A WEB ENVIRONMENT TO IMPROVE CONNECTIONS BETWEEN MATHEMATICS AND STATISTICS LEARNING

S. Mignani, P. Monari, and R. Ricci
University of Bologna, Italy
A. Orlandoni
I.R.R.E - E.R., Italy
monari@stat.unibo.it

The paper describes a project involving cooperative learning of statistical topics implemented in a web environment. The project is the result of a fruitful partnership between the Faculty of Statistics of the University of Bologna and the Institute of Educational Research of the Emilia Romagna Region. The synergy between mathematics and statistics offers a concrete example of theoretical method application in order to investigate the real world. The positive effects of cooperative learning can be broadened to a larger learning-teaching community. It makes it possible to create a virtual place where it is possible to realize a cooperative learning milieu. This methodology is particularly useful for statistics, given both the specificity of the discipline and the fact that teaching of the subject often represents a novelty for mathematics teachers. Problems connected with real life are presented on the web and project participants may communicate through the network.

INTRODUCTION

Cooperative learning can be defined as a methodology involving students who work together as a team to solve an assigned task or to reach a common goal (Garfield, 1993). This learning strategy enables student-teacher interaction and provides an opportunity for a reflective examination of individuals and groups. One very important advantage of this teaching and learning technique is the increase in teacher satisfaction and the greater sense of partnership between students and instructors (National Commission for Cooperative Education, 2003).

Cooperative learning methodology is coherent to the constructivist theory that is broadly and fruitfully applied in mathematics and science education. Constructivism considers that students bring their own ideas and experiences to the group. In this way, they can build up their own skills, rather than acquiring the knowledge conveyed by the teacher.

In this context Mathematics fits well with the main principles of this theory and consequently, those of cooperative learning. In particular, Statistics as a part of Mathematics teaching at school level, offers a fruitful example of cooperative learning application. Working with peers stimulates the comparison between different interpretations of real situations connected with their daily world. This way of teaching Statistics can play a positive role in improving the connection with Mathematics and therefore favours constructive learning of both topics (Ottaviani, 2005). The teaching of Statistics is currently incorporated in mathematical curricula and therefore it is usually illustrated in a very abstract manner. A more compelling way of introducing Statistics can increase the students' interest in Mathematics (Gattuso and Pannone, 2002). Indeed, Statistics makes possible to introduce in a concrete way mathematical concepts that may be difficult to grasp (Mignani and Ricci, 2005).

In order to expand the positive aspects of this kind of synergy between Mathematics and Statistics, a web environment based on the principles of cooperative learning was developed. This overcomes the traditional classroom concept and is replaced by the idea of virtual learning group, so that a larger community is reached. Additionally, teaching Statistics using new technologies is really fruitful (Ottaviani, 1997). Nowadays it is not possible to think about doing data analyses without computers, and communication technologies are closer to the way young people exchange information. Younger generations seem to prefer more informal aids to learning and a cooperative environment on the web may be an interesting solution.

A PROJECT OF THE INSTITUTE OF EDUCATIONAL RESEARCH I.R.R.E.-E.R.

During the 1990s, the Institute of Educational Research of the Emilia-Romagna Region (Italy) developed a pioneering web environment, called *FARDICONTO*, in order to enliven many

traditional mathematical topics at secondary level. The main goal of this application of cooperative learning on the Internet is to invite students to work in collaborative groups and thus favour learning with peers.

On the project website (www.fardiconto.it) a number of arithmetic, algebraic, and geometrical problems are periodically presented. Students and teachers are encouraged to analyse and discuss the proposed issues and to send solutions, suggestions, and remarks. In this way, a learning community is created, a virtual environment where individuals and groups can exchange ideas and find tools to solve problems. In this project, researchers coordinate and encourage discussion and they pose new questions.

Despite the novelty of the methodology, the project was popular with teachers and students and it has become common practice in those schools that usually capitalize on new technologies in the learning-teaching process.

AN EXTENSION OF THE ORIGINAL PROJECT: "STATISTICAMENTE," A WAY OF REASONING ON REAL LIFE PROBLEMS

The General Context

Statistical reasoning has recently assumed a new role within the context of secondary level Mathematics. Indeed, educational science recognises the need for secondary school Maths to be more closely related to citizens' daily life. Surely citizens should be capable of thinking quantitatively, for instance reading a newspaper intelligently, making decisions based on quantitative information (Moreno, 2002).

This new way of considering the citizen's mathematical literacy is a relevant facet of international skill assessment projects, such as the *Programme for International Student Assessment* (PISA-OECD), a three-yearly survey of the knowledge and skills of 15-year-olds in the main industrialized countries. It assesses to what extent students coming to the end of compulsory schooling have acquired the knowledge and skills that are essential for full participation in society (OECD, 2004).

With regard to the above-mentioned issues, the latest reforms of the Italian education system have deeply changed the topics dealt with in Mathematics syllabi. In particular, statistical and probabilistic concepts have been included at every level of schooling. This makes it very important that students and teachers have access to tools that aid their learning-teaching experience in order to handle concrete problems connected with real life (Milito *et al.*, 2001)**,** and first of all, with the scientific disciplines included in their educational curricula, so to stimulate the scientific interests of the students.

Without a statistical intuition of the real world, natural sciences would have never achieved their modern basic ideas: the theory of evolution by natural selection, the physical theory of quanta, thermodynamics and genetics. In natural sciences, as well as in social ones, statistics is something more than a useful *modus operandi*, it is a new *modus intellegendi*: a methodological paradigm for the interpretation of social phenomena, a new language of physical and biological reality (Scardovi, 1982).

The STATISTICAMENTE Project

In order to provide proper tools consistent with this way of teaching of Statistics, I.R.R.E. - E.R, in association with the Faculty of Statistical Science of the University of Bologna has added to the *FARDICONTO* web environment a new project called *STATISTICAMENTE – a way of reasoning on real life problems.* Starting from school year 2004-05, every three months a new problem based on real situations is presented on the website (www.fardiconto.it/statisticamente/). During this period a forum is open. Teachers and students can discuss the problem with the whole virtual community, and make comments on the interpretations and solutions put forward, under the guidance of the project supervisors.

Every quarterly issue consists of one or more problems addressed to students from 11 to 18 years of age. In agreement with the guidelines of the cooperative learning (Garfield, 1993), the assignments proposed set one or more of the following tasks:

- to solve a problem and then compare the different solutions on the website;
- to discuss and compare different concepts or procedures;

- to collect, analyse, and interpret data, and compile a report.

The Problems Put Forward

The problems are concerned with basic Statistics, in particular data collection, graphing and displaying data, measurements of location and variability. The questions always refer to some aspect of everyday life. They have been chosen in order to give the students the opportunity to use mathematical tools by means of statistical reasoning. The problems are expressed in user-friendly language, couched within a fun-to-read vignette, and displayed amusingly on the website.

As far as the abovementioned characteristics are concerned, the first problem published on the web site is shown below. It concerns root mean squares and has been borrowed from an exercise book (Huck and Sandler, 1984):

> *"Mr. and Mrs. Grayson went out for dinner one night with their four children. When they were seated at the restaurant and began talking to their waitress, however, it became apparent to the Graysons that they had failed to make one final, important decision. Before the waitress could pass on their order, the Graysons needed to specify how large a pizza they wanted.*
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> *In response to the waitress's question about the size of the pizza, the Graysons found that they no longer agreed. "I'm starving" said the eldest son, "so let's get four large pizzas." Mrs. Grayson, who was well aware of the family budget, warned against ordering more than the family would actually eat. To assist the Graysons in making their decision, the waitress attracted everyone's attention and pointed to the wall, where six round pizza pans where hung for people to see the pizza sizes available. These pans were clearly labelled as follows: 24 inches, 21 inches, 19 inches, 12 inches, 9 inches, and 5 inches.*
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> *Mr. Grayson was firm believer in resolving all issues – even family ones - in a democratic manner, so he announced that they would take a vote and let the majority rule. Bill spoke up right a way and voted for the 24-inch pizza. Martha voted for 12-inch size, commenting that everyone needed to lose a little weight, and Harry voted for 9-inch size. Mrs Grayson voted for the 19-inch size. Suzie, the youngest member of the family, voted for the smallest size and finally Mr. Grayson voted for the 21-inch pizza.*
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> *It was clear to that the democratic vote had not resolved the issue. "All right," said Mr. Grayson authoritatively, "since each of the six pans on wall got one vote, will take the one that comes closest in size to the average size of all six." Although certain family members realized that they would not be eating the pizza that they had voted for, everyone felt that the decisions had been reached in a fair manner with input from each person.*
>
> *If you had been the waitress, which of the six pans would you have told the chef to use to the Graysons' pizza?"*

This problem is designed to help students know different kinds of measures of centre. This strategy can be useful for making students aware of a mean argued choice.

The second problem consists of two sections: the first aimed at 11-13-year-old students, and second at older students (14-18-year-olds). The assignment in the first section requires reasoning on data tabulation, univariate and bivariate distribution, marginal and conditional frequency distribution. This activity is designed to help students empirically test the soundness of their insight into the association between two variates.

The assignment of the second section is related to variability measurement. The aim is to understand the meaning of variability and its relevant role in real-life situation analysis. The problem posed can be solved using concentration index, which is quite unusual in introductory Statistics courses, but it could be very useful in analysing everyday life situations.

A change in the point of view is given by the third problem. The task is to write a report on the current issue of environmental pollution. The choice of this topic gives students the opportunity to obtain practice with data from official sources and close to their interests using several statistical techniques. Giving students the possibility to analyse real data allows them to put mathematical tools into practice and to explore the efficacy of different methods in order to verify some working ideas. In this way, a *problem solving* strategy is created and its advantages are strengthened in a *broadened* cooperative learning context.

PRELIMINARY RESULTS

As mentioned above, the project has only just began and work is still in progress, so few and preliminary results are presented.

Some respondents gave answers that are not completely adequate. The collaborative discussion on the Internet helped them reformulate better solutions in terms of data analysis. Following a chat session focusing on the doubts of participants (both teachers and students), the given complete solutions have these characteristics with respect to respondents.

As for the age, the answers of the older students prevail, nevertheless the younger respondents did show interest in this project. Most of the respondents are groups of students supervised by teachers, although the percentage of individual answers is substantial.

Concerning the content of answers, the first question stimulated discussions on the use of different measures of location. With regard to this, one student declares, "*We discovered that the spreadsheet computes different kinds of means, not only an arithmetic one.*" This remark also highlighted the relevance of an adequate index for summarizing data. As one teacher underlines, "*Every choice has numbers that it accounts for*," therefore a conscious choice of the tool used is strictly necessary. As pointed out by the student's remark, the use of computers helps the acquisition knowledge as in a workshop situation. This latter aspect is consistent with the constructivism that plays a very important role in Mathematics and Statistics learning. Furthermore, this approach enables participants to work with statistical tools in their natural context, that is, each concept is used to understand a particular aspect of everyday life. Therefore, the suitability of each tool is evaluated in terms of its consistency to phenomena of interest. Students often refer to real life to back up their choices or to reject a statistical concept that seems to be inadequate. For instance, one student says, "*…in this way everyone eats more or less as much pizza as he or she desires*." A workshop-oriented approach also stimulates students to reflect on the limits of each statistical synthesis and the loss of information that a statistical tool causes. One student actually says, "*A statistical law is useful only if the Graysons order only one size of pizza!*."

An adequate selection of statistical methods is even more relevant in the second problem. Indeed, neither sections of the issue consist in an explicit question, but rather they describe a real situation that has to be interpreted and summarized.

With regard to the problem for younger students, the organisation of data was problematic and lengthy discussion among participants was required to find satisfactory solutions. Cooperative learning played a positive role in helping young students who had trouble with data analysis.

Older students found complete solutions to the second section and some groups used data available on the Internet wisely. For instance, one group analysed the problem using population data from the official site of the National Statistics Bureau. Another class applied concentration indexes to explain hypermarket distribution over the national territory more efficiently. Once again for this section it was very useful to work in groups in a broadened cooperative learning context. This work strategy made possible to highlight the importance of several aspects that are connected with the situation presented. Each participant focused his or her attention on a set of data and discussion made it possible to organize a detailed analysis.

As for the third problem of the school year 2004-05 only a few solutions has been sent. This kind of task seems to be more difficult to face with statistical tools that are usually taught at secondary school. Indeed, difficulties turn out in the data analysis (collection, investigation, and interpretation). The latter issue may be a starting point to focus on the necessity to increase the synergy between Mathematics and Statistics. In particular, mathematical tools favour the quantitative reasoning that has to be developed in order accurately depict the real word.

Therefore, the current problem for the new school year 2005-06 consists of some questions, partially pulled out from OCSE-PISA 2003. These questions are easier than a complete data analysis and suitable for the cooperative learning. Several items instead of a unique task would encourage the discussion through the forum, because the problem is faced at different levels of deepening.

DISCUSSION

At the end of the preliminary project set up phase, a number of positive aspects can be identified. For students, the interactive and equalitarian involvement of a virtual space in which they can freely express their attempt to rationalise a real problem breaking it down into a number of objective moments, which can be translated into quantitative and mathematical terms. The choice of real problems to be resolved through statistical reasoning makes it possible to give youngsters a very potent method for better appreciating the phenomenon content of the scientific disciplines that they encounter in their learning process.

For teachers, the constant updating of the subjects involved in the project, who can make use of unconventional teaching instruments to motivate students and to given an operative and experimental meaning to the mathematical subjects presented in the courses. The project also fosters active, ongoing exchange with other subjects interested in the debate that originate from different situations and experiences.

For the research group that supports the project, the possibility of sharing an *in vivo* experiment, in addition to facilitating the contacts between research centres on training and the University and School world at all levels.

To sum up, a successful outcome of the project is related to the participants number. So far the web site has been promoted sending a newsletter to a forum of Mathematics teachers. A further project development is to involve a larger number of education agencies in order to widen the discussion group.

Besides, it should be remembered that a web environment has to be all the time updated. Therefore a constant supervisors effort is required to keep up the participants' interest and the discussion.

As already mentioned, teachers play a very important part in the project success. Their satisfaction and motivation should be assessed in order to evaluate the project efficacy. Therefore, at the end of the current school year, a questionnaire will be administered and according to the analysis results the activity of *STATISTICAMENTE* will be improved.

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