ASSESSING THE INTERPRETATION OF TWO-WAY TABLES AS PART OF STATISTICAL LITERACY

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This paper analyses the interview responses of 29 teachers to a question based on the interpretation of a 2-way table. The teachers were asked to articulate the big statistical ideas behind the problem, to suggest appropriate and inappropriate responses they would expect from their students, and then to indicate how they would respond to two selected specific responses given by students in previous student surveys. Hierarchical rubrics were developed for assessing the teachers' pedagogical content knowledge and examples are given of teachers' responses at each level. Implications for the statistical literacy curriculum, for the use of authentic problems, and for teacher professional learning are discussed.

BACKGROUND

In the statistical component of the school curriculum the focus on association is usually considered based on scaled measurement data and correlation coefficients, often exemplified by scattergraphs (e.g., students' heights and arm spans). In statistics courses, plotted correlation is the main instance seen of association of variables. Although some more advanced courses introduce the chi-squared test for categorical data, the link in terms of association of variables with 2-way tables that might have arisen from conditional probability statements is often not explicit. Seeing 2-way table interpretation problems as a matter of determining probabilities of compound or conditional events, places such questions firmly in the numerical part of some mathematics curricula (Board of Studies NSW, 2002, p. 76). Being able to translate relative cell numbers into ratios or percents of row or column totals, is seen as good experience in applying proportional reasoning skills learned in the middle school. Two-way tables, however, also feature in the GAISE recommendations for statistics in the school curriculum (Franklin et al., 2007). If problems based on 2-way tables are placed in the statistics curriculum, then issues of teaching and assessment become important in helping students appreciate the statistical variables and association involved.

THE PROBLEM

The problem used as a basis for the research is a 2-way table problem arising from conditional probability. It also presents a context that relates to statistical variables and challenges the solvers to think critically about their intuitions about risk. Although Bayes Theorem has traditionally been the theoretical way of solving conditional probability problems, the move to creating 2-way tables, based on chosen numerical equivalents of the required probabilities, has been advocated for some time (Rossman & Short, 1995; Watson, 1995; Pfannkuch, Seber, & Wild, 2002). Examples, such as the taxi problem or false positives and negatives in medical diagnoses, illustrate the difficulties of organising "conditions" and the potential of 2-way tables to assist. In statistical terms 2-way tables represent the association of two categorical variables.

The problem discussed in this report, shown in Figure 1, was devised by Batanero, Estepa, Godino, and Green (1996) as part of a study of high school students' ability to use proportional reasoning in the context of 2-way tables. The analysis of student responses was hence based very much on the numerical techniques used as students struggled with determining relationships among the cells, columns, and rows in the table. Batanero et al. presented 16 different categories of correct, partially correct, or incorrect responses. As part of a wider study of statistical literacy, Watson and Callingham (2003) used the same item and employed a six-level hierarchical rubric, reflecting increasing accommodation of the numbers in the cells of the table. This rubric was used by Watson and Kelly (2006) in a detailed analysis of the responses of 322 students from grade 5 to grade 10. Using a 6-point code from 0 to 5 the average response rose from 0.79 for grade 5 to 2.06 for grade 10, with only 15.5% of student responses coded in the top two levels. These results indicate the need for teachers to assist students in reaching the required level of mathematics and as importantly, to understand the statistical nature of the problem.

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The following information is from a survey about smoking and lung disease among 250 people.

	Lung disease	No lung disease	Total
Smoking	90	60	150
No smoking	60	40	100
Total	150	100	250

Using this information, do you think that for this sample of people lung disease depended on smoking? Explain your answer.

Figure 1. The 2-way table problem presented to students and teachers

UNDERSTANDING PEDAGOGICAL CONTENT KNOWLEDGE (PCK)

A second dimension to this report builds on the growing interest in assessing teachers' pedagogical knowledge for teaching statistics. Derived from the work of Shulman (1987), recent research has focussed on measuring the components of teaching of statistics required for successful student achievement (e.g., Groth, 2007; Watson, 2001). Watson developed a teacher profile that assessed teachers' knowledge of content and students-as-learners by presenting problems and asking teachers for appropriate and inappropriate responses their students would give. To assess teachers' pedagogical knowledge, they were asked how they would address one of the inappropriate responses. A variation on this format was used by Watson, Callingham, and Donne (2008), in presenting teachers with two incomplete student responses to the problem in Figure 1 and asking how they would respond to the students. The two items are presented in Figure 2. Watson et al. (2008) reported on the responses of 44 teachers coded on the basis of a 4-level rubric: irrelevant or no response (0), general strategy, no mathematical input (1), some relevant mathematical content, vague teaching strategy (2), and strategy involving questioning of students, discussing multiple aspects of the problem, introducing proportional reasoning strategies, and/or introducing cognitive conflict (3). The rubric reflected the need for teachers to have the content knowledge and knowledge of their students required to suggest relevant strategies to lead the students to higher levels of understanding. Overall the distribution of teachers' responses across the code levels was disappointing. The relatively short written responses of teachers on the survey led to an extended interview protocol being devised. Its purpose was to gauge teachers' knowledge for teaching statistics generally, and to understand their approach to three sample problems, including the one based on a 2-way table. The interview began and ended with general questions about teaching statistics and, for example, what students liked or found difficult. Figure 3 shows the questions for the 2-way table problem.

Consider each of the following answers and explanations to the problem and discuss how you would respond to the answers.

4.3 Student 3: Yes, 90 who smoked got lung disease.	4.4 Student 4: [No] 60 "no smoking lung disease" and 60 "smoking no lung disease" are the same.

Figure 2. Student answers to the 2-way table problem

The first stage of analysis (Watson, Callingham, & Nathan, 2009) considered an item interpreting a pictograph with the same structure of questioning as in Figure 3. A holistic appraisal of teachers' responses added a component titled "Constructs Shift to General," which displayed awareness of the conceptual underpinnings of the specific problem. For the pictograph item, this component included recognising the difference between the pictograph as a statistical model and as a vehicle representing real data, exploring "majority," exposing the limitations of the data collection, experimenting with alternative data representations, and/or introducing an awareness of language. The component "Constructs Shift to General" was identified as a means to capture more completely the nature of PCK as an analytical framework. It was of interest, therefore, to consider further the relevance of this component by seeing if teachers had a similar ability to broaden the scope of the 2-way table problem for their students. The underlying research question became whether the interviewed teachers identified the 2-way table primarily as a problem of mathematics or statistics. Responses to all the interview questions were important to consider for it was a matter not only of identifying the "Big Idea," but also of demonstrating associated teaching strategies.

STATSMART TEACHER INTERVIEW PROTOCOL

Use "Lung disease / Smoking". Show the problem.

Q2L. What are the big statistical ideas in this problem? (Probe: What answer would you give?)

Q3L. Please can you give an example of an appropriate response and an inappropriate response that your students might give? (Probe: Can you explain why it is appropriate/inappropriate?)

Q4L. What opportunities would this problem provide for you teaching? (Probe: Where would you place it in your lesson sequence? Or in your school's curriculum sequence?)

Show student response: Yes, 90 who smoked got lung disease.

Q5L. A student gave this answer. How would you move this student's understanding forward? (Probe: What would be the next step in learning?)

Show student response: [no] 60 "smoking lung disease" and 60 "smoking no lung disease" are the same.

Q6L. A student gave this answer. How would you move this student's understanding forward? (Probe: What would be the next step in learning?)

Figure 3. Interview protocol extracts for the 2-way table problem

METHODOLOGY

Sample. Twenty-nine teachers from three Australian states were interviewed for this study, with 12 from one state, 9 from a second, and 8 from the third; 27 of the 29 had completed the survey questions in Figure 2. They were involved in a professional learning project in statistics for the middle school. Teachers taught in grades 5 to 12, had teaching experience ranging from 2 to more than 25 years, and had a wide-range of previous tertiary study in mathematics and statistics.

Instrument. The teachers answered the questions in Figure 3.

Analysis. The rubric in Table 1 for PCK Feature 1, Recognises "Big Ideas," applied mainly to question Q2L, and the rubric for PCK Feature 2, Anticipates Student Answers, applied mainly to Q3L. Occasionally information for Q4L was assessed if it addressed these two features. Whereas the rubric for PCK Feature 3, Employs Content-specific Strategies, was used for Questions Q5L and Q6L separately, the rubric for Feature 4, Constructs Shift to General, was assessed across all questions in Figure 3. The responses were coded independently by the two authors using the hierarchical rubric in Table 1 and discrepancies were negotiated until agreement was reached.

Table 1. Rubric for Responses to Two-Way Table Interview

Code	Description
PCK Feature 1: Recognises "Big Ideas"	
0	Responses confused and/or incorrect
1	Response partial, implied: some understanding revealed beyond initial question
2	Immediate grasp of idea, language specific
PCK Feature 2: Anticipates Student Answers	
0	Response irrelevant
1	Appropriate or inappropriate but not both, or unclear
2	Distinguishes both appropriate and inappropriate
3	Demonstrates understanding of students' reasoning and/or includes actual solution
PCK Feature 3: Employs Content-specific Strategies 3A: Yes, 90 who smoked got lung disease. 3B: [no] 60 "smoking lung disease" and 60 "smoking no lung disease" are the same.	
0	Response absent or indicates misleading content or not highly relevant
1	Content knowledge of proportion requisite to initiate a discussion
2	Demonstrates questions or knowledge that might structure a discussion about proportion and/or independent variables
3	Extends discussion by illustrating/referencing beyond the table (e.g., consider representations for percent/ratio/proportions/fractions)
PCK Feature 4: Constructs Shift to General	
0	No shift to general evident
1	Awareness of context: popular beliefs of strong causation intruding on statistical interpretation; numbers vs real scenarios
2	Explores the principles of association between variables; introduces an awareness of language (independence, variable in statistical context)

RESULTS

Table 2 contains a summary of the codes for the PCK Features described in Table 1 for the 29 teachers interviewed.

Table 2. Number of teachers receiving each code for each PCK feature (*n*=29)

	Code			
PCK Feature	0	1	2	3
1	3	17	9	–
2	1	12	7	9
3A	4	12	10	3
3B	6	15	5	3
4	20	7	2	–

PCK Feature 1: Recognises "Big Ideas"

Most teachers (17/29) revealed a partial understanding only of the "Big Ideas" inherent in the 2-way table, and were unable to articulate its learning potential immediately. Responses coded as 2 encompassed the prime idea expressed appropriately. For example:

> Well I guess we're looking at the linking of two different variables. So smoking and non-smoking and then whether people have or don't have lung disease … whether there is an association … how that compares with the proportion with lung and without lung disease. (T1)

Responses coded 1 focussed on fewer aspects of the item, for example, "this is all about proportional thinking obviously" (T2). Responses that only commented on sample size were not considered to have recognised the main big ideas.

PCK Feature 2: Anticipates Student Answers

Some teachers (12/29) had difficulty articulating which answers of students would be appropriate and which not when asked to create student answers (Code 1).

> I think you would end up with an argument and I can imagine smokers versus the non smokers – "my aunty smoked and lived to …" – without even looking at the table you get that …. [appropriate] I think they would analyse sort of – put arguments both way. I think they would see that perhaps it was difficult to come to a conclusion … (T3)

On the other hand some teachers (7/29) could quite succinctly cover the possibilities (Code 2).

> …I would expect them to be looking at your percentages of smoking ones who get lung disease and non smoking ones who get [it] – and comparing … [inappropriate] … it isn't dependent on smoking disease because there are people who have got lung disease who didn't smoke. (T4)

Nine of the 29 teachers could express both an appropriate and inappropriate answer, and reveal an understanding of the mathematics required or an insight into the students' reasoning (code 3).

> – well it is working on two things: the kids' pre-conception since they all know that lung disease is caused by smoking, they are more likely to say yes it does … it kind of tricks you into thinking that more people, that smoking causes lung disease, when the actual percentages are about the same … it is kind of knowing the difference between gross number and percentage … so the smoking lung disease people was the biggest number but as a percentage it is actually the same. (T5)

PCK Feature 3A: Employs Content-specific Strategies for "Yes, 90"

Code 1 responses for Feature 3A tended to display the mathematical knowledge for the task but demonstrated no further strategies for working with students. For example, "well they need to convert all the figures into – well – yes the figures into a percentage" (T6). Code 2 responses (10/29) embodied a greater confidence with the mathematics and an ability to link to a discussion about proportion or variables.

> …I think you'd then go into the fact that looking at how many people that were smoking were surveyed and look at how many that weren't smoking were surveyed and so I'd have a bit of a discussion about the relationship between those values and whether or not if we surveyed an extra 50 would we expect more people to get lung disease … (T7)

Few teachers (3/29) achieved a code 3 response, which required a capacity to extend discussion by considering different representations for the numbers and thus to reference beyond the table.

> It may be that just looking at numbers on a table doesn't connect with anything and so we need to go to physical objects and so let's take 9 people with lung disease and 6 people with no lung disease and, 6 and 4 in physical objects on the table, and say, alright, now what do you think? (T8)

PCK Feature 3B: Employs Content-specific Strategies for "60-60"

A similar distribution of levels of response was observed for Feature 3B. A code 1 response suggested using some form of proportional reasoning: "I'd be asking them to look at what the total was for that particular group. So, I mean this represents a smaller percentage of that sample size than this represents" (T9). Code 2 responses went further to include the notion of variables as well.

> You're not comparing single variables, you're actually comparing two different variables, …so you're actually comparing sort of four combinations of the variables rather than comparing one aspect and seeing whether that has a different result in the other variable. (T7)

Again the top code reflected the complex language and went outside just considering the table.

> There's actually two variables involved here. The language is tricky because it's exactly the same set of four words but just by changing the position of two words, you've changed the conception quite considerably …. I would be going to physical objects where they could see and feel … (T8)

PCK Feature 4: Constructs Shift to General

Many teachers' responses (20/29) did not reflect any shift out of the mathematical purpose of using proportional reasoning to "solve" the problem. Some teachers (7/20), however, appreciated the strong influence that personal beliefs, even their own in some cases, could have on the discussion of the problem and/or discussed contrasting the numbers in the 2-way table with real scenarios.

> The first thing is, I think that the whole context of the question sets up, well certainly for an adult, you've got a perception of what it should be and therefore that may impact or influence your suggestion. (T7)

Code 2 level responses (2/29) were reflected across the interview, exploring the principles of association between variables, showing a strong awareness of the importance of language.

> …I think about the context and perceptions as well, but also the idea of relationships between variables, causation, dependent variables, independent variables… (T7).

DISCUSSION

One of the goals of statistical literacy in the school curriculum is surely to have students appreciate both the mathematical and statistical aspects of scenarios such as presented here. This presupposes that teachers possess the PCK necessary to assist students to critical understanding. Responses to all of the interview questions in this study were dominated by a preoccupation with the mathematics of the 2-way table problem. Most of the teachers appreciated that the problem required the mathematical skill of proportional reasoning and in a general way suggested the need for percentages, fractions, or ratios. However, several teachers commented that the problem was too difficult for their students and only 16 teachers explicitly suggested a mathematical solution. This was somewhat disappointing as the proportional reasoning skills are fundamental to the middle school mathematics curriculum and certainly are a prerequisite to high-level statistical literacy thinking (Watson & Callingham, 2003). Beyond the presentation of the numbers in the problem is the issue of authenticity of the numbers. One teacher said she would not use the item because obviously the numbers were fictitious. The question for statistical literacy, however, is, how do you get students to develop critical thinking skills if they never encounter any situations that challenge their beliefs? Most teachers were not concerned about the actual numbers and some used them in the way expected to get students to question both their beliefs and where the data might have originated.

Although the problem used in this study was created by researchers (Batanero et al., 1996), there are examples in the media that lend themselves to being used with students and teachers in similar settings. A humorous article ("Cheat radar", 2008) about the ability of males and females to detect the infidelity of their partners, provides conditional percentages that can be used to create not only 2-way tables to satisfy the probability curriculum but also a context of variables that allows for discussion of the association of statistical variables (and a subjective element of risk!). The creation of 2-way tables from text about variables is another statistical literacy skill that can be developed in association with the ability to interpret numbers already embedded in tables.

Only two of the teachers articulated the 2-way table problem as one of statistical variables where an association is being questioned. The inability of most teachers to appreciate the inherent statistical nature of the problem may be related to their historical encounters with the school curricula in their respective states. All of the teachers taught high school and hence should have had some familiarity with this type of 2-way table problem. The goal of professional learning for teachers should not be confined to ensuring they can numerically answer the question. It should be driven by assisting teachers to identify and exploit the full educational potential of a statistical problem. Using structured discussions among groups of teachers based on the protocol in Figure 3, would appear a good starting point for increasing the aspects of PCK required to achieve the "shift to general" of the ideal teaching world.

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