AN EXPLANATION OF STUDENT PERFORMANCE USING HIERARCHICAL LINEAR MODEL FOR SCHOOLS IN PERNAMBUCO, BRAZIL

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The hierarchical linear models or multilevels were developed for analysis of data which possess group structure, that is, a structure hierarchy which takes in account the data variability inside and among each hierarchical level. By using data analysis from SAEPE (2002 Educational Evaluation System of Pernambuco), hierarchical models are presented with two levels of evaluation in mathematics and Portuguese language classes applied to the 4th and 8th grades students of fundamental teaching and to 3rd grade high school students. The results in this modeling are more appropriate due to data group structure

DEFINITION OF HIERARCHICAL MODELS

According to Bryk and Raudenbush (1992), hierarchical linear model (HLM) was named by Lindley and Smith (1972) as part of their work in Bayesian estimation of linear models in which they introduced a general approach for grouped data with complex error structure. Until the middle of the 90s, the development of these models were concentrated in the areas of discrete output data, time series models, crossed classification, missing data and nonlinear models (Goldstein, 1995). The hierarchical structure is characterized by the presence of individual observation (people or objects in study) which are grouped in the highest level. The hierarchical models or multilevels provide a theoretical and conceptual ground for treatment and analysis of data with hierarchical structure or grouping. They allow the measurement of how variables at different levels affect the variable output, and to quantify how much variability there has to be at each output level. The hierarchical model involves a statistical integration of the different models specified in the levels of interest. The simple integration leads to added random coefficients in the model, where the regression coefficients of the level of the individual are considered as random variables in the level of the group, where the level of regression coefficient of an individual is derived from a probability distribution. In several studies, the hierarchical model has a different approach where hierarchy is based on the order of variable importance in the study.

OBJECTIVE

In order to show that the hierarchical models are appropriate to identify relevant factors of the student performance, considering they are influenced by inherent characteristics of each student (level 1) and by environment in which they are placed (level group-level 2).

MATHEMATICAL MODELING

The general two-level model is given by:

$$Y_{ij} = \gamma_{00} + \gamma_{10} X_{ij} + \gamma_{01} W_j + \gamma_{11} W_j X_{ij} + u_{0j} + u_{1j} X_{ij} + e_{ij}, \quad i = 1, \ldots, n_j, \; j = 1, \ldots, J.$$

where $\gamma_{00} + \gamma_{10} X_{ij} + \gamma_{01} W_j + \gamma_{11} W_j X_{ij}$ is the fixed-effects part, $u_{0j} + u_{1j} X_{ij} + e_{ij}$ is the random-effects part of the model and,

Y_{ij} is the the score of student *i* from the school *j*;

$\mathbf{X}_{ij}$ is the output variable of student *i* from the school *j*, variables of level 1;

$\mathbf{W}_j$ is the matrix of output variables of level 2;

γ_{00} is the expected value of Y_{ij}; γ_{01} is the difference measured between *j* groups;

γ_{10} is the inclination measured of the *j* groups; γ_{11} is the coefficient of the iteration term $W_j X_{ij}$;

u_{0j} is the random effect of *j*-th group on its intercept value; u_{1j} is the effect of *j*-th group on the inclination;

e_{ij} is random errors of level 1.

The adequate estimation methods for this model are Maximum Likelihood (which includes regression coefficients and variance components in a likelihood function), and Restricted Maximum Likelihood (which includes variance components only in the likelihood function).

APPLICATION

Every year, the Government of Pernambuco´s Department of Education evaluates their student's performances through the so called Educational Evaluation System of Pernambuco – SAEPE. The SAEPE consists of a Portuguese and a mathematic skill´s test. As part of the SAEPE's evaluation process, general information related to the students, their school and their respective principal´s are also collected. In this work, data from the 2002 SAEPE, regarding the 4^{th} and 8^{th} grade middle-school students, and the 3^{rd} grade high-school students, were analysed.

Hierarchical linear models were fitted to the available data set using the *R* language. In order to build the model, variables were selected using a stepwise approach. Fixed effects variables with p-values less than 10% (according to a t-test) and random effects variables with p-values less than 5% (according to a chi-square test) were considered statistically significant.

Some works using HLM in education have already been performed in Brazil like the one by Natis (2000) using student data from public schools in São Paulo (Brazil). In this study, data from SAEPE (System of Educational Evaluation of Pernambuco) from 2002 provided by the Government of Pernambuco´s Department of Education were analyzed. The SAEPE tests of Portuguese language and mathematics were applied to the students together with a personal information form. Information forms from the school and its director were also completed. The grades included in this work were from the 4^{th} and 8^{th} grades of fundamental teaching and the 3^{rd} grade of high school. Model adjustments are available in computational platforms for R language and in *HLM* (program developed specifically for adjustment of models with hierarchical structure by Bryk *et al*. (1996)), which provides very close results. In this work, Portuguese test results using R language and modeling comments from mathematical tests are shown. The dependent variable (Y_{ij}) was defined as the score of student *i* from the school *j* and, according to the variable selection criteria it was chosen as significant level of *t* test less than 10% for fixed effects and a significant level of χ^2 test less than 5% for random effects. Amongst several available variables in the database, the result from model adjustments selected as independent variables of the student performance

Level 1: Variables (students):

$Acelere_{ij}$ (for 4^{th} grade fundamental teaching and 3^{rd} grade high school students): 1 - students in the right grade; 0 - student who has pendency from previous grade;

Sex_{ij}: 1 - male; 0 - female;

$Difidad_{ij}$: student age difference when taking the test and the expected age for attending the grade (for 4^{th} grade: 9 years old, for 8^{th} grade: 13 years old, and for 3^{rd} grade high school: 16 years old);

$Entende_{ij}$: understanding level variable: 1 - understands everything or almost everything; 0 - understands little or nothing.

Level 2: Variables (school):

$Tipo_j$ (for 4^{th} grade): school classification: 1 – city public school; 0 – state public school;

$Conserv_j$ (for 8^{th} grade): school condition variable: 1 - good keeped, 0 - otherwise;

IDH_j (for 3^{th} grade medium teaching): Human Development Index of the city. (IPEA/FIBGE/FJP, 2000).

RESULTS

For 4^{th} grade fundamental students, the selected models include the same variables (and effects) for treating mathematics and Portuguese tests, chosen school classification as the significative variable of group level. The selected model is:

$$Y_{ij} = \beta_{0j} + \beta_{1j}(acerela)_{ij} + \beta_{2j}(difidad_{ij}) + \beta_{3j}(sexo_{ij}) + \beta_{4j}(entende_{ij}) + e_{ij}, \quad i = 1,.,n_j; j = 1,.,J.$$

where: $\beta_{0j} = \gamma_{00} + \gamma_{01}(tipo_j) + u_{0j}$; $\beta_{1j} = \gamma_{10} + u_{1j}$; $\beta_{2j} = \gamma_{20}$; $\beta_{3j} = \gamma_{30}$ e $\beta_{4j} = \gamma_{40}$

The Portuguese evaluation results are presented in Table 1. This table shows that keeping fixed all but one factor, the city's public schools have scores decreased by 1.706 points. Students in acceleration process show a performance 10.186 points lesser than students attending the right grade. All other estimation parameters show performance differences according to gender, schooling deficit and understanding level in class.

Table 1: Estimation parameters of the model for Portuguese scores, 4th grade fundamental teaching

Fixed Effects					Random Effects		
Effects	Estimation	SE (1)	IC_95% (3)	p-value	Effects	SD (2)	IC_95% (3)
β_{0j} –intercept1 γ_{00} –intercept γ_{01} – tipo	 37,527 -1,700	 1,224 0,614	 35,13; 39,93 -2,91; -0,49	 0,000 0,059	u_{0j}– intercept1	5,393	3,46; 8,41
β_{1j} –acelera γ_{10} –intercept	 10,186	 1,091	 8,05; 12,32	 0,000	u_{1j} – acelera	3,853	1,63; 9,12
β_{2j} –difidad γ_{20} –intercept	 -0,808	 0,066	 -0,94; -0,68	 0,000	e_{ij} – level 1	16,28	16,09; 16,47
β_{3j} –Sex γ_{30} –intercept	 -3,506	 0,278	 -4,05; -2,96	 0,000			
β_{4j} –entende γ_{40} –intercept	 4,322	 0,331	 3,67; 4,97				

(1) SE- Standard Error; (2) SD- Standard deviation; (3) IC_95%- Interval with 95% of confidence

For 8th grade students of elementary school, the selected models include the same variables (and effects) for mathematics and for Portuguese tests, selecting the school condition variable as the significative one for the group level, and in mathematics tests, school classification was also considered. The select model for Portuguese tests are presented in Table 2 is:

$$Y_{ij} = \beta_{0j} + \beta_{1j}(difidad_{ij}) + \beta_{2j}(sexo_{ij}) + \beta_{3j}(entende_{ij}) + e_{ij}, \quad i = 1,\dots,n_j, j = 1,\dots,J.$$

Where: $\beta_{0j} = \gamma_{00} + \gamma_{01}(conserv_j) + u_{0j}$; $\beta_{1j} = \gamma_{10} + u_{1j}$; $\beta_{2j} = \gamma_{20}$ e $\beta_{3j} = \gamma_{30}$.

Table 2:Estimation parameters of the model for Portuguese scores, 8th grade fundamental teaching

Fixed Effects					Random Effects		
Effects	Estimation	SE (1)	IC_95% (3)	p-value	Effects	SD (2)	IC_95% (3)
β_{0j} –intercept1 γ_{00} –intercept γ_{01} – conserv	 46,508 0,899	 0,516 0,450	 45,50; 47,52 0,01;1,78	 0,000 0,046	u_{0j}– intercept1	6,822	6,08; 7,66
β_{1j} –difidad γ_{10} –intercept	 -0,997	 0,041	 -1,08; -0,92	 0,000	u_{1j} – difidad	0,562	0,47; 0,66
β_{2j} –Sex γ_{20} –intercept	 -3,954	 0,202	 -4,35; -3,56	 0,000	e_{ij} – level 1	14,66	14,52; 14,80
β_{3j} –entende γ_{30} –intercept	 5,587	 0,206	 5,18; 5,99	 0,000			

(1) SE- Standard Error; (2) SD- Standard deviation; (3) IC_95%- Interval with 95% of confidence

In the case of 3rd grade high school students performance, the selected level 1 variables are the same as the 4th grade, and at level 2 the selected variable was IDH of the city for Portuguese tests. As shown in data of Table 3 the select model is:

$$Y_{ij} = \beta_{0j} + \beta_{1j}(acerela)_{ij} + \beta_{2j}(difidad_{ij}) + \beta_{3j}(sexo_{ij}) + \beta_{4j}(entende_{ij}) + e_{ij}, \quad i = 1, \ldots, n_j,\ j = 1, \ldots, J.$$

Where: $\beta_{0j} = \gamma_{00} + \gamma_{01}(IDH_j) + u_{0j}$; $\beta_{1j} = \gamma_{10} + u_{1j}$; $\beta_{2j} = \gamma_{20}$; $\beta_{3j} = \gamma_{30}$ e $\beta_{4j} = \gamma_{40}$

Table 3: Estimation parameters of the model for Portuguese scores, 3th grade medium teaching

Fixed Effects					Random Effects		
Effects	Estima-tion	SE (1)	IC_95% (3)	p-value	Effects	SD (2)	IC_95% (3)
β_{0j} –intercept1 γ_{00} –intercept γ_{01} – IDH	 23,611 6,181	 1,968 2,663	 19,75; 27,47 0,94; 11,42	 0,000 0,021	u_{0j}– intercept1	2,656	1,66; 4,26
β_{1j} –acelera γ_{10} –intercept	 1,859	 0,373	 1,13; 2,59	 0,000	u_{1j} – acelera	2,742	1,69; 4,45
β_{2j} –difidad γ_{20} –intercept	 -0,266	 0,024	 -0,31; -0,22	 0,000	u_{2j} –difidad	0,189	0,14; 0,26
β_{3j} –Sex γ_{30} –intercept	 -1,746	 0,212	 -2,16; -1,33	 0,000	e_{ij} – level 1	12,11	11,97; 12,26
β_{4j} –entende γ_{40} –intercept	 5,279	 0,213	 4,86; 5,70				

(1) SE- Standard Error; (2) SD- Standard deviation; (3) IC_95%- Interval with 95% of confidence

CONCLUSION

Analyzing level 1 variables from the selected models, it can be concluded that the pending grade students always have a poorer performance if compared with the regular ones who attend the right grade.

The students with age-grade imbalance have a tendency to score poorly. In relation to gender, female students taking Portuguese tests have a better perfomance than the male ones. In mathematics tests, male students perform better than female students. The students who admit to understand little or nothing in class present poorer performance than ones that understand everything or almost everything, mostly in Portuguese tests.

Considering level 2 selected variables, school classification is more important for 4th grade students (city public school students have poorer performance than state public school students); for 8th grade, students tested in mathematics, school classification and school condition are important, while for students tested in Portuguese only school condition matters; for 3rd grade high school students tested in Portuguese, an increase in city IDH where school is localized, produces an increase in student performance.

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