AN ELABORATION OF THE DESIGN CONSTRUCT OF PHENOMENALISATION

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The paper builds on design-research studies in the domain of probability and statistics. The integration of computers into classroom practice has been established as a complex process involving instrumental genesis (Verillon and Rabardel, 1995), whereby students and teachers need to construct potentialities for the tools as well as techniques for using those tools efficiently (Artigue, 2002). The difficulties of instrumental genesis can perhaps be eased by design methodologies that build the needs of the learner into the fabric of the product. We discuss our interpretation of design research methodology, which has over the last decade guided our own research agenda. Through reference to previous and ongoing studies, we argue that design research allows a sensitive phenomenalisation of a mathematical domain that can capture learners' needs by transforming powerful ideas into situated, meaningful and manipulable phenomena.

DESIGNING FOR ABSTRACTION

Our studies of children's use of mathematical microworlds have provided us with fresh insights into the deep relationship between the virtual tools we provide and mathematical thinking. Rather in the spirit of Jean Lave (1988), we have over many years discussed the acute sensitivity on learning of design decisions and the reciprocal manner in which the child's state of knowledge impacts upon how the software is used and therefore in turn on the microworld design. These studies have yielded a proposed model for *abstraction in context*, illustrated through, but not restricted to, students' meaning making for powerful stochastic concepts within a microworld designed for that purpose (Pratt and Noss, 2002). However, we now see this paper as elaborating only half the story. We see, in fact, the dual to *abstraction in context* as *designing for abstraction*. As we move towards a model for designing for abstraction, we plan to reflect in this paper on some critical design decisions in prior studies, suggesting some new constructs that may play their part in the language that needs to be developed for discussing designing for abstraction.

It is reasonable to ask whether design issues are worthy of such attention. The integration of computers into classroom practice has certainly been established as a complex and often difficult process involving *instrumental genesis* (Verillon and Rabardel, 1995), comprising both *instrumentation* and *instrumentalization.* The former refers to the process by which an individual assimilates an artefact (taken to have no meaning in isolation) towards his own state of knowledge. The latter refers to how the individual attributes functionalities within a wider setting to that instrument (Artigue, 2002). Put more simply, how we attend to the tools on offer will depend on the current state of our knowledge and yet that knowledge is at the same time in a state of flux as well-designed tools are likely to bring about thinking-in-change. In the light of such studies, it seems overly simplistic to consider abstraction as independent from situation. At the classroom level, the teacher needs to consider the relationship between her own actions, the student's learning and the structuring resources within the setting that they provide. There is growing evidence that it is no longer acceptable to design curricula and assessment regimes which fail to take into account the learning tools being used. Indeed, designing such tools is in our view *the* most pressing pedagogic challenge.

Not only are we becoming aware of that challenge but new methodologies that build the needs of the learner into the fabric of the product are being developed. Cobb *et al*. (2003) have set out their interpretation of design experiments which in educational research, are concerned with providing a better understanding of a complex learning ecology by the systematic design of its elements through which the emergence of successive patterns in students' reasoning is supported. The purpose is to develop specific theories about both the forms of learning and the means of supporting them. Design experiments always have two faces: prospective and reflective. The nature of the methodology is highly interventionist, with aspirations for educational improvement by engendering new forms of learning, in order to study them through iterative design. Iterative design comprises cycles of design, enactment, invention and revision. The insights gained from each iteration feed into the next iteration. However, theories developed during the process of iterative design are accountable to the process of design. In this sense, we see each design stage as incorporating a set of conjectures about both abstraction in context and designing for abstraction. These conjectures are in effect tested in the next phase of using the designed product. Our approach over many years has been to adopt the approach outlined by Cobb et al in order not only to develop a product but also to put forward new theories on abstraction in context.

Our emphasis here, however, is to exploit those same studies to propose some constructs to inform the notion of designing for abstraction. Much of the literature on design has used the language of *affordances.* The term *affordance* was coined by Gibson (1979) as part of the discourse of ecological psychology. He described an affordance as situated in the environment, whether perceived or not, and arising from what an object does, as in "a *surface* affords *support.*" Gibson stresses that affordances "are neither physical nor phenomenal" and are not the same for all animals. A piece of fruit affords eating for a human but not for a lion. Norman (1988) went on to introduce affordances to the realm of design. He changed the meaning subtly by making an affordance dependant on the observer's culture and experience. The information of an affordance was no longer contained only in light waves but equally in the observer's knowledge. The vocabulary of affordances has since been embraced and built upon by the HCI community. However usage has been shown to be vague and inconsistent (McGrenere and Ho, 2000) and proposed types of affordance abound, including: *perceptible*, *hidden*, *false* and *nested* affordances (Gaver, 1991); *physical*, *real* and *perceived* affordances (Norman, 1999); *cognitive*, *sensory* and *functional* affordances (Hartson, 2003). Furthermore the usefulness of affordances for thinking about practical steps forward when designing computer interfaces is dubious - and the situation is confounded by subtleties and pitfalls when designing specifically to afford *learning*.

We will return to the affordances debate in our concluding paragraphs. For now, we wish to reflect on design decisions as apparent in two products that have emerged from our own design experiments. First we shall describe these two microworlds and subsequently we shall focus on their key design elements.

CHANCEMAKER AND BASKETBALL

ChanceMaker was written as the design strand of the first author's doctoral thesis. Figure 1 shows three of *ChanceMaker*'s gadgets, mini-computational simulations of everyday random generators. The students (11 years of age) were able to simulate the throwing of the gadget either by pulling on the strength control, the black disc beneath the gadget itself, or by clicking directly on the gadget in order to replicate an experiment using the same strength as last time. The students were challenged to decide which of the gadgets were working properly and which were not. The second challenge was to "mend" the broken gadgets using the tools found by opening up the gadget. Figure 2 shows the mending tools inside the dice gadget. The other gadgets were similarly organised. The student was able to continue to play with the gadget itself as if at top-level or repeat quickly many throws of the dice. It was possible to inspect the results or indeed to graph those results. Critically, it was also possible to edit the workings of the gadget which control its behaviour.

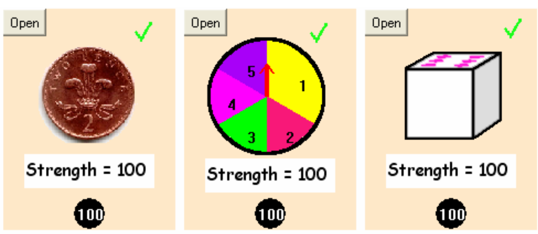

Figure 1: Three gadgets from *ChanceMaker*

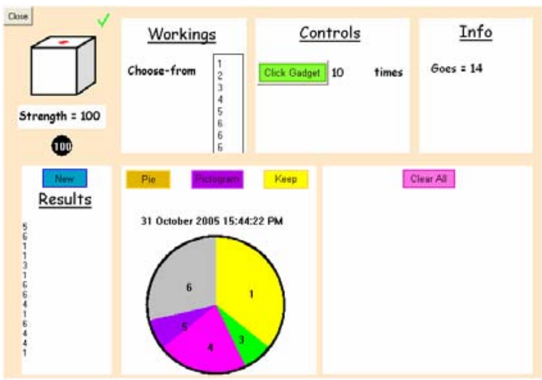

Figure 2: The workings of the dice gadget

Basketball (Figure 3) is ongoing work as part of the third author's doctoral research. The students (between 14 and 16 years) were able to control the animation of the basketball player's throws, using the sliders for release angle, speed, height and distance. Initially, the path of the ball was completely determined by the settings of the sliders. However, the student was able to switch on the error, in which case the angle would be selected from a distribution of values, centred on the position of the slider. Two arrows would appear when the error was switched on. The student was able to move these arrows to increase or decrease the spread of angles around that centre.

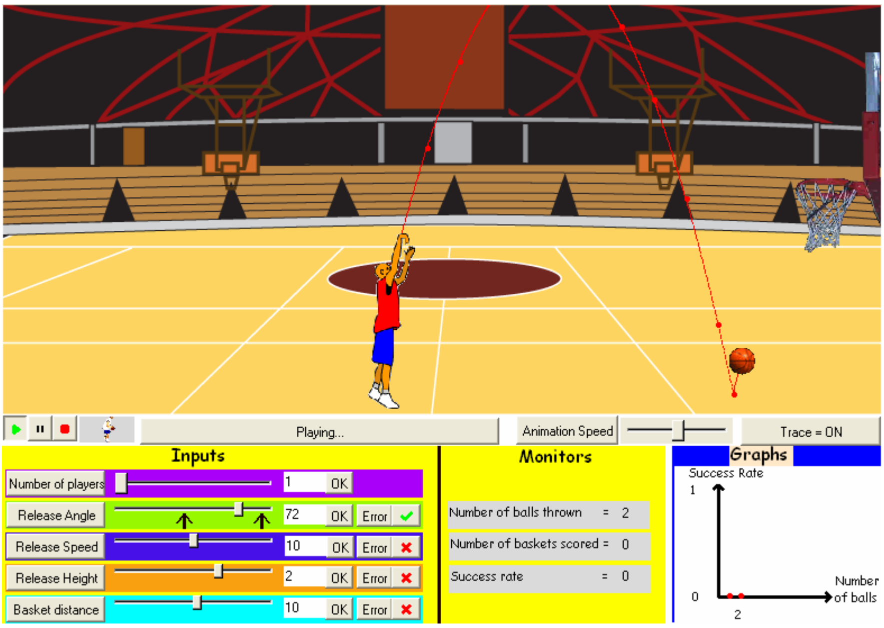

Figure 3: The basketball microworld

Indeed, the asymmetric positioning of the arrows could introduce some skewness into the distribution of throws (as in Figure 4). The student was able to inspect a linegraph of the success rate over time as well as a histogram of the frequency of angles thrown for successful throws or throws in general. In the first instance, the students were challenged to make a successful throw. This they would usually quickly complete. At that point some discussion about the realism of the animation would lead towards consideration of skill levels and the use of the error switches.

Figure 4: When the error is set to be on, the arrows can be moved to control the spread of throws

DESIGN ISSUES IN CHANCEMAKER AND BASKETBALL

Most packages developed to assist the teaching of statistical concepts offer an excellent means for exploring statistical ideas, but do so without any explicit context. For example, if a teacher wants to provide an environment in which he wants his students to explore the Central Limit Theorem, the students are likely to be offered, through any of these packages, a means for representing samples and the sampling process as well as efficient and meaningful methods for graphing the results. The focus however is very much on the statistical or mathematical concept rather than on its context of applicability. The question in focus is "What is the Central Limit Theorem?" and "How does it work?" rather than "Why would I want to know the Central Limit Theorem?" or "What is its possible value to me?" A disadvantage of decontextualising the stochastic concepts is that the students may fail to become engaged and may not develop a sense of the limitations or potentials for the concept. The statistical concept itself can seem disconnected from the rest of the student's life and may fail to arouse his curiosity.

Influenced by the Constructionists' (Harel and Papert, 1991) accent on the affective, we have been concerned to place emphasis on developing tasks in which students are likely to construct *purpose*, while at the same time coming to appreciate the *utility* of the concept (Ainley, Pratt and Hansen, 2006). Along the way, we have discovered just how hard it is to design tasks which make the linkage between purpose and utility. *ChanceMaker* was one of our more successful forays in this direction. Thus, when students used *ChanceMaker*, the task of mending the broken gadgets not only motivated them but also led in most cases to the construction of utilities for distribution and the Law of Large Numbers. The students articulated ideas such as, "The more times you throw the dice, the more even is the pie chart," or "When the workings box is fair, the pie chart is even." We recognise these sorts of statements as *situated abstractions* (Noss and Hoyles, 1996), heuristics for the behaviour of phenomena in particular narrowly defined neighbourhoods. Our claim is that these students were constructing an appreciation of how distribution and the Law of Large Numbers could be useful to them, albeit within the confines of *ChanceMaker*.

The ongoing work with Basketball has thrown up similar challenges. We opted for the Basketball context as a purposeful entry point for some important stochastic ideas. The sliders and the arrows were seen as useful in terms of how they control the behaviour of the player's throwing and we have growing evidence that the students can connect these features with notions of average and spread and the shape of the distribution. Ultimately though we want the students to recognise distribution itself as having utility and this may only be achieved when we find a task that makes the exploration of the basketball player's throwing more imperative.

The design aim is to invent contexts which appear playful and within which students want to engage for their own interest rather than just to satisfy the teacher. As a result, we believe the student will develop a sense of the utility for the concept. There are however other reasons why we think it is important to build the stochastic concept into meaningful contexts. Design experiments are, as was stated earlier, interventionist by nature. Researchers adopting this methodology set out to perturb thinking (Noss and Hoyles, 1996) by offering the student new tools to play with in order that the researcher might not only explore the evolution of the student's thinking but also the impact of the design itself. By providing a context for that thinking-in-change, we open up the possibility of researching the relationship between it and the context. This relationship is critical in the domain of stochastics. Many researchers have shown how stochastic thinking is acutely sensitive to the setting. For example, Konold (1989) has discussed the outcome approach as a tendency to adopt a non-statistical mindset under certain circumstances; Lecoutre (1992) has shown how certain statistical tasks become easier when the randomness is masked even though the mathematical structure remains otherwise identical; Johnston-Wilder (2005) has demonstrated how people's perceptions shift continuously even over the duration of a single a task on randomness. Whether researching or teaching stochastic ideas, the context should not, in our view, be ignored since otherwise the student's attention on the context is likely to remain implicit and unnoticed.

Our approach is to set the exploration in a playful context so that the students' pre-conceptions are made explicit and open to scrutiny by the researcher. In *ChanceMaker*, the students had many decisions to make. Their task was broadly defined in terms of finding out which gadgets do not work. But what did *not working* mean? The students had also to mend the gadgets. But how should a *properly working* gadget behave? This intentional vagueness left space for the students to make explicit their own personal conjectures, which might otherwise have remained private. The students were able to test out their personal conjectures and, knowing that they would learn from the feedback offered by the computer, the students were very prepared to expose their naivety. The *ChanceMaker* took this notion of testing personal conjectures one step further. We anticipated on the basis of the literature and prior iterations that students would expect how they threw the gadget to affect the outcome. Therefore we gave them the opportunity to test out that idea by providing the strength control. In fact, the strength control affects the duration of the animation of the gadget but it has no effect on the result at all. The students did not know this and often pursued explorations of whether and how the strength control affected the outcomes. The strength control is an example of what we have called a *redundant control*, a feature built into the environment in order to promote the likely rejection of a prior conjecture about behaviour within the microworld.

In Basketball, we are most interested in the students' appreciation of the relationship between determinism and the stochastic. Our conjecture is in fact that people use their notions of causality in order to make sense of the stochastic. This notion stands in opposition to the idea that determinism and the stochastic inhabit separate worlds. The playful context of Basketball allowed students to control the player through changing the sliders and, in this sense, the basketball throws were completely determined. This left open the question of how did variation in the real world occur? The variation, which could be perceived through the player's throws or examined through the frequency histograms, was generated in two ways. The students could vary the sliders and observe the animation and the histograms. Such variation caused the students no dilemma. Later when they switched the error on, they explore the situation again by changing the sliders. At first, the variation in the histograms still caused no dilemma since they assumed that it has been caused by their moving the sliders. However, they noticed that this could not entirely explain the variation and began to conjecture what had brought about the rest of the variation. Eventually, they attributed the variation to the arrows themselves. In that sense agency had transferred from their own action to the unseen actions of the arrows. The connection that they made between causality and randomness would not have happened, we believe, without the students having the space to make and test conjectures about the context.

The gadgets in *ChanceMaker* are examples of what Turkle and Papert (1995) have called quasi-concrete objects. Virtual artefacts can be manipulated like material objects but also have connection with abstract formal mathematical ideas. The workings box is another example since it is a non-conventional representation of a probability distribution and yet is editable and impacts on the operation of the corresponding gadget. The workings box in fact fuses control with representation and we noticed, as we worked with students, how they increasingly were able to use the workings box as a representation of the behaviour of the gadget without needing to use it as a control to see how it behaved. Another example of this sort of fusion is in Basketball. The sliders and arrows were initially seen as ways of controlling the throwing of the basketball. Later however the position of the sliders and arrows was sufficient for the students to understand how the player would throw and in that sense the sliders and arrows had become a representation of the act of throwing. The mechanism for this fusion is in fact what Papert (1996) has called the *Power Principle*, students coming to know through use. In contrast, in conventional classrooms, the tendency is for application to be left until very late in the curriculum, if it is tackled at all.

This use of quasi-concrete objects and the Power Principle to fuse control and representation is at the heart of what we have called *phenomenalising*. Indeed, the series of design constructs laid out above constitute phenomenalising as a design activity: the process of making activity purposeful and playful through the use of quasi-concrete objects that reflect both control over and representation of mathematical concepts allows mathematics and stochastics to be treated more like everyday activity, in which one learns about phenomena through use.

PHENOMENALISATION AND THE AFFORDANCES DEBATE

The theory of instrumental genesis has explained how the individual gives meaning to artefacts, turning them into instruments, and accommodates himself to those instruments by inventing how they might be used. The process of phenomenalisation suggests ways in which designs can aim to promote instrumentalization at the same time as instrumentation, even in the most challenging situation where the tools in some sense embody mathematical concepts. How do instrumental genesis, phenomenalisation and affordances relate to each other? The central conflict at the heart of the affordance debate is whether it is appropriate to consider objects as having attributes independently of the observer, whereas instrumental genesis places the learner at the centre of a construction process that turns a meaningless artefact into a meaningful instrument. From the design perspective, the notion of affordances nevertheless has its merits. In our experience, the designer acts as if he were indeed building these attributes into the very existence of the tools. Even as he does this, the designer knows there is every likelihood that in practice the student may not construct the concepts associated with the affordances in the mind of the designer. At the risk of adding to the panoply of types of affordances and confusing the debate even further, we would like to distinguish between *potential affordances* and *realised affordances*. Let us explain how this distinction is at least helpful to us. Based on our reading of the literature and prior iterations, we attempt to build into the tools a set of *potential* affordances, design decisions that we believe at the time are likely to impact on thinking-in-change. Iterative design provides for a mechanism by which potential affordances are tested in practice by focussing on moments of thinking-in-change that occurred when the learner interacted with the software. We refer to the observed attributes of the tool that seemed to have actual impact on thinking-in-change as *realised* affordances. Inevitably, there is a mismatch between potential and realised affordances. Indeed, our approach in the early stages of iterative design is to allow for this mismatch by keeping the investment of resources to a minimum. We begin with just a few ideas and we try them out on only a small number of students. As time goes by the mismatch typically reduces and we feel able to make a greater investment in testing. Design experiments in our experience are characterised by large changes in the design in the early stages and relatively small changes towards the end. In this sense, we think of iterative design as a convergent process in which the design becomes more stable and the mismatch between potential and realised affordances reduces.

REFERENCES

Ainley, J., Pratt, D. and Hansen, A. (2006). Connecting engagement and focus in pedagogic task design. *British Educational Research Journal*, 32(1), 21-36.

Artigue, M. (2002). Instrumentation issues and the integration of computer technologies into secondary mathematics teaching. In H. G. Weigand *et al*. (Eds.), *Developments in Mathematics Education in German-speaking Countries*, (pp. 7-17). Berlin: Fransbecker.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., and Schauble, L. (2003). Design experiments in educational research. *Educational Researcher*, 32(1), 9-13.

Gaver, W. (1991). Technology affordances. *Proceedings of CHI 91*, (pp. 79-84).

Gibson, J. (1979). *The Ecological Approach to Visual Perception*. Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Harel, I. and Papert, S. (1991). *Constructionism*. Norwood, NJ: Ablex.

Hartson, R. (2003). Cognitive, physical, sensory, and functional affordances in interaction design. *Behaviour and Information Technology*, 5(22), 315-338.

Johnston-Wilder, P. (2005). *Learners' Shifting Perceptions of Randomness*. Unpublished PhD thesis. Milton Keynes, UK: Open University.

Konold, C. (1989). Informal conceptions of probability. *Cognition and Instruction*, 6, 59-98.

Lave, J. (1988). *Cognition in Practice*. Cambridge: Cambridge University Press.

Lecoutre, M. P. (1992). Cognitive models and problem spaces in "purely random" situations. *Educational Studies in Mathematics*, 23, 589-593.

McGrenere, J. and Ho, W. (2000). Affordances: Clarifying and evolving a concept. *Proceedings of Graphics Interface 2000*, (pp. 179-186).

Norman, D. (1988). *The Psychology of Everyday Things.* New York: Basic Books.

Norman, D. (1999). Affordance, conventions, and design. *Interactions*, 6(3), 38-42.

Noss, R. and Hoyles, C. (1996). *Windows on Mathematical Meanings: Learning Cultures and Computers*. London: Kluwer Academic Publishers.

Papert, S. (1996). An exploration in the space of mathematics educations. *International Journal of Computers for Mathematical Learning*, 1(1), 95-123.

Pratt, D. and Noss, R. (2002). The micro-evolution of mathematical knowledge: the case of randomness. *Journal of the Learning Sciences*, 11(4), 453-488.

Turkle, S. and Papert, S. (1991). Epistemological pluralism and the revaluation of the concrete. In I. Harel and S. Papert (Eds.), *Constructionism*, (pp. 161-192). Norwood, NJ: Ablex.

Vérillon, P. and Rabardel, P. (1995). Cognition and artifacts. A contribution to the study of thought in relation to instrumented activity. *European Journal of Psychology of Education* 10(1), 77-101.