EXAMINING HOW TEACHERS' PRACTICES SUPPORT STATISTICAL INVESTIGATIONS

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A core aspect of statistical thinking involves engaging in and thinking about the process of statistical investigation, which can be experienced through experiments or simulations. This study examines how middle school teachers' use of probability experiments and simulations can support students in the process of statistical investigations.

Several authors have claimed that engaging in and reasoning about the processes in a statistical investigation is a central aspect of statistical thinking (e.g., Pfannkuch & Wild, 2004; Friel, O'Conner, & Mamer, 2006). Of critical concern to us is how an understanding of probability can support statistical thinking. In their work on describing measures of center as a "signal" in a noisy process, Konold and Pollatsek (2002) propose a shift in attention to the importance of conceiving a sample as being drawn from a global process containing many factors and "noise" rather than drawn from a static population that remains unchanged. The "global process is a probabilistic one, unpredictable at the micro level" (Konold & Pollatsek, 2002, p. 266). Thus, a sample can be conceived as a snapshot of the current stability of the evolving process, a perspective that also promotes thinking about the fundamental variability within the global process and that real-world phenomena cannot be predicted with certainty. Although many curricula and researchers treat statistical inquiry as separate from probability concepts, Jones, Langrall, & Mooney (2007) describe the relationship between statistics and probability as complementary. Statistics uses random processes and probability models to make inferences about that which is unknown. Thus, a useful understanding of probability is that which helps quantify and model random processes. We claim that teachers can develop students' statistical thinking by approaching probability concepts in a way that is directly connected to its use in statistics, particularly attending to the variation in results from a repeated probability experiment (e.g., Franklin et al, 2005; Reading & Shaughnessy, 2004; Saldanha & Thompson, 2002).

One way to heighten the connection between statistics and probability is to promote the processes of statistical investigation, which can be done by conducting experiments or simulations. Generating random data and making predictions or inferences based on data ties closely with the basic principles of statistical investigation as described by Graham's (1987) PCAI model: Posing questions, Collecting data, Analyzing data, and making Interpretations. The *Guidelines for Assessment and Instruction in Statistics Education* (GAISE) *Report* uses the PCAI model in a framework for K-12 students learning of statistics (Franklin & Garfield, 2006), and Friel, O'Conner and Mamer (2006) base their work on PCAI emphasizing the importance of thinking about data as a distribution and making interpretations within a context.

Most teachers have little or no prior experience with using statistical investigation to conduct probability experiments or simulations. Thus, they may have difficulty implementing an experimental approach to teaching probability that is aligned with the process of a statistical investigation (Stohl, 2005; Mojica, 2006). In the past few years, researchers have begun to do more extensive research on teachers' understanding of how to teach probability but not always emphasizing its connection to statistics. Thus, we investigated specifically whether teachers' practices when using probability experiments and simulations were supportive of engaging students in the process of statistical investigation.

FRAMEWORK

Several well-developed frameworks describe the process of statistical investigation that builds a foundation for students' statistical thinking (e.g., Friel, O'Conner & Mamer, 2006; Graham, 1987; Pfannkuch & Wild, 2004). These authors pay attention to the ideas of collecting data through samples (particularly random samples) and the importance of distribution and

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variation in the analysis of data. The four components of the PCAI model may emerge linearly or may include revisiting and making connections among the components. The first component includes posing a question.

Such questions should be about particular contexts and should be motivated by describing, summarizing, comparing, and generalizing data within a context. Collecting the data, the second component, includes a broad range of collection opportunities from populations and samples. Of critical interest to us is the collection of samples from a probability experiment or simulation. The third component of the model, analyzing data, should encompass perceiving a set of data as a distribution when a probability experiment or simulation is conducted. This component also includes describing and analyzing variation, as well as organizing and displaying data in charts, tables, graphs, and other representations. Interpreting the results, the fourth component, involves making decisions about the question posed within the context of the problem based on empirical results in relation to sample size. It may be of interest to interpret these results in relation to the theoretical probability, if such a probability can be estimated. The framework for our study (Table 1) aligns: a) the PCAI model (Graham, 1987; Friel, O'Conner, & Mamer, 2006), b) steps in conducting a probability experiment or simulation (Watkins, 1981; Travers, 1981; Bright et al., 2003), and c) teachers' understandings and pedagogical decisions we view as important within the processes and steps.

Table 1. Framework Connecting Statistical Investigations and Probability Experiments

Process of Statistical Investigation	Steps in a Probability Experiment	Focus on Teachers' Knowledge and Decisions
Pose the questions	1. Understand the problem context to identify the random event(s) of interest.	-Choose problem contexts amenable to collecting samples where probability distribution is either known or unknown. -Define the random event in the problem context.
	2. Identify the possible outcomes for the random event.	-Construct the sample space, if known. -Understand probabilities of each event. -Decide if students should compute theoretical probabilities a priori.
Collect the data	3. Select an appropriate random generating device.	-Choose appropriate device(s) that model the mathematical characteristics of the problem. Either -Use identical device as stated in problem context to carry out experiment or -Use different device to simulate problem.
	4. Determine how to define a trial and sample.	-Define a trial (a single occurrence) and a sample (a collection of trials).
	5. Repeat a number of trials to form samples, possibly repeat sampling.	-Decide on number of repetitions or allow students to make this decision. -Decide whether to structure data collection process and organization of collected data or to allow students this choice. -Understand independence or dependence in repeated trials/samples. -Understand the law of large numbers.
Analyze the distribution(s) of data	6. Analyze distribution of results, including variation across samples, and compute empirical-based probability of random event(s).	-Decide how to use collected data from individual samples and collection of samples, including any private or public displays of tables, charts, etc. -Decide how to help students make sense of empirical results.

Interpret the results	7. Use empirical probabilities to make decision about original problem context.	-Understand the variation expected within and across samples of data, with attention to sample size. -Understand the law of large numbers. -Understand how to interpret results in relation to sample size and probability distribution (if known), or to infer a probability distribution based on empirical results. -Interpret empirical results in terms of the original problem context.

METHODS

Context of Study and Participants

Middle school teachers in this study were participants in a project funded by the National Science Foundation to improve middle grades mathematics teaching and increase the retention of teachers. Twenty-nine teachers from at least nine different counties in North Carolina [USA] participated in a graduate-level course focused on Data Analysis and Probability for Middle Grade Teachers in 2003. One pedagogical objective of the course was increasing teachers' understanding of how to conduct statistical investigations through conducting probability experiments and simulations with concrete materials and technology tools. They were engaged in making inferences from data given to them as part of a context, as well as data generated through random processes in probability contexts. One course requirement was to plan, teach, and reflect on a lesson in which they taught a statistics or probability topic.

Of the 29 teachers in the course, nine chose to conduct a lesson using an experiment or simulation to teach probability concepts. These nine teachers, from seven different counties, include eight female teachers and one male teacher. They range in teaching experience from three to 22 years with most having less than five years experience. Three of the teachers taught a lesson for 6^{th} grade students [ages 11-12], two taught a lesson for 7^{th} grade students [ages 12-13], while four taught an 8^{th} grade lesson [ages 13-14]. The data sources include a 15-minute videotaped episode from the lesson implementation and the teachers' written reflection on their analysis of the episode.

Analysis

Following Powell, Francisco, and Maher (2003), the videotapes were viewed several times and described. Verbatim transcripts of the lessons were created, including reproduction of representations created by the teacher, either on the board or overhead projector. Initial descriptions and researcher impressions of each classroom episode were made. For each of the nine teachers, the components of the framework were used to code the stages in the probability experiments and to describe teachers' decisions and tasks throughout the lesson.

RESULTS

Table 2 contains a brief overview of each lesson, including our claims about teachers' learning goals, which we hypothesized based on their actions in the teaching episodes and on commentary from their reflections. Overall, the lessons illustrated a predominant use of teacher-controlled experiments, where students had little decision-making and choices in collecting data, organizing or displaying results. Our discussion of the results is organized according to the four components of statistical investigation. However, this paper will focus primarily on Analyzing distribution(s) of data and Interpreting results.

Posing the Question(s) and Collecting Data

Of the nine teachers, five had students conducting experiments in a game context with devices such as dice, coins, cards, spinners, and chips in a bag, while four teachers had students simulate a problem context using spinners, chips in a cup, and a graphing calculator as random generating devices. Teachers expressed purposeful decision making in choosing contexts that they perceived would be engaging and/or familiar for students: simulating basketball free throws, engaging in games and investigating fairness. Three teachers purposely designed their lessons to engage students in making predictions based on intuitions and using the data collection and analysis to compare to their intuitive prediction. Only two teachers, however, had a learning goal for students to form an argument about a problem context. All nine contexts had a known distribution and were amenable to computing a theoretical probability. This becomes important in considering what teachers focused on in the lessons.

Table 2. Summary of Lessons

Teacher & Grade	Description of Experiment or Simulation	Main Learning Goals for Students
Kathy 6th	Experiment drawing chips from a bag	Compare data across samples; compare theoretical to empirical probability; provide a context to compute probabilities
Pam 6th	Simulate gumball machine with 10 chips in a cup	Predict and then compare to data to test intuitions
Wanda 6th	Simulate die rolls to test fairness of die with a graphing calculator	Importance of sample size; compare theoretical to empirical; collect data to form argument about fairness,
Frank 7th	Experiment rolling die to test likelihood of rolling doubles	Compare theoretical to empirical probability; provide a context to compute probabilities
Whitney 7th	Simulate basketball shots with 1 spinner	Compare data across samples; provide a context to compute probabilities
Megan 8th	Simulate basketball shots with 2 different spinners	Compare theoretical to empirical probability; provide a context to compute probabilities
Helen 8th	Experiment two-die game of “Odds and Evens” to evaluate fairness	Predict and then compare to data to test intuitions; provide a context to compute probabilities
Marsha 8th	Groups had different experiments with devices such as spinners, dice, chips, etc	Collect data to form argument about fairness; provide a context to compute probabilities; compare experiences across contexts
Morgan 8th	Experiment of Montana Red Dog card game	Predict and then compare to data to test intuitions; provide a context to compute probabilities

A few teachers used a worksheet designed for students to record their data during data collection, but many allowed students to record data in their own way. Teachers typically instructed groups of students to conduct trials with small sample sizes, with little opportunities for repeated sampling within the groups. In seven lessons, groups of students conducted 40 or less trials. One teacher instructed groups of students to conduct 100 trials, and only one teacher gave students control of data collection and choice of sample size.

Analyzing Data and Interpreting Results

A critical component of statistical thinking occurs in the analysis of distributions of data and considerations of variability. In order to examine distributions of data from a probability experiment, data needs to be examined in small groups as well as across groups. The ways in which data are represented are important in considering how such data are analyzed, particularly with a focus on variability. Looking across the lessons, teachers focused on making two major types of comparisons: 1) comparing expectations to empirical results and 2) comparing empirical results across collections of samples.

Comparing expectations to empirical results

Seven of the teachers use the experiments as a vehicle to have students attend to the differences between what is expected based on theoretical probabilities and what is obtained from empirical results. Three of these teachers began their lesson with students making predictions based on their intuitions. To help students attend to comparisons, teachers promoted and used a variety of representations. A common strategy was to instruct students to record a tally mark for each occurrence of the possible outcomes, sum the tallies, and write the empirical probability for each possible outcome as a fraction or percent. Graphical displays were rarely used, with only three teachers instructing students to construct bar charts. These instances appeared to be more of an exercise in graphing than contributing to examination of a distribution. The predominant use of tallies and condensing of results into empirical probabilities in the form of fractions (often expressed in lowest terms, and thus losing contextual information about sample size and frequency) seems to be the most common instructional pattern to focus students on comparing experimental results with expectations based on a probability distribution.

Comparisons between empirical results and expectations were dependent on students' computing either empirical or theoretical probability or both. Thus, explicit instruction and considerable time spent on computation were also major themes across these lessons. Although all of the contexts used in the lessons had a known distribution that was amenable to computing a theoretical probability, only two of the teachers (Frank and Kathy) had students compute a theoretical probability before doing any experiment. However, theoretical probability was explicitly used as a comparison base in these seven lessons. Teachers overwhelming privileged the theoretical probability as the best or most reliable estimator of probability, often using empirical differences across groups to justify this preference.

During these seven lessons, the teachers predominately expressed an expectation that results from an experiment should be different from expectations based on a theoretical probability and appeared to use the comparisons between empirical and theoretical probability as an example of this difference. Except for Wanda who encouraged students to consider sample size in their arguments about fairness of a die, there was little to no evidence of discussions that promoted an awareness of the effect of sample size and its role in considering the differences between empirical and theoretical probabilities. One exception to this occurred during Megan's lesson when a student asked, "How does the number of spins affect the outcome?"

Comparing across samples

Comparing data across samples seemed to be a major focal point in the instructional activities in the lessons of Kathy and Whitney. However, several other teachers drew attention to differences across samples at least briefly during their lesson (Frank, Helen, Pam, Megan). Only three of the teachers (Whitney, Frank, Helen) displayed data publicly in such a way that students could attend to variability across samples of the same size. These teachers appeared to pay particular attention to variability in experimental results across samples. But again, variability seemed to be mostly used to signify to students that empirical results are not "reliable" because they differ across samples. Public representations of data across groups were typically recorded as an unordered list of fractions for empirical probabilities or tallies of the frequencies of results in a certain data range. These types of representations are not supportive of considering distributions of sample proportions. Only one teacher (Whitney) publicly recorded the data across samples in a tabular form that could be supportive of considering a distribution of sample statistics (i.e., proportion of baskets made).

SUMMARY AND IMPLICATIONS

Although these teachers are engaging students in statistical investigations through probability experiments, we found that the teachers often missed opportunities for deepening students' reasoning. In particular, these missed opportunities occurred during the analyzing and interpreting components of the PCAI model. Teachers' approaches to using empirical probability to estimate uncertainty do not foster a conception of probability as a limit of a stabilized relative frequency after many trials. Teachers almost exclusively chose small samples sizes and rarely pooled class data or used representations supportive of examining distributions and variability across collections of samples. Teachers' beliefs that experimental and theoretical probabilities are always different, along with conducting experiments without regard for sample size, fail to address the heart of the issue: When should estimates of probability, using an experimental or theoretical approach, be similar? What variability should be expected in results from repeated trials within a sample, and across a collection of samples? More importantly, why?

More work is needed for teachers to develop conceptions of the connections between statistics and probability and useful classroom practices (e.g., pooling class data, representations of a distribution) for promoting such connections. Teacher education efforts need to include experiences with authentic statistical inquiry that includes use of simulation tools and modeling pedagogical practices that are useful for examining and discussing data collected during an investigation.

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