DESIGNING OPEN QUESTIONS FOR THE ASSESSMENT OF CONCEPTUAL UNDERSTANDING

BROERS, Nick J.
Maastricht University
The Netherlands

The theory of statistics is composed of highly abstract propositions that are linked in multiple ways. Both the abstraction level and the cumulative nature of the subject make statistics a difficult subject. A diversity of didactic methods has been devised to aid the student in the effort to master statistics, one of which is the method of propositional manipulation (MPM). Based on this didactic method, a corresponding assessment method has been developed.
Basically, in using MPM for assessment purposes, the student is instructed to construct arguments using subsets of elementary propositions. In effect, the assessment procedure demands the student to display knowledge of the interrelationships between the propositions in a particular subset. Analysis of the student responses allows for scoring purely propositional knowledge, as well as conceptual understanding. In this paper we discuss research on the effectiveness of this assessment method, relative to assessment of conceptual understanding using concept mapping.

INTRODUCTION

In the past few years, Broers has conducted a number of studies investigating the potential of a didactic method for stimulating directed self-explanation (Broers, 2002; Broers & Imbos, 2005; Broers, Mur & Bude, 2005). Research showed this method to be of value in stimulating students to probe the interrelationships between various important concepts. Self-explanation has long been established to be of value for students in their efforts to master the learning material. (see e.g. Chi, Bassock, Lewis et al., 1989; Chi, DeLeeuw, Chiu et al., 1994). Because self-explanation is recognized as so important, various methods have been developed to try to stimulate this in activity in students, such as concept mapping (Novak & Gowin, 1984; Novak 1998; Schau & Mattern, 1997), worked-out problems coupled to teaching expectancy (Renkl, 1995) and partially worked-out problems (Stark, 1998). The method that was developed by Broers, and has since been referred to as the method of propositional manipulation (MPM) or the method of directed self-explanation, is comprised of three steps. In the first step, the instructor deconstructs the study material into a finite number of constituent propositions. These propositions pertain to all the important concepts, principles and basic ideas that the student needs to master. In the second step, the list of propositions is converted into a list of questions pertaining to these propositions. This list of questions is handed out to the students as a checklist. It will alert them to all the important questions they must in the end be able to answer. But the learning objective will usually be more ambitious than simply obtaining knowledge of key propositions. Usually we will want our students to develop a cognitive network of interrelated concepts, enabling them to understand the ideas behind the numbers. In order to help them achieve this goal, the third step is to provide them with a number of true-false statements, coupled to a number of questions pertaining to a subset of propositions. The student is then instructed to construct an argument showing the statement to be either true or false; an argument that contains all of the propositions to which the questions pertain. This way the student is directed to reflect on the connections between these various propositions.

As Hubbard (1997) noted, students will tend to learn in anticipation of the exam questions. If the exam only probes superficial, surface level knowledge, many students will not be motivated to invest time and effort to come to a deeper understanding. In the prior research on MPM, motivational problems proved to have a negative impact on the efficacy of this didactic method. Obviously, the motivation of students to work seriously on MPM tasks will be greatly enhanced if the assessment at the end of the course reflects this particular type of training. It is to this end, that we now wish to examine the possibility of using MPM to construct items to assess conceptual understanding of students.

DESIGNING OPEN QUESTIONS USING MPM

As a preliminary to the design of tasks or questions for the assessment of conceptual understanding, the first thing the examinator needs to do is to determine the collection of concepts and their interrelationships that the instructor wishes the student to master. An overview of this conceptual network can be obtained by first compiling a list of pertinent concepts, followed by the construction of a concept map showing how these concepts relate to each other. Suppose our instructor has been teaching the students elementary statistics, and with regard to the theory on estimation wants them to be able to demonstrate comprehension of at least the following network of interrelated concepts:

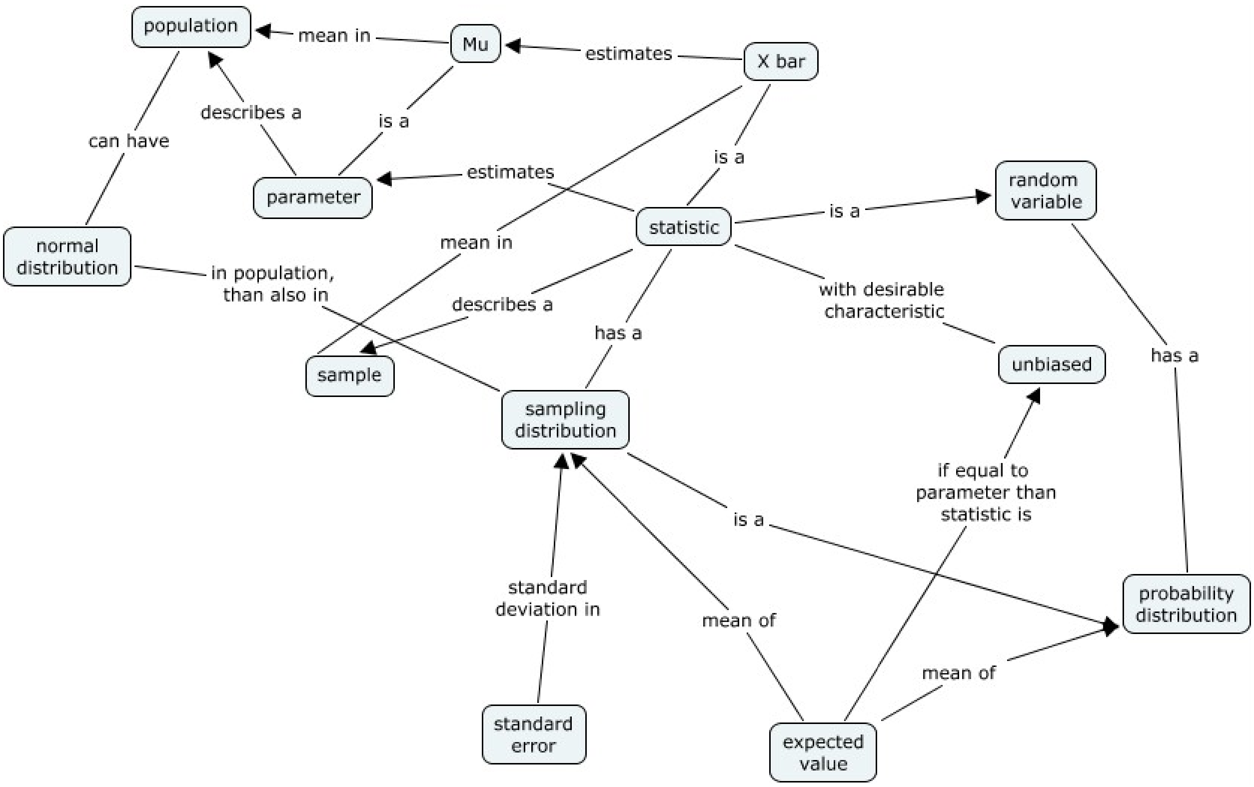

Figure 1: A network of important basic concepts related to estimation

This concept map shows the relations between 13 different concepts. In reality of course, in teaching elementary theory of estimation we would be considering many more concepts. For the purpose of this exposition however, we will restrict ourselves to these 13 concepts. Many more relations and concepts are left implicit in this map. For example, definitions of random variable and sampling distribution are not given.

The next step for the instructor will be to compile a list of all the propositions that pertain to the individual concepts in this map, for example:

- A random variable is a variable of which the outcome is determined by chance
- A statistic describes a sample and is used to estimate an unknown parameter
- An unbiased statistic has an expected value that equals the value of the parameter to be estimated

For the concept map picture above, this would give us 13 propositions. These propositions are subsequently translated into question form, as follows:

- What is random variable?
- What is a statistic?
- What do mean by an unbiased statistic?

Having created an overview of relevant interrelationships with the concept map, and having compiled a list of all the propositions pertaining to the concepts involved, the instructor is now able to design questions that will assess the ability of students to perceive the relevant interrelationships. For instance, the instructor wants the student to demonstrate his or her views on the interrelationships between the concepts “statistic”, "parameter", “random variable”, "probability distribution", “sampling distribution”, “expected value” and “unbiased”.

To this end, in line with MPM, a statement is produced that can be either true or false, for example:

MPM 1: “To judge whether or not a statistic is unbiased, one will have to study its sampling distribution”.

The student is instructed to demonstrate the verity or falsehood of this statement by creating an argument that contains answers to each of the following directing questions:

- What is a statistic?
- What is a random variable?
- What is a sampling distribution?
- What is an expected value?
- What do we mean by an unbiased statistic?
- What is a probability distribution?
- What is a parameter?

A valid argument could run as follows: “A *statistic* is a quantity that describes a sample and which we use to estimate the value of an unknown population *parameter*, which is a quantity that describes a characteristic of a population. A *random variable* is a variable of which the outcome is determined by chance. Each random variable has a *probability distribution*, which gives the probabilities of each of the possible outcomes of the random variable. A statistic is a random variable because the value of the statistic will be determined by the random sample which has been selected from the population. Different samples will yield different values for the statistic. The probability distribution of the possible values that the statistic could take up over an infinite succession of equal sized random samples taken from the same population, is known as the *sampling distribution*. The mean value of this sampling distribution is known as the *expected value* of the statistic. If this expected value equals the parameter value that we wish to estimate, the statistic is said to be *unbiased*. In order to determine whether a statistic is unbiased, we therefore need to know the sampling distribution of the statistic. The statement is therefore true.”

In this form, the student would demonstrate his or her ability to perceive interrelationships between the given concepts. The responses of the students to this MPM task will permit a scoring of purely propositional knowledge as well as a scoring of conceptual understanding. Especially with regard to conceptual understanding, it will not always be clear cut whether or not the student has actually grasped particular relationships. The MPM design allows the construction of multiple items that partly tap the same relationships, thus providing the possibility of cross validation. An example would be the construction of the following item:

MPM 2: “The number that you face after rolling a die is a random variable, just like the mean you will find in a sample. Both variables therefore have a probability distribution.”

Here the student is instructed to demonstrate the verity or falsehood of this statement by creating an argument, that contains answers to each of the following questions:

- What is a statistic?
- What is a random variable?
- What is a sampling distribution?
- What is a probability distribution?
- What is μ?
- What is $\overline{X}$?

In this assignment the student has to profess comprehension of partly the same relationships that he or she had to demonstrate in the previous item.

By using assessment items such as the ones outlined above, a close correspondence is established between the assessment of conceptual understanding and the way it was trained, when use was made of MPM as an instruction method.

A QUALITATIVE STUDY

Subjects and procedure

14 psychology students who in a previous year performed well on an elementary statistics exam, volunteered to participate in this study. The 14 students were randomly allocated to one of two possible groups. Both groups participated in three sessions, which were organized during three consecutive weeks.

During the first session, all students were first presented with the two true-false statements that were mentioned in the previous section. The statements were at this stage not accompanied by any of the directing questions (pertaining to key propositions). Instead students were simply asked to comment on whether they believed the two statements to be either true or false. In addition, as a second task during the fist session, all the students were presented with a list containing the 13 concepts that were displayed in the concept map described in the previous section. The students were asked to construct a concept map showing how they believed these 13 concepts to be interrelated. The purpose of this first session was to probe the potential of concept mapping in making explicit the conceptual understanding of students. The true-false statements would later on be used in an MPM format (with the accompanying directing questions), and it was necessary to establish the amount of information that the true-false statements would yield in themselves, so that any additional information that would be revealed when the true-false statements were accompanied by the directing questions could be more uniquely ascribed to the MPM format.

In the second session, the first group of seven students was presented with the same true-false statements that they responded to in the first session, but this time accompanied by the seven directing questions (see the previous section). The second group received a list containing the seven concepts to which these directing questions pertain, and was asked to construct a partial concept map showing the perceived interrelationships between these seven concepts. (Partial concept map, in the sense that the students were now asked to construct a concept map on the basis of a subset of concepts from the 13 concepts they used for constructing the complete concept map during the first session. The objective of having students construct such a partial concept map, was to establish whether concept mapping would be a more accurate and revealing method for establishing conceptual understanding if fewer concepts were to be connected, than in the case that a larger collection of concepts was to be connected. The additional purpose of having this second group construct a partial concept map on the basis of the same seven concepts that were involved in the first MPM question, was to be able to directly compare the potential of MPM questioning to that of concept mapping in revealing conceptual understanding.

In the third session, the first group was now asked to construct the partial concept map, whereas the second group was now presented with the MPM questions.

Preliminary results

For MPM1, we wished to establish an understanding of at least the following eight relationships:

R1: A random variable has a probability distribution
R2: A statistic is a random variable
R3: A statistic estimates a parameter
R4: A statistic has a sampling distribution
R5: A sampling distribution is a probability distribution
R6: The expected value is the mean of the sampling distribution
R7: A random variable has an expected value
R8: If the expected value equals the value of the parameter, the statistic is unbiased

For the data of the first two sessions, we scored the number of these relationships that were displayed by respectively the total concept map of the first session, the MPM tasks of the second session and the partial concept map of the second session. We found that on average, the total concept map displayed 3.6 relationships, MPM displayed 5.1 relations and the partial concept map displayed 4.4 relations.

Apart from identifying correct relationships, each of the three methods also permitted us to scan for the presence of erroneous relationships. On average, the total concept map suggested 0.7 errors, MPM revealed 2.1 errors, and the partial concept map showed 0.4 errors. Although these patterns of means suggest that MPM may be superior in revealing conceptual understanding, the means do not differ significantly when analyzed with nonparametric statistical tests. Since the number of observations is small (7 in each group), the quantitative analyses have limited power and therefore only limited use. A qualitative look at the data will provide us with important additional information. To this end, we will look at the data from our first subject.

In response to the first true-false statement (without the accompanying, directing questions), our first subject wrote

> "Correct. If the mean of the sampling distribution equals the population mean, then the statistic is unbiased. If it strongly deviates from the population mean, than it is biased".

It should be noted that most of the students were very familiar with the idea of using the sample mean for the estimation of the population mean, but were less accustomed to thinking more generally in terms of using a statistic (any statistic) for estimating a corresponding parameter. This tendency to concretize thinking on estimation in terms of estimating the population mean is apparent in the above response, which mentions that the expected value should equal the population mean, which of course is only true when this is our parameter of interest. Note that none of the relationships that we wish to see reflected in the answer, is explicitly mentioned. In order to qualify this students' response as correct, we would have to assume a lot of implicit meaning.

The concept map of this student incorporated all 13 concepts, but only a minor subset of links (arrows) were accompanied by comments. Most links, like "Expected value--→ Unbiased" or "sampling distribution--→ expected value", were left unspecified, which makes it hard if not impossible to decide whether or not the subject truly understands the relation between the specified concepts. It is difficult to understand why this subject commented on some of the links, but not on most of the others. Of the eight relationships we wish the student to demonstrate, three are explicitly manifest in the concept map, whereas an additional two relationships can be reasonably inferred from the pattern of arrows.

In response to the first MPM question, the subject responded:

> "A statistic is a variable with which we want to approximate the true parameter value μ. This μ is an unknown value that represents the mean of all scores in the population. A statistic is a random variable, the precise value of which is the result

of a random phenomenon. The probability distribution of this statistic, the distributional curve that couples a density to every interval of probabilities, is the sampling distribution. A sampling distribution is an imaginary probability distribution that would result if we redrew the statistic infinitely often. A statistic is unbiased if the expected value, the mean of the sampling distribution, equals the population mean. If we then study the sampling distribution and we find that $\mu_{\bar{X}} = \mu_X$, than our statistic is unbiased. The statement is therefore correct."

In this argument, it is again clear that for this student, "a statistic" means the sample mean, and the parameter is exclusively thought of as the population mean. For the rest, the argument is correct, all the relationships are clearly demonstrated and in addition, all the individual propositions (such as "what is a sampling distribution?", "what is a random variable", etc.) are correctly defined and understood. For this particular subject at least, the MPM tasks reveals a wealth of information that was not provided by either the concept map or the isolated true-false statement.

CONCLUSIONS

Although here we have only discussed the work of a single subject, study of all 14 subjects revealed a consistent pattern of observations that can be summed up as follows. First of all, it takes less time for a subject to draw a concept map than to answer an MPM type of question, and in a single drawing a multitude of relationships between concepts can be shown. The downside of this economic assessment tool is the fuzzyness of the information it provides. Contrary to instruction and (admittedly superficial) training, several subjects did not place any comments next to the arrows, leaving it to the test administrator to decide whether a subject had actually grasped a particular interrelationship or not. Furthermore, even where students had placed commentaries, these were often obscure, making it difficult to evaluate them.

The advantage of MPM over concept mapping is that the subject is forced to make his view of interrelationships explicit in an argument. If the argument falters, this immediately suggest that the student fails to perceive a number of important connections. On the whole, subjects tend to be more explicit in their responses to the MPM questions than in their concept map construction. In addition, MPM requires the students to demonstrate their knowledge of individual propositions, making it possible to score not only understanding of complex relationships, but also knowledge of more basic facts and principles. Although responding to an MPM question is more time consuming than drawing a concept map, the amount of time required to dedicate oneself to this task in a serious and meaningful way is not unreasonably large. Scoring the quality of the responses of MPM seems easier than scoring the quality of a concept map, although further research is desirable to permit a quantifiable comparison between the two assessment methods in this respect.

Some students do have difficulty in constructing a proper argument, but the same goes for concept mapping, which seems a fairly trivial task but is not an easy technique for a novice.

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