TEACHERS' BELIEFS ABOUT USEFULNESS OF SIMULATION WITH THE EDUCATIONAL SOFTWARE *FATHOM* FOR DEVELOPING PROBABILITY CONCEPTS IN STATISTICS CLASSROOM ®

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Here we present a report of a study carried out with six high school teachers who participated in a workshop of simulation activities using Fathom –dynamic software for teaching statistics-. At the end of eight weekly work sessions of three hours each, participant teachers were asked to answer a questionnaire related to their opinions about aspects of using the technique of simulation in teaching. We analyzed their answers bearing in mind four general aspects: the role of simulation in teaching; the different steps to follow in a simulation; the complexity of starting situations; and the most important concepts which take part in simulation activities. The results show that teachers deem as important only certain aspects of simulation but neglect others, which are also fundamental in teaching.

INTRODUCTION

This work is a part of a project whose aim is to influence high school teachers in our country to use, in an appropriate way, the didactical software of statistics *Fathom* in their Statistics and Probability classes. From the variety of topics that the discipline has and which can be introduced or strengthened with the software, we have chosen that of *computational simulation* because of its potential didactical possibilities. The initial question that motivates this research, is investigating what software-simulated aspects are relevant to teachers and how they think the teaching, with the use of this kind of tool can be carried out.

Hawkins (1996) has mentioned some 'myths' which have accompanied the innovative intentions to introduce computational technology in teaching statistics: "Technology enhances the teaching and learning of statistics" (p. 1), "Computers have changed the way we teach statistics" (p. 3), "Students learn statistics more easily with computers" (p. 7), are some of the myths pointed out by Hawkins and which frequently we accept as true in our projects about using computational technology in teaching. Maybe we ought to add to her list that of the role of the teacher: "Teachers will be able to draw out didactical consequences and to carry out activities with technology in order to achieve their students' learning, once they have learned its functioning".

Hawkins (1996) does not deny that the myths she refers to are partly true, but she points out that there still is a lack of empirical evidence about specific points concerning processes in which technology-in-teaching is involved. "Introducing technology effectively requires exactly the same kind of planning and understanding about how students learn, and how best to teach them, that we should use to plan any other non-technologically-based teaching" (p. 6). In this sense, it is convenient to obtain evidences about how teachers see software potential and its possibilities in the classroom. In this work we will show some of the beliefs given by a few qualified teachers on this matter.

FATHOM SOFTWARE

Fathom (Finzer, Erickson & Binker, 2000a) is a dynamic statistics software, made specifically for teaching, using the opinions of some specialists in statistical education (Finzer et al., 2000b). Some of the outstanding features of this software are its possibility to move objects, the possibility to have the representation and the simultaneous coordination of data in tables and in graphics, and tables with the most important statistical features of a data set. The frame of a *case table*, which contains the basic data and from which the job is achieved, is that of a calculation sheet which provides an immediate access to a series of functions and some other representations which provide a broad flexibility for statistical work. It has various RANDOM functions, which allow generating random data of diverse distributions. With these functions, a great deal of simulation activities can be conceived.

THE OBSERVED SUBJECTS

A Fathom course-seminar was carried out with six teachers and the aim was to learn to use the software. Eight three-hours sessions per week were devoted to solving simulation problems: problems involving tossing dice, roulette, conditional probability, expected value, the classical problem about the birthday, etc. Also in those sessions there was discussion of ideas about how simulation techniques can enhance probability learning.

The participant subjects are high school teachers who have taught probability and have shown interest in mathematics education, i. e., they are committed to reading, and have studied, research reports related with this theme, have thought about the situation of teaching probability and have made efforts in looking for alternative ways for the teaching statistical concepts. Each session consisted of an exposition and discussion of some problem that was previously assigned and solved by the presenter. This work was also accompanied by inquiries about the commands and functions of the software. Just before finishing the sessions there was the opportunity to comment about applications of what was seen related to teaching probability.

These teachers are involved in a project that aims at generalizing software use; in particular, they carry out training workshops with other teachers. Having finished the simulation sessions, the six teachers were asked to answer an inquiry questionnaire in which they gave their opinions about some aspects of the theme. They were told the purpose of the questionnaire was to compare their beliefs about simulation. The questionnaire and a summary of the answers given by the teachers are shown in the final part of this work.

FRAME OF ANALYSIS

We will use the following distinctions about simulation procedure to analyze participant teachers' opinions; we will distinguish those points in which participants put more attention and those which weren't taken into account. The results will allow us to identify aspects of simulation which were favored by the teachers when they became aware of the software's capabilities and those which don't attract their attention in spite of being emphasized by researchers as important.

Biehler (1991) has presented an extensive analysis about the capabilities and limitations of simulation techniques in teaching statistics; he points out that "the different roles, goals and pedagogical perspectives for simulations have not yet been clearly analyzed and distinguished". He suggests a basic distinction between "the use of simulating as a method for solving problems, similar to the professional use outside school, and the use of simulation to provide model environments to explore, which compensate for the 'lack of experience'" (p. 183).

The process of computational simulation relates to problems and situations with computer programs, which generate (pseudo) random numbers through models, from which the urn model is a paradigmatic one. Biehler (1991) points out that it is possible to distinguish four aspects of simulation: 1) Formulating the model; 2) Simulating the model; 3) Analyzing the results of simulations; and 4) Validating the model. The difficulty to achieve each of the prior steps will depend on the problem or situation that is desired to be solved by a simulation method. Particularly, point 2) will be very liable to problems according to following levels:

First level of starting situations: When the situation is located (or almost located) in an urns context; in this case, modeling is reduced to the choosing of an appropriate combination of urns.

Second level of starting situations: When problems are in a context, and the subject has to transform the situation to an equivalent situation using the urns model or another appropriate model.

Third level of starting situations: When problems are posed in actual situations in which intervene stochastic elements; for instance, the voting problem mentioned in the reference handbook.

On the basis of the above remarks, we have formulated the following list of points that serve as a reference in our analysis:

- The function assigned to simulation
  - Simulation for solving problems
  - Simulation to create exploration environments

- The stages or aspects of simulation
  - Formulating the model
  - Simulating the model
  - Analyzing the results of simulations
  - Validating the model
- Levels of complexity in the starting situations
  - Situations in urns context
  - Situations in a context different from urns
  - Situations in the student environment
- Relevant concepts which take part in simulation
  - Frequency type probability
  - RANDOM function
  - Probability distribution of a random variable

The purpose of the analysis is to search in the teachers' answers, which of the above points are and are not borne in mind. When two or more teachers mention one of the above points, it could be inferred that that aspect is potentially easy to cover. If it is mentioned by only one of them, we could think that that aspect is difficult but could be improved by means of certain activities. Finally, points that are neglected by all of them indicate a need for special emphasis and are out of a spontaneous approach to simulation didactics.

ANSWERS' ANALYSIS

Before analyzing the questionnaire answers, it must be said that during the activities, all participant teachers manifested their enthusiasm about the software's strength and flexibility. In the following paragraphs, we present the relevant aspects of the opinions given by the teachers commented with reference to our Frame of Analysis, and discerning in the teachers' opinions, the presence or absence of the aspects pointed out in our Frame.

Participant teachers admit to having understood better certain probability concepts with the help of Fathom (question 1). Particularly, three of them mentioned the *variability* concept of relative frequencies. They also admit that there were problems that they cannot solve through theoretical methods but which they can outline and solve using a simulation technique; three of them mentioned the expected value with one of them adding the *elevator problem*[1], and another mentioned the *appointment problem*[2].

Our subjects recognize simulation as a technique useful to solve problems, even those of hard solution through analytical methods. This acknowledgment gives a typical value to simulation, not only as a didactical resource but also as a tool for solving problems. All teachers consider that simulation activities are very useful in the classroom (question 2); one of them emphasizes their potentiality in solving problems, another mentions the usefulness for giving meaning to concepts; yet another mentions the usefulness for enhancing the understanding of concepts, for example, the concept of hazard. Among other opinions, one teacher emphasizes the possible applications and another the possibility of simulating actual life situations.

That which is most important to achieve by working with simulation activities, is the understanding of concepts. However, teachers did not mention how to accede to such understanding; this can be due to a belief that often can result in non-desirable didactical consequences: the belief that if the teacher succeeds in understanding something by some means of focusing or using an instrument, then the student also will succeed in reaching that understanding. In relation to the way in which the use of simulation ought to be lead in the classroom (question 3), the following was mentioned:

- Beginning with physical simulations, analyzing them and reproducing on the computer
- Posing and solving problems with and without the use of technology
- Studying games of chance and then simulating them
- Beginning with training in the use of the software and then making a carefully design of activities
- To carry out simulation of problems which can also be solved by classical means in order to make comparisons

- Beginning with easy problems, then verbalizing them in order to take them to an urn model and, finally, simulating them

The answers given by the teachers reflect their concern about the two first aspects of simulation pointed out by Biehler (1991): that of formulating a model and that of simulating it. Maybe the teachers take for granted the aspect of analyzing the simulation results; however, its worth deserves to be mentioned.

We must also note the absence of references from the teachers to the aspect of validation. Maybe the fifth point, mentioned by one teacher (Santiago), could reflect his concern in that sense. However, these two aspects are very important whether considering simulation to solve a problem or its focus towards understanding concepts. The concepts that were mentioned at least once, considering they could be introduced or strengthened with the help of simulation (question 5), are: probability (classical and frequency types), chance, variability, independence, and random phenomenon.

Teachers relate simulation activities basically with the concept of frequency type probability. Chance, variability and random phenomenon concepts are but another expressions of the same concept, i. e., in depth, these concepts are related with the unforeseeable feature of particular outcomes and the convergence of relative frequencies at long term. It is clear that simulation techniques give another dimension to frequency type probability focus. This is a fact that appears to be meaningful in simulation. We must note the absence in the teachers' opinions of an important concept in simulation: the concept of *distribution*. Random variable and distribution of probability are concepts that are introduced in high school. Simulating a random variable provides this concept with a dynamic significance and it is the technical basis of Montecarlo method. In relation to the difficulties in applying simulation activities to classroom (question 6), there were two kinds of answers: one, which made reference to didactical-cognitive aspects and the other to technical aspects.

- The didactical-cognitive difficulties are: the interpretation of tables and graphics, the confidence in simulation as an expression of the phenomenon and *verbalization* of how to achieve the simulation of the experiment
- The technical difficulty: teachers training in using the software.

Teachers are prone to place a great deal of difficulties in modeling, i. e., and the passage from the problem to the model. However, looking at the answers to all the questions, they only mention problems from the first and second modeling levels, i. e., they don't pose everyday problems or reality problems related to the pupils. In neglecting these types of problems, one of the most exciting and meaningful applications of simulation is overlooked.

CONCLUSION

In relation to the role of simulation for teaching, the participant teachers are inclined to confer to it the role of formation or elucidation of students' concepts. However, teachers don't specify the skills that could lead to that elucidation. For example, they don't mention aspects related to explorations made with given models, such as changing parameters and observing behaviors. In this sense, it must be noted that they have centered their attention in aspects like formulation of a model and its simulation but neglect aspects like the analysis of results and validation.

None of the teachers suggested initial situations belonging to the third level of problems, i. e., situations close to students' reality. The participant teachers could have mentioned activities such as queuing in a bank or in a supermarket, time spent in a gas station, or the survey or poll to predict the result of voting, problems which are well known by students, , but they were inclined to choose problems from the textbooks. This absence could be explained by the dynamics of the work sessions and the types of problems solved in the workshop but also by the teachers' vocational training and their traditional way of teaching.

In relation to the concepts teachers believe need to be clarified with simulation activities, there is a prevalence of frequency type probability. It is attractive to carry out a great deal of tests of an experiment and "see" the convergence of probability. But there are two very important concepts in simulation which ought to be borne in mind: *random* and *distribution* but teachers

appear to overlook these concepts. The above observations will be very useful in developing our project to influence high school teachers to use and exploit technology capabilities in teaching statistics. But a research question also emerges: Which teachers' opinions, developed from the experience with simulation can be beliefs that are present or arise in a more general form in the teachers community?

NOTES

1. *The problem of elevator*. Seven people get up an elevator in a seven-story building. What is the probability of exactly one person getting off in each story?

2. *The problem of appointment*. Two friends make an appointment for a meeting in a restaurant between 9 and 10 a.m. They decide that when one arrives he will wait for the other only 15 minutes. What is the probability of their meeting?

3. *The problem of points*. Pascal and Fermat, in letters, discussed a simple game: They toss a coin. If the coin comes up heads, Fermat receives a point. If tails, Pascal receives a point. The first player to receive four points wins the game. Each player stakes fifty francs, so that the winner stands to gain one hundred francs, and then they play. Suppose, however, that the players need to end the game before a winner is determined. Further, suppose this happens at a moment when Fermat is ahead, two points to one. In their letters, Pascal and Fermat discussed the question: How should the 100 francs be divided?

4. *Answers to the questionnaire*

1.Is there some probability concept you have understood better because the simulation activities?

Three teachers (Miguel, Santiago and Humberto) answered they understood better the concept of *variability*, i.e., the idea that succession of relative frequencies vary a lot when the number of essays is small and it stabilizes when it grows. Román judged that he understood better the "streak" phenomena when tossing coins.

To Gabriel, the concept of *conditional probability* was clearer, particularly because it depends on the quotient of two quantities, which do not depend neither on time nor causality. Roberto pointed out that Fathom helps to understand better the *large numbers' law*.

2.Is there some problem of probability, which you cannot solve theoretically, but, nonetheless, you can solve it with the help of simulation?

Gabriel answers that some problems about hope of random variables are very difficult to solve analytically but using simulation are easier. Humberto admits he cannot solve the problem of the elevator analytically but he can do it with simulation. Roberto points out some problems, one of them about expected value: How many times, in average, must a die be thrown in order for the six faces to appear? Another problem about continuous variable is the *problem of the appointment*.

Miguel and Román point out that there are problems which are easy to solve using simulation and difficult to do it analytically but they do not specify any.

3.What do you think about the usefulness of simulation activities using Fathom in classroom?

Gabriel thinks that using simulation can be useful for students to find meaningful the mathematical models of chance situations.

Miguel judges that simulation activities enhance the understanding of probability concepts, as that of the chance.

Humberto replies that Fathom is useful because it is possible to simulate a lot of applications.

Román points out that Fathom is useful but depends on the availability of computers and on the teachers' training, as much as in the operation of the software and the relationship with the content of curriculum.

Santiago points out the software usefulness for making simulations of real life situations in the classroom.

Roberto judges it is useful to solve problems using frequency focus, besides the fact that students find interesting using computers.

4.How do you think simulation should be implemented in teaching probability?

Gabriel points out that it is necessary to begin making simulations with physical things (roulettes, coins, dice, etc.); then to analyze such processes and, finally, to take the same problems

to the computer. He explains that the idea is leading activities toward understanding probability concepts so that the results also have a *symbolic representation*.

Humberto suggests beginning with problems that can be solved with simulations with and without using technology.

Miguel suggests beginning with chance games experiences and, finally, simulating those experiments. He points out that the passage from an experiment to its simulation could be a hard problem for students.

Román points out that the beginning must be the training in the use of software and the teachers must plan carefully the activities to carry out in classroom.

Santiago suggests that simulation be studied in relation to classical focus, checking how the obtained results by means of simulation coincide with those provided by means of classical definition, but also considering problems in which the concept of frequency is underlying and which are not easily solved by means of classical definition.

Roberto points out that the start must be with simple problems in order for students to achieve verbalization and be able to take them to an urn model or another equivalent; then to take them to Fathom bearing in mind the teaching of the use of the software.

5.Do you think that simulation activities can help to understand probability concepts? Which concepts? How? And, Why?

Gabriel points out the very concept of *probability*.

Miguel points out the concepts of *chance*, *frequency type probability*, to make a contrast with *classical probability*.

Humberto answers that simulation helps to understand such concepts as *range*, *variability*, and *collection*.

Román proposes the concepts of *chance* and *independence*.

Santiago proposes: *frequency type probability*, *conditional probability*, *sample distributions*, and the *theorem of central limit*.

Roberto points out the very concept of *random phenomenon*.

6.What problems do you think are the most important to overcome for implementing simulation activities in teaching probability?

Gabriel points out the difficulty of the programming for modeling problems as well as that of reading and interpreting tables and graphs.

Miguel points out the difficulty of equipment availability in schools and statistics teachers' attitude and training.

Roman points out the difficulty for students to trust that simulation reflects what is going to be simulated. He adds also the difficulties of equipment availability and the lack of trained teachers.

Roberto points out the students' difficulty to verbalize how the simulation of the experiment will be accomplished and then translating it to Fathom language.

Santiago points out the difficulty that students surmount the idea that with a few essays is achieved a frequency next to probability.

Humberto misunderstood the question. His answer made reference to problems that can be solved by means of simulation.

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