THE ROLE OF CONTEXT IN THE DEVELOPMENT OF STUDENTS' INFORMAL INFERENTIAL REASONING

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The role of context is discussed in the setting of an extended curriculum development and research project in primary school designed to develop and study students' reasoning about statistical inference. Qualitative research methods are used to critically dissect the roles of context in the emergence of sixth grade students' informal inferential reasoning (IIR). Context is examined as part of a complex network of themes, such as inquiry, norms, knowledge of statistical concepts and tools, beliefs and expectations, and meaning making and explanations. The paper analyzes and discusses these themes and the role context plays in the emerging inferential reasoning of these students.

OVERVIEW

In this paper we study the role of context in learning to make Informal Statistical Inferences (ISI), which are probabilistic (non-deterministic) generalizations from data (Makar & Rubin, 2009). We briefly present a case study of a small group of sixth graders (age 12) working within an inquiry-based and technology-rich learning environment that was designed to promote students' Informal Inferential Reasoning (IIR, Ben-Zvi, Gil & Apel, 2007), the reasoning that underlies ISI. We briefly review the literature on IIR and context in statistics education. We use qualitative analysis methods to describe the role of context in the students' emerging IIR. When we discuss the results, we suggest that the classical distinction between data and context, although useful, represents only a partial picture of the complex processes of statistical reasoning.

LITERATURE REVIEW

This review presents two main issues: Informal Inferential Reasoning (IIR) and context in a statistical investigation. Reasoning about data and context in various stages of a statistical investigation are an integral part of IIR, the reasoning processes that lead to generalizations from a sample to a population. In drawing statistical inferences, statisticians employ their real-world experience and use statistical and other knowledge bases to conjecture about the characteristics of the population beyond the data at hand and try to explain their actions and findings (Wild & Pfannkuch, 1999).

Informal Inferential Reasoning

Statistical inference aims to draw conclusions about a particular population from a sample while taking into consideration the variability in the data. Statistical inference "adds an emphasis on substantiating our conclusions by probability calculation" (Moore & McCabe, 2006, p. 382). Difficulties in studying formal inference in tertiary and high school courses that are abundant in the statistics education literature (e.g., Garfield & Ben-Zvi, 2008) have motivated suggestions to consider *Informal Statistical Inference* (ISI) as an overarching concept for pre-formal statistics instruction (Ainley & Pratt, 2008). ISI can become a "bridge" between exploratory data analysis (EDA) and formal statistical inference (Ben-Zvi et al., 2007). While the purpose of EDA is unrestricted exploration of the data, searching for interesting patterns in the data, and about the data at hand, the purpose of statistical inference is to answer specific questions from a sample regarding the population from which the sample was drawn. In drawing inferences informally, the emphasis is on reasoning from distributions of data and comparing them (Pfannkuch, 2006), conceptual understanding and the provision of explanations, rather than on statistical procedures and mathematical calculations. In addition, an informal statement of confidence in the inference can replace the formal confidence interval.

Informal Inferential Statistical Reasoning (IIR) refers to the cognitive activities involved in informally drawing conclusions (generalizations) from data (samples) about a 'wider universe' (the population), while attending to the strength and limitations of the sampling and the drawn inferences (Ben-Zvi et al., 2007), and "articulating the uncertainty embedded in an inference"

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(Makar & Rubin, 2009). Rubin, Hammerman and Konold (2006) relate to IIR as statistical reasoning that involves consideration of multiple dimensions: properties of data aggregates, the idea of signal and noise, various forms of variability, ideas about sample size and the sampling procedure, representativeness, controlling for bias, and tendency. Bakker, Derry and Konold (2006) perceive the meaning of statistical inference in a broader way, and allow more informal ways of reasoning to include human judgment based on contextual knowledge.

Context in a Statistical Investigation

Context is used in the educational literature in multiple ways referring to the educational setting, learning environment, reality or background that is represented in a problem (the problem context), and more. In this paper, we refer mainly to contextual knowledge in the meaning of the knowledge about the real world accessed by students during IIR (Gal, 2002). According to Wild and Pfannkuch (1999), a constant shuttle between the statistical and the context spheres exist in the statistical investigation process. This shuttle is apparent in each stage of the investigation. For example, in the formation of a statistical question, the way from the context sphere to the statistical sphere is accompanied by interrogation of the background of the problem, while the passage from the statistical sphere to the context sphere might involve questions that promote explanations, such as, "what does this mean or why is this happening?" (p. 228).

In the process of making meaning during a data investigation, students are involved in many contextual related activities, such as, reflecting on their real-life knowledge and experience and using it to support their inferences and accessing and evaluating beliefs they hold about the real-world. Although researchers stress the importance of students investigating real-world scenarios to help them reason about data and make it easier for them relate their findings to the real phenomena (e.g., Cobb, 1999; Pratt et. al., 2008), this is by no means an easy task for students (Konold & Higgins, 2003). For example, Masnick, Klahr and Morris (2007) studied young students' understanding of error and data variability in three different learning contexts that differed by their previous knowledge (their expectations of causal factors of phenomena). The researchers found out that when young students poorly understood the context of the scientific investigation, they had difficulty in interpreting the data, particularly when the interpretation of the data contradicted their beliefs. However, about half of the participants in their study developed theoretical explanations for the data and speculated on potential causes for the variation in the data. In the current study, we further focus on students' use and consideration of context in IIR processes, which are sometimes not apparent in their final inference statements.

METHOD

Our study aims at examining the nature and role of context in the emergence of students' informal inferential reasoning (IIR). In the context of open-ended, inquiry-based learning environment for sixth graders, we ask: *What role does context play in IIR?*

In our *Connections* Project (grades 4–6, 2005–2007), we studied students' evolving ideas of statistical reasoning within an empirical extended statistical inquiry cycle in a computerized learning environment. Students actively experienced some of the processes involved in experts' practice of data-based inquiry by working on data scenarios, and investigated via peer collaboration and classroom discussions. The sixth grade learning trajectory (Gil & Ben-Zvi, 2007) provided ample opportunities for students throughout the five-week intervention to account for, describe, explain and argue as they made informal inferences. Students generated and (re)formulated the questions they wished to investigate about a population, (re)formulated hypotheses, analyzed additional samples of data, interpreted the results and drew conclusions about the population. A central feature of the learning environment was the use of *TinkerPlots* (Konold & Miller, 2005). Students investigated data that was collected from all students in grades 6 and 7 in their school using a 17-item questionnaire about gender and age, issues related to transfer from primary school to middle school (e.g. homework load), and sports (e.g., long jump results, favorite sport). A pedagogical decision was made in order to nurture students' reasoning about sampling and randomness: hide the population data (n=206) from students but allow them to sample randomly from this population to make inferences.

In the current study we follow in great detail Odi, Eli and Asi (pseudonyms, males, age 12) who are very good mathematics students (Asi was a gifted student) in a science-focused magnet primary school in Haifa, Israel. They participated in the *Connections* lessons in fourth and fifth grades. These previous encounters made them fluent with the software and basic informal statistical ideas, language, skills and perspectives. Articulate students were chosen to provide a rich source of information about their learning and reasoning as they worked independently (Patton, 2002). In the first investigation the three students compared homework loads of sixth and seventh graders in two different random samples. Unsatisfied with the triviality of this investigation, they decided to look for a more challenging and interesting research topic: a comparison of long jump results between sixth and seventh graders in relation to favorite sports. They randomly drew two samples (n=20) and analyzed them.

To examine the nature and role of context in IIR, students' inquiry sessions were fully videotaped and transcribed. The analysis of the videotapes was based on interpretive microanalysis (Meira, 1998): a qualitative detailed analysis of the protocols, taking into account verbal, gestural and symbolic actions within the situations in which they occurred. The goal of such an analysis is to infer and trace the development of cognitive structures and the sociocultural processes of understanding and learning. Explanations and informal inferences that included contextual considerations were carefully identified by us. We discussed, presented, and advanced and/or rejected hypotheses, interpretations, and inferences about the students' cognitive structures. Advancing or rejecting an interpretation required: (a) providing as many pieces of evidence as possible (including past and/or future episodes and all sources of data as described earlier) and (b) attempting to produce equally strong alternative interpretations based on the available evidence.

RESULTS

Odi, Eli and Asi compared long jump results between sixth and seventh graders in relation to favorite sports. As they interpret the sample data, they encounter a surprising finding that seems unreasonable to them: sixth graders jump farther than seventh graders. Triggered by this unreasonable result that contradicts their contextual knowledge and expectations, Asi suggests gender as a possible confounding attribute. Their discovery that there are more girls in the seventh grade group is taken by them as a possible explanation for the conflicting situation, one which does not remove their sense of doubt. In a response to a researcher's question about the unreasonable results, Asi explains: *Because the sixth grade long jump mean can't be greater than the seventh grade mean, even though we can explain that: boys probably jump farther than girls, and there is only one girl in the sixth grade sample*.

Asi seems able to link the research question, the evidence provided by the data, and their conclusion. He first acknowledges the conflict between the sample data and his contextual assumption – seventh graders should jump farther than sixth graders. He then suggests a resolution to this puzzle by inspecting gender and making a claim based on another contextual assumption that boys jump farther than girls. Throughout this explanation he is making meaning by comparing his assumptions about the world with the data interpretation. The purpose for him and his peers is to make sense of the investigation and present evidence and a reasonable explanation for their inferences in relation to their contextual knowledge. The group further explained their expectations when they presented their inquiry in front of the class: *Our hypothesis was that seventh-graders jump farther because they are apparently stronger and bigger* (Odi)… *And they are also more experienced than us. For example, they practiced last year, like we did and they also practiced this year. Therefore, we thought they'll jump farther* (Asi).

Asi's attempt above is to resolve a conflict between his expectation based on contextual knowledge and a suggested inference by indicating the small number of girls in the sixth grade sample. His explanation combines contextual (*boys probably jump farther than girls)* and data-based statistical arguments (*there is only one girl in the sixth grade sample*). The explanation is *contextual*, because it refers to his contextual knowledge, that assumes that girls jump a shorter distance than boys. The explanation is *data-based* in that they checked the distribution of gender across the grades. It is *statistical* in the sense that it focuses on issues of samples and sampling, and discusses a possible confounding factor. While the first concepts were the topic of instruction in sixth grade, confounding was an unfamiliar idea to students and was "reinvented" by Asi here (in the sense of diSessa's reinvention, 2008). This explanation and their continuing sense of doubt led them to take a second random sample to try settling the unresolved conflict between data and their contextual expectations. However, the second random sample that had almost equal number of girls between the groups did not resolve the conflict since the mean difference remained the same. Thus, the inference that sixth graders in their school jump farther than seventh graders was reinforced, as Asi explains later: *It doesn't make sense even more if we know that girls jump to a lesser distance. One can base it on.., it even doesn't make more sense if in grade 7 that their mean is lower… it could be explained if they had more girls* (Asi).

The three students kept asking 'why' until the conflict was resolved by providing new abductive explanation (e.g., the awkward explanation that of sixth graders are physically fitter than seventh graders). In these examples, context plays an important part in students' explanations, as they compare the results to their previous knowledge and expectations and try to resolve the conflict, find causes and meaning for the investigated phenomena and data, and increase their confidence in their informal statistical inferences.

DISCUSSION

We used here a microgenetic interpretive analysis to identify, analyze and discuss the role of context in the emerging statistical understandings of sixth grade students as they developed their ideas and skills about IIR. Context had a role in sense making of the sample data, in resolving conflicts between expectations and data, and served as a vehicle to break through unclear or contradicting points in understanding of a graph. An unclear trend in the data or a conflict between data and context was resolved in many cases by a contextual-based explanation (abductive explanation, see Gil & Ben-Zvi, in press) and was substantiated sometimes by a statistical argument. However, this study provides some preliminary evidence that students' explanations, when they solve a statistical problem or investigate a research question, are sometimes embedded not in the statistical sphere or in the contextual sphere but rather in a combination of the two in a way that can't be easily separated. For example, Asi's explanation of the presence of one girl only in sixth grade, can explain the unreasonable long jump results in the first sample. Similarly, it has been suggested that the dichotomy between context and statistics can be resolved through the lens of "space of reasons" (Bakker et al., 2008).

Students' explanations as well as their use of context and previous knowledge and experiences differed significantly between groups, not presented in this short report. Several explanations come to mind: (1) the varying level of motivation, involvement and interest of the students in the inquiry topic, (2) the learning style of a student which affects the dynamics of the inquiry in the group; and (3) the norms and working habits can lead to different learning ecologies between groups or even classes. For example, the norm of listening to a peer and presenting clarifying questions, can enhance verbalization of relatively profound and diverse statistical reasoning.

Drawing conclusions that contradict previous knowledge and conceptions is a challenge for children as well as adults (cf., Batanero et al., 1996). We have demonstrated this challenge by providing a description of Asi's struggle to overcome his contextual assumptions (that girls jump to a lesser distance then boys) when gender could not continue to explain the difference between the sixth and seventh grades long jump results.

Several questions are prompted by this study: What makes context a part of a student's explanations and reasoning tools? Is it a way of thinking, to see the relevance of one's world to the data, or is it induced by a true interest in the data to reveal a story about the real world? What will make one student believe the data and another check it thoroughly by initiating statistical procedures and providing explanations or further substantiation? Does this reflect a level of thinking or level of understanding of statistical concepts? Some of the uses of context seen in this study are by no means the type one would expect to come from sixth graders regularly. We speculate they are embedded in a web of settings that include the design of the learning environment, norms and habits of work and inquiry, motivations and interest, strong sense of purpose, the technological tool used, etc. These circumstances can allow for growth of statistical reasoning and explanations and deserve further investigation, which is beyond the scope of the current paper.

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