MODELS OF TEACHER PREPARATION DESIGNED AROUND THE GAISE FRAMEWORK

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Statistical literacy is a must have competency for any citizenry to thrive in an information rich society. Sound statistical reasoning skills take time to develop and cannot be honed in a single general-purpose statistics course. To acquire proficiency in statistics foundational concepts should be introduced and nurtured in the elementary grades, and strengthened and expanded throughout the middle, high school and post-secondary grades. In this paper, examples will be presented that illustrate developmental learning trajectories for statistical concepts as proposed in the American Statistical Association's Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report: A Pre-K-12 Curriculum Framework (2007). The GAISE Report describes a cohesive and coherent framework for statistical education at Grades PreK-12 in the United States. Models of teacher preparation that integrate both the content and pedagogical knowledge as proposed in the GAISE Framework will be described.

INTRODUCTION

"Every high-school and college graduate should be able to use sound statistical reasoning to intelligently cope with the requirements of citizenship, employment, and family and to be prepared for a healthy and productive life." This quote from the *GAISE* Report (Franklin et al., 2007) sets the foundation for why statistical literacy is essential for all. Utts (2003) proposed seven important topics that provide benchmarks for statistical literacy including: knowing when is it appropriate to make cause and effect inferences, knowing the difference between statistical significance and practical significance, recognizing sources of bias in surveys, and understanding variability is natural and that 'normal' is not the same as 'average'. Garfield and Ben-Zvi (2008) define statistical literacy as "understanding and using the basic language and tools of statistics; that is, knowing what basic statistical terms signify, understanding the use of simple statistical symbols, and recognizing and being able to interpret different representations of data."

Statistical literacy cannot be achieved by a single experience in one introductory general-purpose statistics course. The development of sound statistical reasoning skills takes time and requires that foundational statistical concepts be introduced and nurtured in the elementary grades, and strengthened and expanded throughout the middle, high school and post-secondary grades. In order to provide every high-school graduate with sound statistical reasoning skills, the traditional path of preparing and leading all students to calculus must be rethought. The mathematician Arthur Benjamin (2009) argues, "Calculus is the wrong summit of the pyramid to direct our students in mathematics. The correct summit should be probability and statistics. Very few people use calculus in meaningful ways in their daily lives. But statistics is used in everyone's daily lives." A recent newspaper article in the New York Times (2009) had as its headline, "For Today's Graduate, Just One Word – Statistics". As noted in the article, "We're rapidly entering a world where everything can be monitored and measured. But the big problem is going to be the ability of humans to use, analyze and make sense of the data." This paper describes how a framework proposed in the *GAISE* Report promotes the development of statistical literacy and discusses issues associated with preparing teachers to teach statistics in the schools.

OVERVIEW OF THE GAISE REPORT

The goals and objectives of the *GAISE* framework are to: (1) promote and develop statistical literacy, (2) discuss the differences between mathematical and statistical thinking, (3) clarify the role of probability in statistics, (4) illustrate concepts associated with the data analysis process, and (5) present the statistics curriculum for grades Pre-K-12 as a cohesive and coherent curriculum strand. Statistics focuses on variation in data and, in contrast to mathematics, context is critical in statistical problem solving. The primary role of probability in statistics is that of randomization – a sample is *selected at random* from a population or units are *randomly assigned*

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to treatment groups in an experiment. *GAISE* identifies three developmental levels for learning statistic in the schools (Levels A, B, and C) and points out that students must progress through these levels in order to develop sound statistical reasoning skills. *GAISE* also describes a conceptual structure for statistics education as a two-dimensional framework model with one dimension defined by the statistical problem solving process (Formulate a question that can be addressed with data, Design and implement a plan for collecting data, Analyze the data, and Interpret the data) together with the nature of variability in data, and the second dimension being comprised of the three developmental levels.

THE NEED TO PREPARE TEACHERS TO TEACH STATISTICS

Over the past 30 years there has been increased emphasis on data analysis, statistics and probability in school curricula and standards internationally. Consequently, there is an increased expectation for teachers to both teach and assess learning in probability and statistics. *The Mathematical Education of Teachers (MET)* report (Conference Board of Mathematical Sciences, 2001) notes that teachers should gain "both technical and conceptual knowledge" of the statistics and probability content that appears in the curriculum for their students and that secondary teachers, in particular, need to "appreciate and understand the major themes of statistics" Unfortunately, few teachers today in United States are adequately prepared to teach statistics in the schools as envisioned by the *GAISE* Report or as recommended in the *MET* Report. How do we address the issue of preparing teachers to successfully teach statistics in the schools?

The *MET* report emphasizes the necessity of teacher education being a shared responsibility between mathematical scientists and education faculty. Franklin and Mewborn (2006) suggest that this collaboration be expanded to include statisticians and discuss the importance of building a nucleus of teachers who can effectively teach data analysis by improving the preparation programs for both pre-service and in-service teachers. They also promote the development of statistics courses for teachers where content, pedagogy, and assessment issues are an integrated part of the course. Ideally, these courses would be taught in collaboration with statisticians. Ball (2003) proposes that the mathematical knowledge needed for teachers must be based upon three principles: (1) teachers need to know what we would expect educated members of society to know and even more, (2) knowledge for teaching mathematics is not the same as the mathematical knowledge needed for other mathematics based professions, and (3) the mathematical knowledge needed for teaching must be usable for such challenges as interpreting a student error, using multiple forms to represent a mathematical idea, and developing alternative explanations. We believe these same principles apply to the statistical knowledge needed for teachers of statistics.

Teaching statistics is challenging for most of K-12 mathematic teachers in the United States who are struggling with fundamental notions of teaching, learning, and the nature of statistics as a discipline. One of the concerns often expressed by K-12 teachers is a lack of sufficient statistical content knowledge that will allow them to address questions from students. Based on our experiences in preparing teachers, this perceived inability to apply statistical knowledge to different contexts (where context is crucial in statistical thinking) and not being able to predict what students' questions will be, is a major issue for these teachers.

A CONJECTURED LEARNING TRAJECTORY FOR TEACHERS OF STATISTICS

An important component of successful teaching for any subject matter is that teachers not only have a sound understanding of the content appropriate for students at their grade level, but also understand the relationship of this content to associated concepts at other developmental levels. Specifically, in the case of statistics, the teaching of Level A (or B) concepts requires an understanding of the concepts students will encounter at Level B and/or Level C. At the same time, teaching at Level B (or C) requires an understanding of students' prior experiences from the foundational levels, Levels A and/or B. Thus, the preparation of teachers of statistics for the schools must include learning experiences based on concepts from more than one developmental level.

Clements and Sarama (2004) describe a "complete hypothetical learning trajectory" as having three components: (1) the "learning goal" with (2) a "developmental progression of thinking and learning" and (3) a "sequence of instructional tasks" to support the learning goal. As classroom facilitators, teachers are expected to implement (and sometimes design) hypothetical learning trajectories. In the following example we propose a hypothetical learning trajectory for use in teacher preparation to help teachers better understand the overarching statistical notion of *distribution*. In statistics, when examining the distribution for numeric data, there are three important aspects of the distribution to consider: (1) the shape of the distribution, (2) the center of the distribution, and (3) the degree of variation or spread in the distribution. The sequence of instructional tasks described below focuses on the latter two aspects (center and variation) of the distribution. These activities illustrate a progression of thinking and learning about the mean as a measure of the center of a distribution, and quantities that measure variation (spread) about the mean from the three developmental perspectives described in the *GAISE* Report.

Understanding the Mean and Variation from the Mean

Level A students learn that the mean corresponds to the "fair share" value, and Level B students learn that the mean is the "balance point" of the data distribution. At Level C students make the distinction between the mean of a population and the mean of a sample, and are introduced to the notion of a sampling distribution and using information from a sample to make *inferences* about the population. The notion of variability in data is fundamental to statistical thinking, and various ways to quantify variation in data from the mean should go hand-in-hand with the different ways students view the mean. The following sequence of learning activities illustrates the progression of these ideas. Ideally, teachers are able to transfer these experiences to address the individual developmental needs of their students.

Cobb and Hodge (2002) point out that an important aspect of learning statistics involves "students' development of a sense of who they are in relation to statistics," and that development of "effective data based arguments" in students requires realistic and legitimate problems and data sets. In preparing teachers to teach statistics, teachers should explore problems similar to those of their students. There are numerous investigations involving the collection of discrete data that are useful for developing the mean as the fair value or as the balance point. In companion articles Franklin and Mewborn (2007) and Kader and Mamer (2007) describe two such investigations involving scores from soccer games at Levels A and B, respectively. Russell and Corwin (1989) and Kader (1998) discuss Level B investigations on the question of "family size."

Exploring the Mean at Level A/Task 1

Students are introduced to the notion of "fair share" early in their education. For example, if a class has 25 students and there are 50 pieces of candy in a bag, then the "fair share" for each student is 2 pieces of candy. The notion of fair share is related to the mean of discrete numeric data. A statistical question that can be addressed at Level A is: *How large is a typical family for our class?* In this investigation each family size is represented with a tower of cubes. Figure 1 shows this representation for nine students. Students should notice that not all family sizes are the same (i.e., the family sizes vary). A question we might ask is: *Based on all the members from these nine families, what would be the family size if all families had the same size?*

At Level A, students first learn to find the "fair share" value by "redistributing" the cubes. They would disconnect all the cubes, and redistribute them one at a time to the 9 students until all cubes have been allocated. For these data, this results in nine towers with 6 cubes in each tower as shown in Figure 2. Thus, the fair share family size is 6. Eventually, students learn to "level off" the towers, finding the fair share value by removing cubes from highest towers(s) and placing them on the lowest towers(s) until all towers are the same height (with a possible remainder).

Exploring the Mean at Level A/Task 2

Students are asked to create a tower cube distribution for nine family sizes with a fair share family size of 6, and to discuss their strategy for determining their distribution. One common strategy is to begin with the fair share distribution in Figure 2 and to redistribute the cubes among the nine towers. Using this strategy, they come to recognize that any cube distribution for nine families with a fair share of 6 must have a total of 54 cubes distributed in any way among the nine towers. This recognition leads to developing the algorithm for computing the mean. The nine towers in Figure 3 illustrate another cube distribution with a fair share value of 6.

Consider the two tower cube distributions in Figures 1 and 3. In each distribution there are nine family sizes and the fair share family size for each is 6. Since the fair share value for each distribution is 6, we cannot distinguish the two distributions based on their fair share values. A question we might ask is: *Which distribution is* ***closer*** *to being fair?* One way to measure fairness is to count the number of "steps" required to make the distribution fair, where one *step* occurs when a cube is removed from a tower higher than the fair share value and placed on a tower lower than the fair share value. The number of steps required to make a cube distribution fair provides a measure of the degree of fairness in a distribution. The distribution with fewer steps is the one closer to being fair. The horizontal line in Figure 3 is drawn at a height of 6 (the fair share value) and makes it easier to determine the number of steps to fair. Note that it takes 8 steps to make the data in Figure 1 fair and it takes 9 steps to make the data in Figure 3 fair. Consequently, the data in Figure 1 is closer to being fair than the data in Figure 3. That is, based on the *Number of Steps to Fair* there is less variation from the fair share value for the data in Figure 1 than there is for the data in Figure 3. Note that many students will suggest the distribution with the most sixes as the one closest to fair.

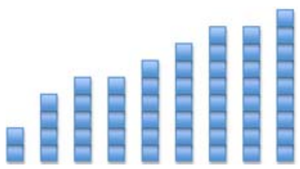
Figure 1. Nine Family Sizes

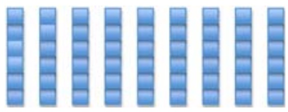
Figure 2. Fair Share

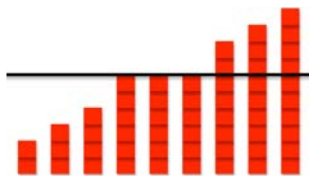
Figure 3. Nine Family Sizes

As it relates to this activity, students completing Level A understand (1) the notion of the fair share value for a set of discrete numeric data, (2) the fair share value is also called the mean value, (3) the algorithm for finding the mean, and (4) the notion of "number of steps" to make fair as a measure of variability in data about the fair share value. Students transitioning to Level B are developing notions of proportional reasoning. One such notion is that when comparing two groups of numerical data of different sizes, the fair share/mean value adjusts the total for the different groups and provides a *fair* basis for comparison.

Exploring the Mean at Level B

The activity at Level B begins with the same question as in Level A; however, Level B students are asked to summarize their data in a dotplot using stick-notes. The dotplot shown in Figure 4 is the data distribution corresponding to the cube distribution in Figure 1. Students are next asked to create a dotplot distribution for nine family sizes with a fair share family size of 6, and to discuss their strategy for determining their distribution. At Level B, students will often rely on the algorithm for the mean in determining their distribution. That is, they recognize that the total for all 9 family sizes must be 54 and they use this information in distributing the stick-notes. Another strategy often used is to begin with the dotplot shown in Figure 5, which corresponds to the fair share distribution in Figure 2. Students recognize that if a stick-note is moved to a value above 6 this will increase the total to more that 54. In order to offset this increase in the total, there must be a corresponding move to one or more values below 6 in order to return to total to 54. This approach is foundational in developing the idea of the mean as the balance point of the distribution and the notion of the deviation in a data value from the mean value. The dotplot in Figure 6 corresponds to the cube distribution in Figure 3. The number on each stick-note in Figure 6 indicates the distance the corresponding family size is from the mean family size of 6.

Consider the data distributions in Figures 4 and 6. In each distribution there are nine family sizes and the mean family size for each is 6. Since the mean for each distribution is 6, we cannot distinguish the two distributions based on their means. How does the degree of variation in the data from the mean compare between the two groups? That is, in which group do the data vary more from the mean? Students first address this question based on a visual inspection of the dotplots. Next, they are asked to quantify the degree of variation by examining "how far" each data value is from the mean. Using the distances indicated in Figure 6, the total distance from the mean for the nine family sizes is 18. In a similar manner, the total distance from the mean for the nine family sizes in Figure 4 is 16. In general, the total distance for a set of numeric data from the mean is the Sum of the Absolute Deviations in the data from the mean. That is,

$$\text{SAD} \equiv \sum_{i=1}^{n} | x_i - \bar{x} |$$

The SAD is the first step in arriving at a measure of the degree of variation in data from the mean. It is interesting to note that for discrete data with a whole number mean, the SAD corresponds to twice the number of steps to fair in the cube tower representation for the data. When group sizes are different, comparing SADs is not valid. In this case, the SADs are adjusted by finding the Mean of the Absolute Deviations, or the MAD. The MAD is a precursor to the standard deviation, the more standard statistical measure of the degree of variation in numerical data from the mean. As it relates to this activity, students completing Level B and transitioning to Level C understand the mean as the balance point of a distribution, the mean as a "central" point of a distribution, and the SAD, the MAD, and the standard deviation as measures of variation about the mean.

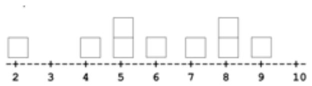

Figure 4. Nine Family Sizes

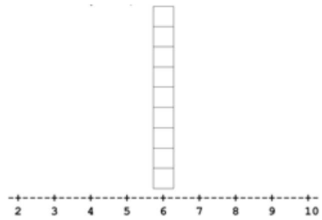

Figure 5. Fair Share

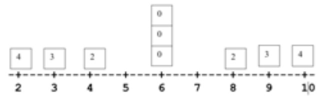

Figure 6. Nine Family Sizes

Exploring the Mean at Level C

At Level C students learn to make the distinction between populations and samples and between parameters and statistics. In most statistical investigations, we do not have access to the entire population, so we use the value of the statistic from a sample (e.g., the sample mean) to estimate the value of the corresponding parameter in the population (e.g., the population mean). A essential question in estimation is: *How close can we expect a sample mean to be to the population mean?* The following, adapted from an activity described by Chance and Rossman (2006), illustrates this idea using the problem of sampling words from *The Gettysburg Address*, a famous speech given by President Abraham Lincoln in 1863. The statistical question of interest is: *What is the mean length of the words in the Gettysburg Address*? In this case, the population consists of the 268 words in the Gettysburg Address and the variable of interest is the word length (number of characters). As this is a relatively small population, the population mean ($\mu = 4.3$) can be computed easily. The goal of the activity is to illustrate the nature of variation in the sample mean from sample to sample with respect to the population mean, 4.3. Each student is asked to perform the following tasks:

Task 1: Select 10 words from the Gettysburg Address you consider to be representative of the varying lengths of the words and determine the sample mean word length. Create a dotplot for the various sample means.
Task 2: Select a *simple random sample* of 10 words from the Gettysburg Address and determine the sample mean determine the sample mean word length. Create a dotplot for various sample means.

The results in Figure 7 show the sample means from samples selected by 50 students. It is evident from the dotplot that the self-selected samples tend to produce sample means greater than the population mean of 4.3. That is, the means from the self-selected samples tend to overestimate the population mean. This is an illustration of the notion of bias in the sampling procedure. The results from random sampling are unbiased since the sample means from repeated sampling are on average around the population mean of 4.3. Additionally, as randomness is utilized in selecting a simple random sample, probability provides a way to predict the long-run behavior of the values for the sample mean if samples are repeatedly selected. Probability provides the mechanism for describing the sampling distribution of the sample mean. The simulated sampling distribution in Figure 7 for the sample means from simple random sampling is the link for understanding two important concepts in statistical inference – margin of error & statistical significance.

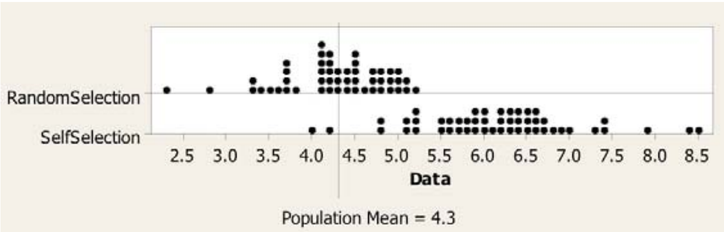

Figure 7. Means from 50 Samples

CONCLUSION

Through the use of designed tasks and activities, as modeled in the *GAISE* Framework, our goal is to help teachers develop confidence in their ability to apply statistical knowledge in their teaching and confidence in the pedagogical methods for delivering statistical content. The teachers will most often work through the activities and tasks in groups, experiencing the type of thinking, progressions and learning their students will experience.

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