in a process over a long period of time. The JT model was used to describe how students who had been exposed longer to learning about how to think with statistical tools reached higher levels of thinking. The HS model describes how process knowledge increases through a cyclical synergy between data and contextual knowledge.

Models can simultaneously be utilitarian and theoretical. The utilitarian approach is exemplified in the HS and JT models where their purpose is to be useful to other people. The HS model shows people how to do statistical thinking and acts as a guide for people to follow when they are in the process of problem solving. The JT model shows teachers the level at which their students are working in terms of their thinking capability and hence acts as an indicator for teachers to follow when planning learning tasks. These models are intended for use as thinking tools to improve processes either within business (HS) or in the classroom (JT). Although both models are utilitarian in intention they are based on well-developed theories and philosophies about learning. The HS model is based on management and systems theories about how one learns more about a process whereas the JT model is based on the SOLO theory about assessing the current state of learning for an individual.

The other two models are not intended for particular purposes, as they are simply explanations of what has been empirically observed. They give a global account of thinking processes that are used by students (BF) or by statisticians (WP). Their use is solely for the originator to enable an encapsulation of ideas and to communicate those ideas to others. The authors do not state that they are based on any particular theoretical stance although implicit theories about thinking appear to be present in that the BF model delineates developmental stages of thinking and the WP model is non-hierarchical and non-linear. The differences could be that the former two models are grounded in a developmental history and hence have undergone much

iteration whereas the latter two are emergent models. These emergent models may develop new theories, be grafted onto existing theories, or may simply die, whereas the other models may continue for as long as they are used by other people.

When considering the statistical thinking featured in these models it seems that the JT model emphasizes describing, representing, organizing, reducing, analyzing and interpreting data whereas the BF model focuses on the ongoing search for meaning and interpretation, critical thinking, the meaningful handling of multiple representations, metacognitive abilities and creative thinking, and recognizes that the empirical cycle provides the connections for students to seek deeper meaning. The WP model focuses on the empirical enquiry cycle, the dispositions required to drive the thinking, an interrogative cycle acting in the metacognitive sense as an evaluator for all incoming information, and specific types of thinking such as seeking explanations, consideration of variation and integrating the statistical and the contextual. The HS model emphasizes that understanding the problem is a key component along with subject matter knowledge. Its focus is on looking for causes, improving a process, using data for decision-making, and highlights the understanding of variation as a core element. These models feature quite different representations of statistical thinking and are probably context-dependent.

The JT model is within the context of the types of examples typically used in a school classroom while the HS model is for the types of problems that people will need to solve within a business organization. The other two models are somewhere in between. In the case of the BF model the context was students solving authentic–type problems with real data whereas the context for the WP model was an aggregation of the experiences of statisticians from many sub-disciplines not just the business discipline. Furthermore the HS model is designed to promote or stimulate productive intellectual activity while the others describe intellectual activity. This may account also for the different types of statistical thinking featured.

The models, however, convey some similarities about ways of thinking in statistics. The similar messages appear to be that the thinking focuses on data, there is interdependence between contextual and data-based reasoning and that data is used for learning in the context sphere. This learning is continuous and occurs over a period of time. The learning starts with a problem, whereby the learner engages in a mental and physical process of sifting information and empirical data to find out what the data are really saying. Within that process the learner needs to know how to reason with specific statistical tools and representations, how to manipulate and reorganize data, and how to represent the data in multiple ways. The process is analytical, involves synthesis and is evaluative. Above all the main message is that statistics has its own characteristic ways of thinking and these ways of thinking must be learnt by people.

Other messages implicit in the models raise many issues and questions. The educators' implicit message is that teachers need to know how their students are thinking and give a picture of what students can do whereas the statisticians' message is that students need tools other than the standard analysis tools to think with, that tools are a way of fostering thinking. Does this mean for the educator that more thinking tools should be developed for their students? That preliminary tools to support thinking before the conventional tools are introduced are possibly useful? Does this mean for statistics that the type of tool produced should be more in tune with human thought processes? That possibly a range of tools should be developed to cater for different thought processes in people? The statisticians believe that models that are useful are simple and parsimonious, highlight from four to seven elements, and integrate tools with the model (Snee, 1999). Does this mean educators should rethink the models they are producing for use by teachers and students? The educators point out that metacognitive processes, as well as creative thinking that may produce non-conventional representations for communicating insights into data, are involved in statistical thinking. Should statisticians produce more evaluation models to check thinking processes? Can they find more or other ways of valuing and nurturing creative thinking in statistics? Educators find that some statistical representations are difficult for students to interpret. Should the statisticians develop better tools or should intermediate tools be developed for the learning process such as the computer-based mini-tools developed by Cobb (1999), in conjunction with Gravemeijer, Doorman and Bowers. The statisticians have created tools that emphasize a way of thinking. Should educators further develop such tools for students? Or should they question whether such tools will broaden, channel or mislead students' thinking? The models

for statistical thinking appear to be context-dependent and give different representations of statistical thinking. Is this because of different viewpoints on the nature of statistics and statistical thinking? Or is it because of knowing appropriate learning experiences for different age groups?

The different perspectives between education and business can be illustrated in the type of models constructed. In education the phenomena that are being modeled are basically teaching and learning phenomena: "they are models of what people actually do when they learn or teach mathematics" (Kieran, 1998, p. 219). In business the models are for a "disciplined quantitative approach for improvement of defined metrics in manufacturing, service or financial processes" (Hoerl & Snee, 2001, p. 476). The education model is for explaining the current situation while the business model is for improving the situation. The educators might argue that by explaining the current situation this can lead to improvements in the way we teach and conceptualize statistics. The statistician might argue that by using a model for improving the situation this leads first to explaining or understanding the situation and secondly to methods of improving the process.

CONCLUSION

All the models are describing a process of how people are enculturated into a way of thinking statistically and into a community of statistical practice. What we have learnt in comparing these models is that in the construction of a model we should be clear about why we are constructing it, its purpose and its usefulness. We should question whether it is in a form that will lead to discussion, to improvement in teaching or business if that is desired, whether it will help ourselves and other people think productively, and what is being communicated to other people. We should look at the types of models being produced in education and statistics and see whether we can draw on some features to improve our models in our respective fields. The theories on which these models are based should be considered to determine the commonalities and differences between education and statistics and whether one theory can inform the other. We should also look at the statistical thinking being communicated and see whether there are some universal messages between educators and statisticians. Both educators and statisticians emphasize that models provide a vocabulary for communicating with other people and this suggests that statisticians and educators should develop a common language through their models for communicating statistical thinking.

Conceptualizing, characterizing, and promoting a statistical way of thinking are the foundations on which these models have been constructed. Educators and statisticians will continue to develop more models to communicate their insights into a statistical way of thinking. Neither should ignore the potential rich source of ideas emanating from the other.

REFERENCES

Ben-Zvi, D., & Friedlander, A. (1997). Statistical thinking in a technological environment. In J. Garfield and G. Burrill (Eds.), *Research on the role of technology in teaching and learning statistics* (pp. 45-55). Voorburg, The Netherlands: International Statistical Institute.

Biggs, J., & Collis, K. (1982). *Evaluating the quality of learning: The SOLO taxonomy (Structure of the observed learning outcome)*. New York: Academic.

Box, G. (1979). Some problems of statistics and everyday life. *Journal of the American Statistical Association*, *74*(365), 1-4.

Brown, M. (1998). The paradigm of modeling by iterative conceptualization in mathematics education research. In A. Sierpinska and J. Kilpatrick (Eds.), *Mathematics education as a research domain: A search for identity,* Vol. 2 (pp. 263-276). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Cobb, P. (1999). Individual and collective mathematical development: The case of statistical data analysis. *Mathematical Thinking and Learning, 1(1),* 5-43.

Deming, W.E. (1986). *Out of the crises*. Boston: M.I.T. Center for Advanced Engineering Study.

Fischbein, E. (1987). *Intuition in science and mathematics.* Dordrecht, The Netherlands: Reidel.

Hoerl, R.W., & Snee, R.D. (2001). *Statistical thinking: Improving business performance*. Pacific Grove, CA: Duxbury.

Jones, G., Thornton, C., Langrall, C., Mooney, E., Perry, B., & Putt, I. (2000). A framework for characterizing children's statistical thinking. *Mathematical Thinking and Learning*, *2(4),* 269-307.

Kieran, C. (1998). Models in mathematics education research: A broader view of research results. In A. Sierpinska and J. Kilpatrick (Eds.), *Mathematics education as a research domain: A search for identity* Vol. 1 (pp. 213-225). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Rubick, A. (2000). *The statistical thinking of twelve Year 7 and Year 8 students*. Unpublished masters thesis, The University of Auckland, New Zealand.

Senge, P. (1990). *The fifth discipline: The art and practice of the learning organization*. New York: Doubleday/Currency.

Snee, R. (1999). Discussion: Development and use of statistical thinking: A new era. *International Statistical Review*, *67(3),* 255-258.

Wild, C. J., & Pfannkuch, M. (1999). Statistical thinking in empirical enquiry (with discussion). *International Statistical Review*, *67(3),* 223-265.

Yoon, C. (2001). *An analysis of students' statistical thinking.* Unpublished masters dissertation, The University of Auckland, New Zealand.