ANALYSIS OF DIDACTIC SITUATIONS SUGGESTED TO DISTINGUISH DISJUNCTIVE EVENTS AND INDEPENDENT EVENTS

Adriana D'Amelio de Tari and Angela Diblasi
Universidad Nacional de Cuyo, Argentina
adamelio@fcemail.uncu.edu.ar

The concepts of disjunctive events and independent events are didactic ideas that are daily used in the classroom. Previous observations of attitudes and assessment given to students at university level who attend the introductory Statistics course have helped detect the confusion between disjunctive events and independent events, and indicate the spontaneous ideas that students tend to elaborate about both concepts in the different situations in which these notions have to be considered. However the relation between these ideas and their formal definitions is not known in detail. In this work, we use Didactic Engineering as methodology to analyze students' misconceptions, their persistence, and the process by which the student confronts his misconceptions by applying theoretical concepts. The aim is to improve the teaching of these topics.

INTRODUCTION AND CONTENT

This work, carried out with 2nd year students from Statistics I in Accountancy, is an analysis to the responses to a problem were there is the concept of mutually exclusive events and independent showing the confusion they have about these concepts.

According to Sanchez (1996, Ph.D. thesis about independent events) the problem starts with:

- the beliefs that independent events and disjunctive events are the same;
- confusion between independent events and independent experiences.

Besides, it is understood that independence is only quantitatively proved by the product rule.

These concepts are simple as they are defined. However it was proved through interviews at other colleges that the confusion persists in many university students attending Statistics I. The phenomenon appears in students with different Mathematics backgrounds.

Are there techniques of teaching and learning good enough to take into account the spontaneous concepts of the probability notions while developing their formal knowledge?

Some studies about attitudes and responses in exams indicate that students use intuitive ideas to analyze independent events and mutually exclusive events in different situations where these notions play a role. But the relationship between these intuitive concepts and the formal definitions is not known.

HISTORY AND EPISTEMOLOGY

The concept of independence emerges in the analysis of hazard games "without replacement" given by De Moivre (1718-1756) and by Bayes (1763). Before them Bernoulli had used this concept to show his theory without realizing it.

There was no change with the intuitive concept of independence with the improvement made by Laplace and Moivre. The concept of independence was understood only in the context of independent experiences as is shown with the definitions of classic authors as De Moivre (1756):

> "Two events are independent when there is no connection between them and what happens in one of them does not occur in the other one."
>
> "Two events are dependent when they are connected in such a way that the probability that one occurs is altered by the occurrence of the other."

Laplace does not define in an explicit way independent events and their properties. In this period drawing with and without replacement in successive trials was identified with independent and dependent events respectively. At present difficulties derive from classic authors' concepts.

Von Mises (1964, p. 38) rejects the formal definition of independence. He considers that in the axiomatic theory of Kolmogorov there are events that are independent but are not seen as independent one of each other, in the intuitive sense that "they do not influence each other."

> "When two characters are considered to influence each other or not, it is given a notion of independence. Nevertheless, a definition based on the multiplication rule is no more than the weak generalization of a concept full of meaning."

This problem of the inversion of content and the mathematics definition plays an important role in the teaching process. In some books the deduction of the independence formula appears as a consequence of conditional probabilities. This generates considerations of analogy with the theorem of addition.

Boge says: "The difficulty reflects the relation between mathematics and reality."

The difficulties appear in the historic development of the concept. During the eighteenth century, the task was to legitimate and delimit the object of mathematical studies; the most diverse methods were accepted to analyze this object. In the nineteenth century, the relation was inverted. The object became arbitrary, and the task was to confirm the methods and define strict procedures to allow the abstraction of the objects and so, an extension of the applications. Feller (1983) comments:

Generally the correct intuition that certain events are stochastically independent is felt, because if it is not like that, the probability model would be absurd. Nevertheless [...] there exist situations in which the stochastic independence is discovered just from calculus. (p. 137)

Turan-Turan (1996) says:

"*the difficulty is due to that in the first definition, two or more random experiments are considered, while the present definition, in elementary texts, only considers events from the same sample, generally associated with a unique experiment.*"

Steinbring analyzes the historic development of stochastic independence from an epistemological perspective, to find elements for a didactic perspective. In the historical development there is an inversion of content of the concept and its mathematics definition.

- Firstly, there is an association of concrete representations of the dependency of real facts.
- Secondly, the concepts have been defined formally in mathematics by the multiplication rule.

These statements are usually not properly connected. Consequently, it may produce a confusion about the concepts of independence (or dependence).

METHODOLOGY

An exam was developed on the topics of probability and random variables. In the first exercise there were concepts of probability organized in items. One of them included the concepts of mutually exclusive and independent events. The students were asked if the proposition was false or true and to justify their answer. The statement was false.

The analyzed exercise is the following:

Let (S, P) be a probability space, A and B events in S so that P(A) >0 and P(B)>0. Decide whether the following statement is true or false. If it is true, justify your conclusion; if it is false, state the right expression.

"If A and B are mutually disjunctive, the probability of at least one of them occurring is: P(A).P(B)."

RESPONSES

Among the responses, the student is given the total mark for the exercise if he or she answers "false: and writes the correct expression: that is, the following:

"The probability that at least one of them occurs, when the events are disjunctive, is:

$$P(A \cup B) = P(A) + P(B)."$$

Table 1 shows the results of the 97 students.

This apparent easy response was answered correctly by 14 students, of whom 10 passed. As it is seen, 12 students do not answer, of whom only 4 passed. Incorrect, with response true, 14. Responses F but with wrong justification or without justification and had no total mark were 57, only 19 students passed.

Our target is to analyze the false answers which were wrongly justified.

student	correct	no answer	False not justified	False justified wrong	False justified regular	True justified	True not justified	Total
passed	10	4	1	14	5	3	0	37
failed	4	8	6	26	5	7	4	60
Total	14	12	7	40	10	10	4	97

Table 1

ANALYSIS OF RESPONSES OF THE STUDENTS

An analysis was made of the responses of each student, particularly of the 40 that considered F and justified incorrectly. Nearly half of them (17) justified it in this way:

$A \cap B = \varnothing \Rightarrow$ *the probability of occurrences is given by P(A).P(B); that is*

$$P(A \cap B) = P(A).P(B).$$

This was the most repeated mistake in the justifications.

The confusion of the concepts followed the patterns proposed by Sanchez. The question was not direct, which can produce misunderstanding. This shows that the students make mistakes systematically because they do not have a clear concept of both notions.

A second type of response that was presented was:

If A and B are disjunctive $A \cap B = \varnothing$ *then* $P(A)= 1-P(B)$ *or* $P(B) = 1 - P(A)$; *then*

$$P(A) . P(B) = P(A \cap B) = \varnothing$$

The third answer that also was present with several students was:

$$P(S) = P(A)+P(B).$$

We can say that the second and third responses are associated because the students consider that the sample space is formed by two sets. They do a Venn diagram including these sets in *S*. According to Duval there is a problem in the translation from graphs to symbols. Students represent one thing and write another.

This problem of symbol representations is repeated among the answers that we can associate because of the wrong symbolic representation. Half of the students wrote $P(A \cap B) = \varnothing$.

Other students wrote $P(A) \cup P(B)$, $A \cap B = 0$.

They tend to misunderstand symbols. On the one hand they considerer union of probabilities. On the other hand they associate the empty set with the number 0 (zero). They equate the probability of intersection of disjunctive events to the empty set, and they equate the intersection set to zero. It is recurrent in students from all the carriers.

SOME CONCLUSIONS

The concepts of disjunctive events and independency persist in the students in a mistaken way. In a way, these come from games of chance but they have a more complex relationship in probability calculus. From the historic studies with De Moivre (1756) a wrong concept of independent events may be inferred, if it is not analyzed exhaustively: "*Two events are independent when they have no connection to each other and what happens to one does not affect the occurrence of the other.*"

When we say they have *no connection*, we are talking improperly; this persists at present. The difficulty in the case of independence is to place the concepts in opposition. On the one hand, there is a theoretical mathematical definition. On the other hand, there are numerous intuitive representations.

The symbolic representation associated with the graph presents difficulties too. Although the students know the formal symbolic definition of each concept separately, in exercises such as the ones we have analyzed, they cannot distinguish one from the other.

Teachers must be conscious that the idea of independence has a meaning only in a probability context while the one of disjunctive events may be considered with no knowledge of this.

REFERENCES

Feller, W. (1983). *Introducción a la Teoría de Probabilidades y sus Aplicaciones* (vol. 1). México: Limusa-Wiley.

Guzmán, I.(1999). *Fundamentos Teóricos de la Didáctica de las Matemáticas*. Lecciones para un curso del Programa de Magister ECDIMAT (Magister en Enseñanza de las ciencias con mención en Didáctica de la Matemática).

Hernández, R. and Joffre, M. (2000). C*oncepciones de los Estudiantes de Educacion Superior Acerca de la Nocion de Limite.*Tesis de Magister en Matemática,Mención Enseñanza de la Matemática República bolivariana de Venezuela Ucla-Upel-Unexpo.

Kilpatrick, J., Gómez, P. and Rico, L. (1995). *Educación Matemática.* Iberoamericana: Grupo Ed.

Sánchez, E. (1996). *Conceptos Teóricos Ideas Espontáneas Sobre la Noción de Independencia Estocástica en Profesores de Bachillerato: Un Estudio de Casos.* México: Tesis de Doctorado, Departamento de Matemática Educativa, Cinvestav-IPN.

Sánchez, E. (1996). Dificultades en la comprensión del concepto de eventos independientes. In F. Hitt (Ed.), *Investigaciones en Matemática Educativa*, (pp.389-404). México: Grupo Ed. Iberoaméricana. Departamento de Matemática Educativa, Cinvestav-IPN.

Sánchez, E. (2000). Investigaciones didácticas sobre el concepto de eventos independientes en probabilidad . *Recherches en Didactique des Mathématics*, 2(3), 305 - 330.

Steinbring, H.(1986). L´Independence stochastique. *Recherches en Didactique de Mathématiques*, 7(3), 99-118.