Developing Consideration of Variation: Case Studies from a Tertiary Introductory Service Statistics Course

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1. Introduction

An important issue in statistics education is how to help students develop statistical thinking, reasoning and literacy. Research in this area is extensive (e.g., Chance, 2002; Garfield, 2002; Rumsey, 2002). Wild and Pfannkuch (1999) identified consideration of variation as one of the fundamental types of statistical thinking, while MacGillivray (2004) reinforced the importance of variation by introducing the notion of statistics as the 'science of variation'. Also, as Bakker (2003, p.3) explains, 'If students do not expect any variability in a particular context ... they neither have an intuition of why one would take a sample or look at a distribution.' Other recent research also highlights the importance of understanding of variation to the development of students' statistical thinking (e.g., Meletiou-Mavrotheris & Lee, 2002; Reading & Reid, 2004; Reading & Shaugnessy, 2004; Torok & Watson, 2000).

Much of the research to date on the role of variation (or lack of) in statistical reasoning in education has been at the pre-tertiary level. There has been concern that too little emphasis has been placed on the notion of variation by educators at this level (e.g., Torok & Watson, 2000; Meletiou-Mavrotheris & Lee, 2002). Reading and Shaughnessy (2004) asserted that there is an emphasis on measures of location, to the detriment of consideration of variability, and Meletiou-Mavrotheris and Lee (2002) were concerned by the negative impact that a deterministic approach in the mathematics curriculum would have on statistics instruction. This lack of stochastic awareness may leave students embarking on their tertiary statistics education ill-prepared to consider the more advanced notions of the statistical model as a combination of both systematic and random effects. Reading and Reid (2004) suggested that without an appreciation of the complete statistical model, students may view statistics as a list of techniques to be learned in isolation. A sound understanding of variation could help promote a more comprehensive approach to learning statistics.

How does a student's understanding of variation develop at the tertiary level? There is comparatively little research that explores the development of students' perceptions of variation at this level. DelMas and Liu (2003) focused their research on students' interpretations of the standard deviation. Lann & Falk (2003) found that when students in a first year service course were explicitly asked to consider variation, their intuitive notions varied. However, a greater proportion of students chose the range than any other single measure of spread to summarise the variability in a data set. A broader study of students' consideration of variation was presented by Meletiou-Mavrotheris and Lee (2002). They developed a college level introductory statistics course based on students' experience that incorporated variation at its core. By the end of that course, more students' had an increased awareness of the need for information regarding the spread of a distribution, in addition to a measure of location. Meletiou-Mavrotheris and Lee (2002, p.33) recommended using a range of assessment tasks to examine students' understanding of variation since '… assessment of thinking about variation is heavily reliant upon both the types of assessment tasks employed and the contexts in which the tasks are situated.'

2. Research focus

Reading and Reid's research project, *Understanding of Variation,* was based at a regional Australian university and explored the development of tertiary students' consideration of variation in an introductory service statistics course where variation was treated as a core concept. Reading and Reid explored what understanding of variation was demonstrated as science students engaged in various learning activities and assessment tasks. The project aimed to refine hierarchies being developed to assess students' understanding of variation and to investigate how this understanding develops. Such information is critical in developing curriculum and evaluating its effectiveness. The complete project included analysis of student responses to a range of tasks: pre-study and post-study questionnaires, follow-up interviews of selected students, four separate minute papers, one question from a class test, and one question from an assignment. Reading and Reid (2004) described the analysis that developed a hierarchy of levels of consideration of variation based on responses to the minute papers. Reid and Reading (forthcoming) incorporated the results from the other assessment tasks to further refine the hierarchy being used to describe the students' developing consideration of variation. For each task key concepts in the consideration of variation were demonstrated in student responses and provided the basis for describing four levels in the hierarchy; *No*, *Weak, Developing,* and *Strong Consideration of Variation* (summarised in Table 1). Is this hierarchy useful for tracking students' developing consideration of variation during the course? Case studies are now presented to highlight important aspects of the hierarchy, and address this question.

Table 1: Hierarchy of levels of consideration of variation.

No	does not display any meaningful consideration of variation
Weak	acknowledges variation with poorly expressed descriptions; incorrectly explains variation; does not use variation to support inferences
Developing	clearly describes at least one source of variation; correctly interprets some factors explaining variation; uses variation to support inference
Strong	clearly describes all major sources of variation; correctly interprets all factors explaining variation; recognises the link between variation and tests of significance

3. Case studies

Coded responses from particular students across all tasks were examined to build up a picture of students' progression during the course. Ideally we would like to see students demonstrating some progress in terms of the hierarchy. However, there were some students who gave no evidence of improved consideration of variation, and others who showed improvement in some, but not all, tasks. The following case studies provide exemplars of three students, Sarah, Alex and Rachel (pseudonyms assigned to the students) across the assessment tasks. Sarah's responses remained at the same level as the course progressed, the level of Alex's responses were inconsistent from one task to another; while Rachel's responses demonstrated improvement as the course progressed.

3.1 Sarah: Consistent

Sarah's responses consistently demonstrated a weak consideration of variation. Poor interpretation of questions, compounded by difficulties with statistical concepts and notation, meant that the level of Sarah's responses did not improve. Sarah was limited in her expression, and tried to use concepts, notation and formulae that she had seen previously, but apparently without any real understanding. She did provide some good examples in one question, illustrating different causes of variation, and displayed some sound intuitive understanding in two other questions in the pre-study questionnaire. These responses may possibly have been based on her contextual knowledge (personal experience). However, Sarah was unaware of the need to consider variation in tasks involving comparisons of data sets given at the beginning of the semester. Later in the course, Sarah's responses suggested an attempt to consider variation in tasks given but she was unclear about the difference between within-group variation and between-group variation. Sarah's lack of understanding of statistical notation is evident in one task, in particular, where she provided a confused mix of the standard normal Z-score with a formula for the standard error of the mean. In addition, in her responses to questions in the post-study questionnaire, she incorrectly referred to statistical concepts of which she obviously had poor understanding. Sarah's responses to the pre-study questionnaire suggest that she had a sound but limited intuitive understanding of the concept of variation. However, responses later in the course suggest that she was unable to develop this further, instead replacing it with a confused representation of the curriculum themes.

3.2 Alex: Inconsistent

Alex, like Sarah, demonstrated a sound intuitive understanding of variation at the beginning of the course, with a good use of examples demonstrating different sources of variation. However, his response to the same question in the post-study questionnaire was not as well developed. Furthermore, his responses for those tasks requiring proportional reasoning were poor and inconsistencies appeared in his responses to other tasks. There were a number of tasks comparing distributions, where Alex did refer to both within-group and between-group variation, demonstrating a developing level of consideration of variation. Yet, in a later task that required an intuitive analysis of variance (e.g., examine these distributions – are the means different?), Alex's response was weak because he only discussed between-group (systematic) variation. Nevertheless, in a post-study interview, Alex did discuss the need for consideration of both systematic and random variation although he did not use those terms explicitly. Alex demonstrated an awareness of the need for consideration of variation, yet the level of his consideration had not stabilised sufficiently to ensure that it formed a consistently sound basis for all of his responses.

3.3 Rachel: Improved

At the beginning of the course, Rachel demonstrated only a weak level of consideration of variation and could not provide a detailed explanation of variability nor any useful illustrative examples of variation in the pre-study questionnaire. In tasks involving data set comparisons, she only considered the range of the distributions and, in a later task, suggested that a large amount of within-group variation indicated a real difference in means, without consideration of the between-group variation: 'There is a real difference in group means because there is a lot of scatter in [each] data set.' In other words, she was focusing on the random variation in isolation from systematic differences. This was a common mistake made by a number of students. However, by the end of the course, Rachel was able to explain and use between-group variation in detail, in conjunction with within-group variation, to draw valid inferences.

4. Discussion

Reading and Shaugnessy (2004, p. 223) asked, 'How can intuitive notions [of variability] be harnessed to develop a more sophisticated notion of reasoning about variation?' The above has demonstrated that some students were able to build on their intuitive notions of variation, through the learning and teaching activities provided in the course. However, as we saw with Sarah's case study, others may discard the limiting notions of variation that they bring to the course and yet are unable to replace them with more sophisticated reasoning from the course curriculum, even when variation is treated as a core concept throughout the course.

It is important for tertiary students to be able to deal with more sophisticated concepts of variation. In each of the case studies, the student was challenged by the more complex tasks, for example, those requiring a comparison of distributions. The development of each student's consideration of variation was restricted (consistently or occasionally) by the tendency to focus on either variation due to systematic effects or random effects, rather than a consideration of both. Sarah struggled throughout the course with basic concepts and notation and so was unable to assimilate the core message of the need for consideration of variation. Rachel and, to a lesser extent, Alex understood the message and often were able to successfully integrate it in their responses to tasks of varying complexity.

It is clear that it takes time for students to develop a strong consideration of variation that can be successfully applied in a variety of situations. It is not unreasonable to expect students to achieve a higher level in the hierarchy in some tasks by the end of the course, but the study suggests that consistently high achievement across all tasks will take longer. Pfannkuch (1997, in Meletiou-Mavrotheris & Lee 2002, p. 24) recognised that '… the concept of variation would be subject to development over a long period of time, through a variety of tools and contexts.' By giving students a range of tasks in different contexts, educators can help students' develop a sound consideration of variation. Hopefully, by building up the complexity across time, suitable learning activities may enable students to build on their intuitive notions of variation and thus develop a more sophisticated consideration of variation.

5. Summary

As consideration of variation is one of the fundamental types of statistical thinking, the success of students studying statistics will depend on how well they can develop this consideration and apply it in different contexts. The hierarchy that has been developed is useful for describing the level of consideration of variation evidenced by students' in their responses to a variety of learning tasks. The case studies have demonstrated that students begin their tertiary education with varying levels of consideration of variation and

that progression in the hierarchy is not a linear process nor is it the same for each student. This hierarchy may help educators focus on students' developing consideration of variation, identifying stages of that development. Providing such direction for learning and teaching activities helps students advance in the hierarchy thus developing a strong consideration of variation which can be applied in a variety of contexts.

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RÉSUMÉ

En tant que l'un des éléments de base de la pensée statistique, la considération de la variation touche la statistique dans toute son étendue. L'article présent offrira une vue d'ensemble des résultats des recherches les plus récentes sur le raisonnement des étudiants universitaires au sujet de la variation. En particulier, les résultats de nos recherches actuelles décriront le développement de la considération de la variation chez des étudiants universitaires pendant un cours préliminaire de statistique générale. Reading et Reid (2004) ont établi une hiérarchie de niveaux de considération de la variation basée sur les réponses à de courts questionnaires qu'a produites un groupe d'étudiants inscrits dans un cours préliminaire de statistique générale. Les chercheurs s'intéressaient à toute considération de la variation (c'est-à-dire aux expressions de la variation et à l'utilisation de ces expressions) provenant des réponses écrites de ces étudiants. Des recherches supplémentaires (Reid et Reading, à paraître) ont incorporé les résultats d'autres exercices afin de mieux préciser cette hiérarchie. Dans l'article présent certains dossiers individuels souligneront des aspects importants de l'hiérarchie.