LINKING PROBABILITY TO REAL-WORLD SITUATIONS: HOW DO TEACHERS MAKE USE OF THE MATHEMATICAL POTENTIAL OF SIMULATION PROGRAMS?

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In Quebec secondary schools, the teaching of probabilistic concepts is mandatory, but most teachers feel insufficiently prepared. Furthermore, middle school students are confronted to gambling activities at a young age, without necessarily understanding the underlying probability. In order to analyze the development of a more realistic and mathematically correct understanding of gambling activities, we conducted a one-year design experiment involving 4 high school teachers. We trained the participants in various concepts of probability and accompanied them to prepare classroom situations, which they used in their classrooms. In this paper, we analyze how the participating teachers used a simulation software we provided them. Mainly, the teachers used the software to show that gambling activities do not pay off over the long term. However, it was more difficult for the teachers to discuss probabilistic concepts through the simulation software.

INTRODUCTION

In Quebec, since the implementation of the new primary and secondary school curriculum (Government of Quebec, 2001, 2004), teaching probability and the concept of chance has been mandatory starting at the beginning and especially at the end of elementary school (grades 5 and 6; Government of Quebec, 2001) straight into the beginning of secondary school (grades 7 and 8). However, teaching probability rarely builds upon authentic contexts, and predominantly uses a theoretic approach to probability, rather than a frequentist one. Hence, pupils often develop conceptions (Konold, 1995) about probability based on a deterministic reasoning system and deterministic conceptions (Savard, 2008), which are used in various real-life situations and influence both their in-school performance and their everyday behavior (Musch and Ehrenberg, 2002). In fact, teenagers are frequently exposed to gambling advertising in which gambling is omnipresent and socially accepted (Griffiths, 2003). They are sensitive to its popularity and to the glamour represented by the World Series of Poker, lottery advertising and online casinos. In another part of the research we documented gambling activities among teenagers with a questionnaire administered to 256 students in three different schools. These students were taught by one of the 4 teachers involved in this project. Our preliminary results show that almost all of them know someone in their family who uses some strategies to find the lottery numbers on which they are betting. In the meantime, while younger students believe that chance plays an important role in gambling activities, older ones believe that they can control the outcome of these games, a conception called "illusion of control" (Langer, 1975). It is interesting to note that 41% of them play poker and 15% gamble online.

In order to cope with these problems, we developed a research program in which we trained several secondary school teachers (grades 7 and 8) to teach probability. The objective of this project was twofold: first, to allow the pupils taught by these teachers to develop a better understanding of probability through activities rooted in a gambling context; second, to develop a more realistic approach to gambling activities.

DESCRIPTION OF THE SIMULATOR

In this research, we provided the teachers a probability simulator that we developed. It simulates gambling activities of different nature. We intentionally chose activities that solely rely on probability, requiring no particular skill component, as would be the case for poker, for instance. Seven games were included in the program: Black jack, a magic wheel with ten different cases, a 6/49 lottery (in which the participant must chose 6 out of 49 possible integers - the jackpot goes to the participant who has 6 correct numbers), dice, heads or tails, roulette, as well as a simulation of the Monty Hall Paradox. In all these games, the simulation software allowed a high number of

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trials in a very short time and showed various results: number of wins, percentage of wins, amounts of money lost or won, a graph with the evolution of the percentage of wins, etc.

METHODOLOGY

During this research, we conducted a design experiment–and more specifically an in-service teacher development study (Cobb et al., 2003)–during which we trained 4 middle school teachers to develop and implement in their classroom probabilistic problems in a gambling context. We accompanied the participating teachers while they were elaborating problems that were designed to promote the development of a more accurate understanding of underlying probability concepts and to address the students' misconceptions about probability. To do so, six whole-day meetings were held with the participating teachers. During these meetings, we provided them with some theoretical background on the teaching and learning of probability. We also provided them with various tools that could be used to teach probability, among which were television commercials by the provincial lottery agency that had some mathematical interest and a simulation program. Finally, we assisted these teachers in developing mathematical problems that they would use in their classroom.

At the end of the school year, the teachers applied the learning situations they had developed in their classroom. These lessons were videotaped and transcribed. The transcription data were then analyzed with a qualitative methodology. In this paper, we will concentrate on the way the teachers explored the mathematical potential of the simulators we provided them. (Teachers' names have been replaced by pseudonyms in order to maintain their anonymity).

RESULTS

We are currently analyzing the transcriptions of the lessons in which the teachers used the simulation software with their pupils. In this paper, we present the preliminary results of this analysis. We will discuss the following issues: a) the teachers used the simulation programs mainly to show their pupils that gambling activities are not in favour of the gambler in the long term, b) the teachers had difficulties making the most of other probabilistic concepts that could potentially have been taught through the simulation software.

Using the simulator to show that gambling makes you lose money in the long term

All 4 teachers who used the simulation software in the classroom used the software to show the students that lottery games do not pay off for the gambler in the long run. In Justin's class, the students chose the heads or tails game and Justin ran 1,000 trials through the simulation software. The game went as follows: 3 coins were used, you won if you got 3 identical symbols. Each time, you bet $1. If you won, you got $2. Afterwards, Justin explained that the game is not interesting for the gambler in the long run:

> Unfortunately, as I lost 751 times, this means I lost 751 dollars, for a total loss of 253 dollars [after subtracting the $2 wins from the 249 times he had won]. And, here, (shows the simulator), you can find the expected loss per game. It is 0.25 here. This means that, in the long run, if I continued, this means I would lose 25 cents per game. So this is not a winning game for me.

In this situation, the conclusion about the outcome of the game was not developed by the students, but explained by the teacher. The concept of expected loss was not calculated by the students–the simulator software showed it directly on the screen–nor explained explicitly during this lesson, although this might have been done in one of the preceding lessons.

In Michelle's class, the students also discussed the return on investment that comes out of lottery games. To do so, Michelle used a simulation of a 6/49 game, which was run 1,000 times. Faced with these results–according to which the students would have won $130 (on 13 occasions, they got 3 correct numbers, which in Quebec would give $10 each), for a total payment of $2,000 ($2 for each game)–she asked them whether this was a good investment or not.

In a similar way, Emma and Christina also used 6/49 lottery simulations in order to find out whether this game allows the gambler to win more than he invested. Their students had to draw conclusions about the interest of these games after a certain number of simulations and compare the wins these numbers would have returned in real life with the initial investment. Hence, the teachers directly and explicitly tried to show their students that lottery games do not pay off in the long run. To achieve this, they essentially used the simulation software's ability to generate a high number of trials in a very short time. Most of them did this through a realistic use of a 6/49 game, where the simulation software allowed them to recreate exactly the same conditions than those in the real-life game. Only Justin chose to use a different game (heads or tails), which is not offered by the national lottery agency. It is interesting to note that the teachers did not use probabilistic reasoning in this case. Thus, the concepts of randomness and uncertainty were not discussed. The simulators were essentially used to determine money wins and losses and to convince students that gambling is not a winning activity in which to earn money.

Difficulties using the simulation software to teach other concepts than the law of large numbers

Even if the teachers managed to work on the long-term return on investment of gambling activities, it was much more difficult for them to work on other probabilistic concepts through the use of the simulation software. For instance, both Emma and Christina wanted their students to find out whether certain strategies used to choose numbers in a 6/49 game (anniversary dates, lucky numbers, etc.) would result in a higher chance of winning. In principle, such activities could have been an opportunity for the teachers to discuss whether such strategies are efficient – a misconception held by a number of students. However, the way these questions were tackled in the classroom made it difficult for students to come up with answers and to eventually change their conception. In Christina's class, the students were asked to come up with six numbers by using various strategies they had invented (diagonals on the lottery form, lucky numbers). The numbers were run through the simulation software by the teacher on her computer (200 to 500 trials) and the students had to find out whether they got more money back than they invested. Of course, such a teaching strategy demonstrates that, even with numbers that are chosen for certain reasons, lottery activities do not pay off. However, it does not allow them to compare results that would have been obtained by random numbers with those obtained by reasoned numbers. Randomness and illusion of control were not discussed either. In conclusion, students who hold the belief that they have better chances to win if they chose their own numbers will not change their belief through this activity.

In Emma's class, the students were separated in two groups. One group had to run groups of 500 simulations on the 6/49 simulation software using random numbers. Another group had to do the same, but using numbers they had chosen according to a certain strategy (anniversary dates, lucky numbers, etc.). They then had to compare which of the strategies first triggered a result with 5 correct numbers. In this case, students do not think about what strategy gives more results with 5 correct numbers. Furthermore, since obtaining 5 correct answers or more happens rarely in the 6/49 lottery, choosing which strategy allows this event to come up first is not an adequate indication of whether one strategy has a higher chance to obtain 5 correct numbers than another. In addition, approaching the questions this way leads the students to think about a single event instead of analyzing the situation more broadly.

The preliminary analysis also reveals that it was difficult for teachers to create probabilistic knowledge different from the general inefficiency of lotteries available to the students. For instance, in the preceding example, the idea of comparing strategies is not mentioned anymore in the lesson. In the 3 other teacher's lessons, it is also apparent that they often finish the lesson in small groups without implementing afterwards a discussion with the whole group. Such a discussion would allow students to work through the concepts they learned about and to institutionalize their newly acquired knowledge.

Another general characteristic of the teachers' lessons is that they solely relied on frequentist probability and did not establish a relationship between them and the theoretical probability. In itself, the shift from frequentist to theoretical probability was important during our project, because, at the beginning of the year, the teachers told us they solely relied on theoretical probability to teach these concepts. However, the absence of a link between both is problematic insofar as it might lead the students to see both kinds of probability as separate entities. This difficulty might be due to the way the teachers used their situational constraints. For instance, when working on the roulette software, both Michelle and Justin gave the students the freedom to decide whether they would bet on even or odd numbers or black or red numbers. In both classes, pupils ended up using the 4 possibilities in disorder, which made it impossible for them to draw conclusions about the equiprobability in this situation. The same goes for the possible link between theoretical and frequentist probability. Since the students did not use a systematic method to generate their results, it became difficult to draw conclusions about the results obtained and the theoretical probability of winning. With a more systematic approach, these situations could potentially lead to discussing with the students the possible variation between the theoretical probability of winning and the results that are generated by making a certain number of draws.

CONCLUSION

The preliminary results show that the participating teachers used the simulation software primarily to make the students understand that lottery games do not pay off in the long run. In this sense, the use of the simulation software allowed them to go beyond what would have been possible with a paper and pencil approach. In the same way, they managed to reorient their teaching strategies toward more inquiry-oriented strategies. However, the teachers had a harder time making use of the simulation software to teach other probabilistic concepts. The situational constraints often made it difficult to build such concepts and no link was established between theoretical and frequentist probability.

In order to avoid these difficulties it might be useful to reinforce certain parts of the training we offered the teachers. For instance, even if we mentioned some of the misconceptions often held by students, it could have been useful to have a more in-depth discussion about these beliefs. It might also have been useful to discuss more explicitly the simulation software' potential.

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