STATS 10X WORKSHOP

EXAM PREP 3: CHAPTERS 8 & 9 PROPORTIONS

SAT 16 & TUE 26 OCT 2010

Students **MUST REGISTER** for all workshops with
The Student Learning Centre, 3rd Floor, Information Common

Statistical help available at the SLC

The Student Learning Centre (SLC) offers help for STATS 10x by offering:

- one-on-one tutoring help, and
- a number of workshops

One-on-one help over S2 2010 including exam period

One-on-one assistance for STATS 10x is available at the SLC. Check appointment availability and book at SLC reception in person (third floor, Information Commons building) or by calling 373-7599 ext. 88850.

Note: SLC tutors are not allowed to help students complete their assignments.

SLC STATS 10x Exam Prep Workshops

Any questions regarding STATS 10x workshops should be forwarded to:

Leila Boyle; SLC Statistics Co-ordinator: l.boyle@auckland.ac.nz

These twelve workshops (six different sessions, each repeated twice) are held prior to the exam, from Saturday 2 October until Monday 1 November 2010 (inclusive).

These workshops concentrate on questions reviewing the **basic concepts**, rather than questions on finer details. They are designed to assist students to achieve a pass and **don't cover all material**.

The timetable for these workshops is available at this workshop, at SLC Reception and on Leila's website. Please enrol in each of your preferred workshops by EITHER:

- ***Dropping by*** **the SLC Reception to enrol in person (Room 320, Level 3, Information Commons Building, 11 Symonds Street) OR**
- ***Emailing*** **slc@auckland.ac.nz with your name, ID number, and the name, date and time of the workshop/s you wish to attend OR**
- ***Calling*** **the SLC Reception on 373-7599 ext. 88850 and book over the phone.**

Useful Websites

- SLC webpage: **www.slc.auckland.ac.nz**
- Cecil: **https://cecil.auckland.ac.nz**
- **Leila's website for STATS 10x SLC workshop handouts & information: www.stat.auckland.ac.nz/~leila**

Recall: **Chapter 8 – Confidence Intervals**

Look at blue pages for good notes and test/exam questions for practice

- A **confidence interval** gives a range of plausible values for the parameter of interest that is consistent with the data (at the specified level of confidence). It determines the **size** of the effect or difference.

- You can do all kind of CI's, 90%, 95%, 99%...

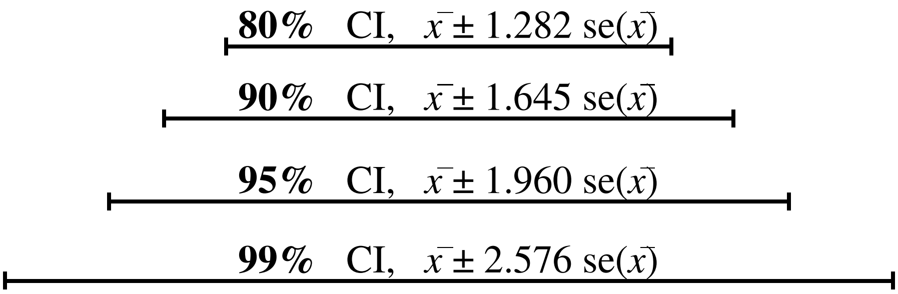


Figure 8.1.3 The greater the confidence level, the wider the interval

From *Chance Encounters* by C.J. Wild and G.A.F. Seber, © John Wiley & Sons, 2000.

- Increasing the confidence level will **increase** the width of the interval.

- Increasing the sample size will make the confidence interval more precise.

- To double the accuracy of the confidence interval we **need 4 times** as many observations.

- To triple the accuracy of the confidence interval we **need 9 times** as many observations.

- 95% confidence interval

  ✓ Range of plausible values for the parameter of interest that contains the **true value** of our parameter of interest for 95% of samples taken.

  ✓ 5% of samples taken will not have the parameter within the calculated confidence interval.

  ✓ We do not know if the sample we have taken is one of the 95% that contains the true unknown parameter. All we can say is that 95% of the time it will.

✓ If you take 1000 samples, based on the same sampling protocol, then you can expect approximately 950 of these samples will contain the true value (e.g. true mean, true proportion, true difference between means, true difference between proportions) of the population.

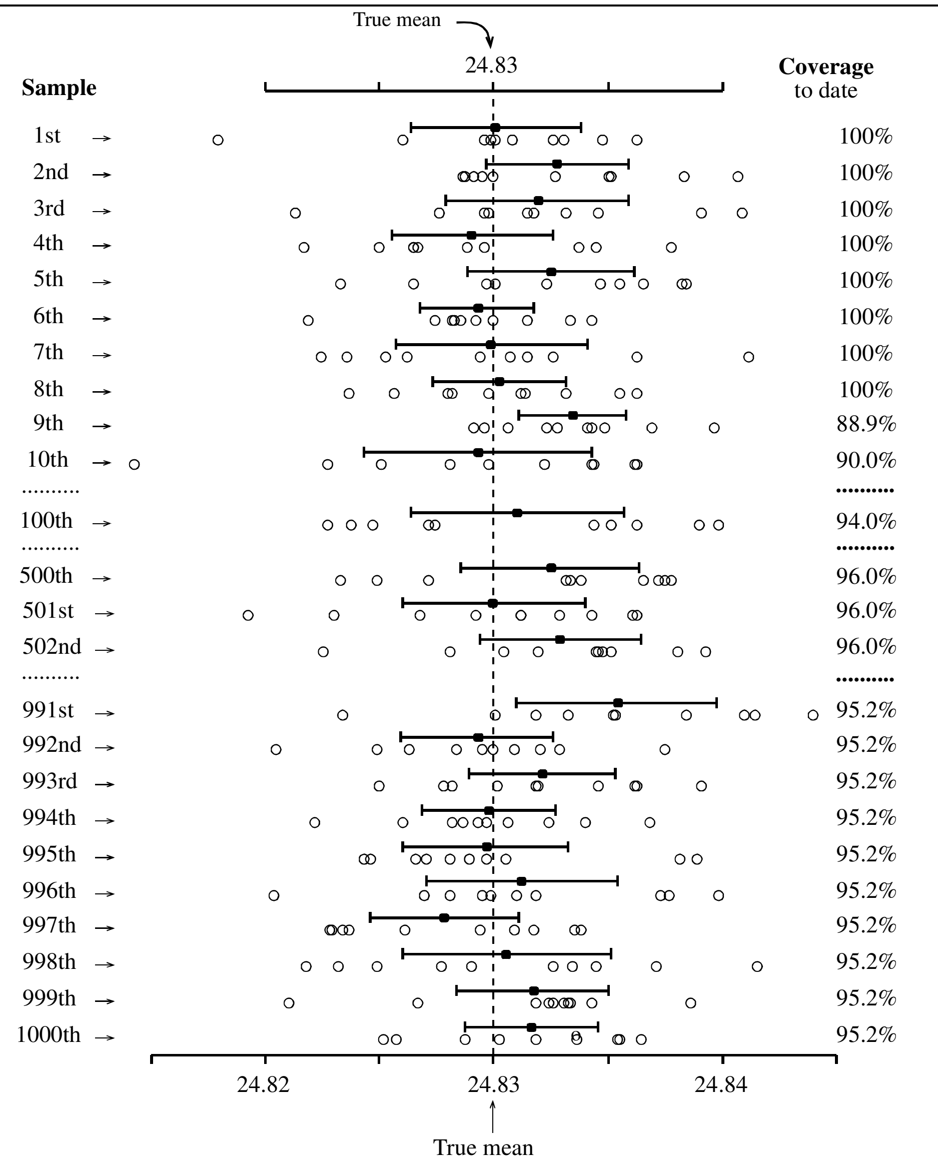


Figure 8.1.2 Samples of size 10 from a Normal(μ=24.83, s=.005) distribution and their 95% confidence intervals for μ..

From *Chance Encounters* by C.J. Wild and G.A.F. Seber, © John Wiley & Sons, 1999.

Step-by-Step Guide to Producing a Confidence Interval by Hand

1. State the **parameter** to be estimated (symbol and words).
Is it μ, p, $\mu_1 - \mu_2$, or $p_1 - p_2$?

2. State the **estimate** and its value.

3. Write down the **formula** for a CI, ***estimate* ± *t* × se(*estimate*)**, from the Formula Sheet

4. Find the appropriate **standard error** from the Formula Sheet.

5. Find the appropriate ***df***.

6. Find the appropriate ***t*-value.** This is called the '*t*-multiplier'.
Need to know the confidence level and *df*.

7. Calculate the **confidence limits**, the end points of the confidence interval.

8. Interpret the interval using plain English.
Use the confidence limits to construct an answer to the original question in plain English.

- There are four different types of problem covered in Chapter 8:
  1. Single mean 2. Single proportion. 3. Difference between two means
  4. Difference between two proportions:
     Situation (a) ***Proportions from two independent samples***
     Situation (b) ***One sample of size n, several response categories***
     Situation (c) ***One sample of size n, many yes/no items***

- The **estimate** is based on the **parameter** of interest we are investigating:

Parameter	**Estimate**
1. Single mean $\boldsymbol{\mu}$:	*estimate* = $\bar{x}$
2. Single proportion $\boldsymbol{p}$:	*estimate* = $\hat{p}$
3. Difference between two means $\boldsymbol{\mu_1 - \mu_2}$: (independent samples)	*estimate* = $\bar{x}_1 - \bar{x}_2$
4. Difference between two proportions $\boldsymbol{p_1 - p_2}$:	*estimate* = $\hat{p}_1 - \hat{p}_2$

- The ***t*-multiplier** (means) / ***z*-multiplier** (proportions) is based on:
  - ✓ Whether we are investigating means or proportions
  - ✓ The desired level of confidence
  - ✓ The degrees of freedom

Estimate	Degrees of Freedom
1. *estimate* = $\overline{x}$	$df = n - 1$
2. *estimate* = $\hat{p}$	$df = \infty$
3. *estimate* = $\overline{x}_1 - \overline{x}_2$	$df = minimum(n_1 - 1, n_2 - 1)$
4. *estimate* = $\hat{p}_1 - \hat{p}_2$	$df = \infty$

i.e. for proportions, assume the degrees of freedom is infinity, hence replace *t* with *z* score (i.e. the standard Normal).

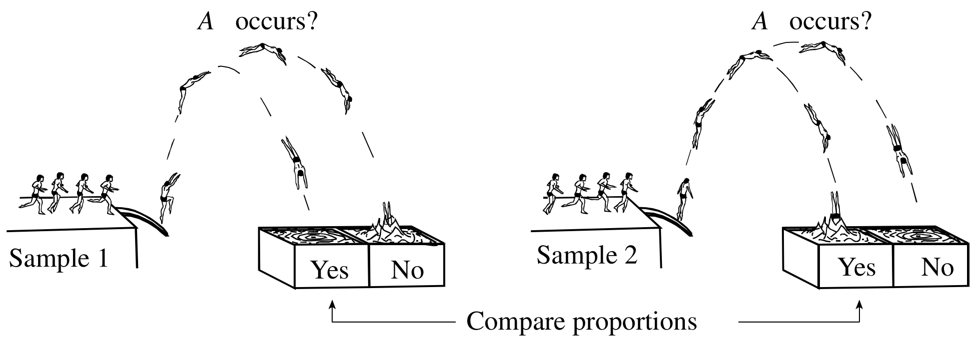
✓ The **standard error** is based on the estimate, the number of samples and sample size(s).

Estimate	se(estimate)
1. *estimate* = $\overline{x}$	$se(\overline{x}) = \frac{s}{\sqrt{n}}$
2. *estimate* = $\hat{p}$	$se(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
3. *estimate* = $\overline{x}_1 - \overline{x}_2$	$se(\overline{x}_1 - \overline{x}_2) = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$
4. *estimate* = $\hat{p}_1 - \hat{p}_2$	**Situation (a)** ***Proportions from two independent samples*** $se(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$ **Situation (b)** ***One sample of size n, several response categories*** $se(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{\hat{p}_1 + \hat{p}_2 - (\hat{p}_1 - \hat{p}_2)^2}{n}}$ **Situation (c)** ***One sample of size n, many yes / no items*** $se(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{\min imum(\hat{p}_1 + \hat{p}_2, \hat{q}_1 + \hat{q}_2) - (\hat{p}_1 - \hat{p}_2)^2}{n}}$ where $\hat{q}_1 = 1 - \hat{p}_1$ and $\hat{q}_2 = 1 - \hat{p}_2$

- CIs for the difference between two means/proportions:
  - ✓ If the CI contains 0 (i.e. one negative and one positive number), there may be no difference between the two means/proportions.
  - ✓ If CI is positive, then μ_1/p_1 is higher/larger than μ_2/p_2.
  - ✓ If CI is negative, then μ_1/p_1 is lower/smaller than μ_2/p_2.

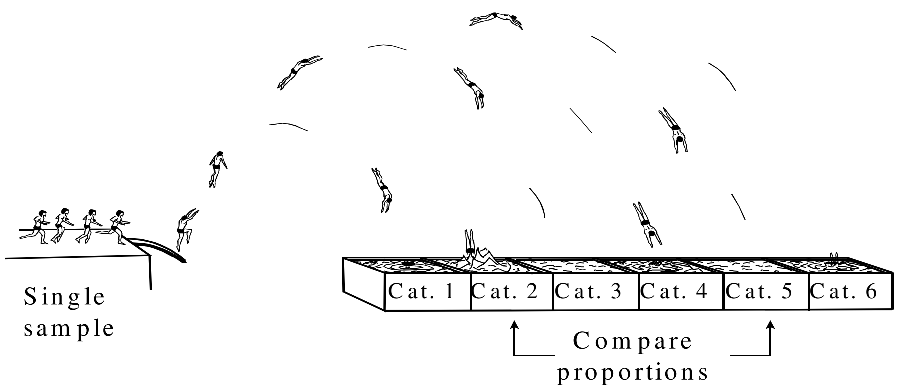
- **3 sampling situations for the difference between two proportions**
-

Situation (a) *Proportions from two independent samples*



From *Chance Encounters* by C.J. Wild and G.A.F. Seber, © John Wiley & Sons, 1999.

Situation (b) *One sample of size n, several response categories*



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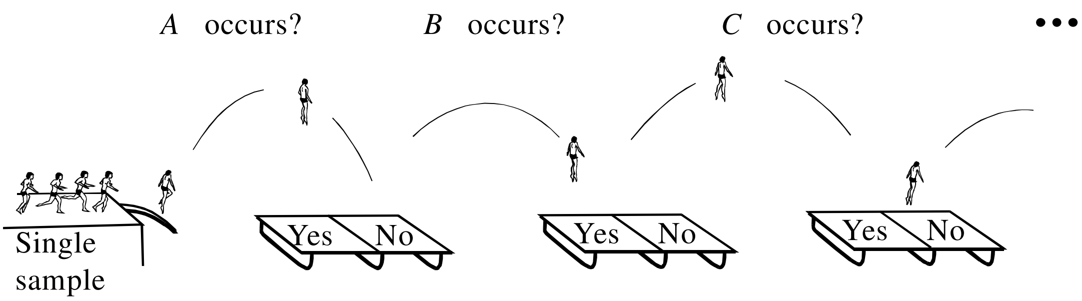
Situation (c) *One sample of size n, many yes/no items*



From *Chance Encounters* by C.J. Wild and G.A.F. Seber, © John Wiley & Sons, 2000.

Chapter Nine

In hypothesis testing and confidence intervals we use **sample** data to make inferences (draw conclusions) about **population** parameters.

- A **confidence interval** gives a range of plausible values for the parameter of interest that is consistent with the data (at the specified level of confidence).
- A **significance test,** tests one possible value for the parameter, called the **hypothesised** value. We determine the strength of evidence provided by the data against the null hypothesis, H_0.

A significance test determines the **strength** of the evidence **against** the **hypothesised** value, while a confidence interval determines the **size** of the effect or difference.

Significance testing is a method used to deal with the **uncertainty** about the true value of a parameter caused by the sampling variation in estimates.

H_0 denotes the **null hypothesis** whereas H_1, denotes the **alternative hypothesis**.

The null hypothesis, H_0

- ✓ It is our best guess as to what we think the parameter of interest is – a single plausible value.
- ✓ General form: H_0: *parameter = hypothesised value (some number)*
- ✓ The hypothesised value is **not** the parameter of interest. Remember that the parameter of interest is an unknown quantity.

The alternative hypothesis, H_1

- ✓ Specifies the type of departure from H_0 that we expect to detect.
- ✓ Corresponds to the research hypothesis.
- ✓ There are three different types:
  - H_1: *parameter ≠ hypothesised value (some number)*
  - H_1: *parameter < hypothesised value (some number)*
  - H_1: *parameter > hypothesised value (some number)*
- ✓ ONLY use a one-sided alternative when you have prior information or a theory.

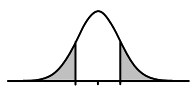
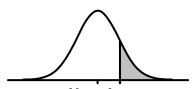
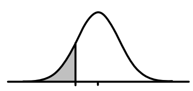
The *t*-test-statistic

- ✓ Is the number of standard errors our estimate is from the hypothesised value.
- ✓ We calculate it using: $t_0 = \dfrac{\textit{estimate} - \textit{hypothesised value}}{\textit{std error}}$

The *P*-value

- ✓ Is the probability that sampling variation would produce an estimate that is further away from the hypothesised value than the estimate we obtained from our data, assuming that the null hypothesis is true.
- ✓ It is a measure of evidence **against** H_0.
- ✓ We calculate the *P*-value using the *t*-test statistics and the appropriate Student's *t* distribution.
- ✓ We interpret the size of the *P*-value using the following table:

P-value	Evidence against H_0
> 0.12	None
≈ 0.10	Weak
≈ 0.05	Some
≈ 0.01	Strong
< 0.001	Very Strong

Alternative hypothesis	*P-value* = area of shaded region $T \sim Student(df)$
H_1: parameter ≠ hypothesised value (2-sided)	 2-tailed test
H_1: parameter **>** hypothesised value (1-sided)	 1-tailed test
H_1: parameter **<** hypothesised value (1-sided)	 1-tailed test

Step-by-Step Guide to Performing a *t*-test by Hand

1. State the **parameter**, being discussed (symbol/s and words).
   Is it μ, p, $\mu_1 - \mu_2$, or $p_1 - p_2$?

2. State the **null hypothesis**, $\boldsymbol{H_0}$.

   i.e. H_0: parameter = *hyp. val.*

3. State the **alternative hypothesis**, $\boldsymbol{H_1}$.

   i.e. H_1: parameter ≠ *hyp. val.*
   OR H_1: parameter < *hyp. val.*
   OR H_1: parameter > *hyp. val.*

4. State the **estimate**, and its value.

5. Calculate the ***t*-test statistic**:

   - Use: $t_0 = \dfrac{\text{estimate} - \text{hypothesised value}}{\text{std error}}$ (from the Formula Sheet)

     Write down the estimate and the hypothesised value.

     Find the appropriate standard error and the *df* using the Formula Sheet.

   - Calculate t_0.

6. Calculate the ***P-value***.

   See *Finding P-values*, pages 9-12, *Excel* Tutorial, Section F – Lecture Workbook

7. **Interpret** the *P-value*.

   See *Interpreting the P-value*, page 11, Chapter 9 Notes, Section B – Lecture Workbook

8. Calculate the **confidence interval**.

9. **Interpret** the confidence interval using plain English.

Statistical significance

- ✓ Relates to having evidence of the **existence** of an effect or difference.
- ✓ Determined by examining the ***P-value*** of your significance test.

Practical significance

- ✓ Depends on the **size** of the effect or difference.
- ✓ Determined by examining the **confidence interval** in relation to the research in question.

Chapters 8 & 9 – Questions

1. Which one of the following statements about hypothesis testing is **false**?
   (1) The larger the *P-value,* the stronger the evidence against the null hypothesis.
   (2) The *P-value* is the probability that, if the null hypothesis were true, sampling variation would produce an estimate that is further away from the hypothesised value than our data estimate.
   (3) We cannot establish a hypothesised value for a parameter, we can only determine whether there is evidence to reject a hypothesised value.
   (4) H_0 is typically a sceptical reaction to a research hypothesis.
   (5) The *P-value* measures the strength of evidence against the null hypothesis.

2. Which **one** of the following statements about significance tests is **false**?
   (1) Formal tests can help determine whether effects we see in our data may just be due to sampling error.
   (2) The *P-value* associated with a two-sided alternative hypothesis is obtained by doubling the *P-value* associated with a one-sided alternative hypothesis.
   (3) The *P-value* says nothing about the size of an effect.
   (4) The data should be carefully examined in order to determine whether the alternative hypothesis needs to be one-sided or two-sided.
   (5) The *P-value* describes the strength of evidence against the null hypothesis.

3. Which **one** of the following statements about hypothesis testing is **false**?
   (1) We make hypotheses about sample estimates.
   (2) In the t-test, the null hypothesis, H_0, always involves an "=" sign.
   (3) We investigate whether a hypothesised value is plausible in light of our sample data.
   (4) If we get a t-test statistic with a value of 3, we know the sample estimate is 3 standard errors above the hypothesised value.
   (5) A large *P-value* does not imply that H_0 is true.

Questions 4 and **5** refer to the following information.

A survey of 1,146 New Zealanders was published in the 23 March 1992 issue of Time magazine. In response to the question "Is it a good time to buy a major household item?" 585 respondents replied "Yes", 332 replied "no" and 229 replied "don't know".

4. The proportion of the sample who think that it is a good time to buy a major household item is:

   (1) 0.20

   (2) 0.29

   (3) 0.49

   (4) 0.51

   (5) 0.95

5. A 95% confidence interval for the proportion of New Zealanders who think that it is a good time to buy a major household item is:

   (1) [0.43, 0.50]

   (2) [0.45, 0.57]

   (3) [0.48, 0.54]

   (4) [0.50, 0.52]

   (5) [0.61, 0.67]

6. Which one of the following statements is **true**?

   (1) A point estimate is preferred to a confidence interval because the interval summarises the uncertainty due to sampling variation.

   (2) The standard error used to construct the interval will be identical for all samples of the same size.

   (3) If I plan to do a study in the future in which I will take a random sample and calculate a 90% confidence interval, there is a 90% chance that I will catch the true value of *p* in my interval.

   (4) The size of the multiplier *t* depends on only the sample size and not the desired confidence level.

   (5) The process of using a population parameter to construct an interval for the data estimate is an example of statistical inference.

Questions 7 to **10** refer to the following information.

Four single-sex and two co-educational schools in Melbourne, Australia, were asked to participate in a recent study designed to examine adolescents' attitudes towards confidentiality in the school counselling situation. All six schools were private schools. Three of the single-sex schools agreed to take part; one of the single-sex schools and both of the co-educational schools declined to take part in the study.

The students were advised that participation was voluntary and anonymous, and that they were free to withdraw from the study at any time.

Questionnaires were completed in school. Some results from the study are given in Table 4 below. It shows the percentage of students (aged 14–18 years) agreeing, disagreeing, or unsure as to whether the school counsellor should tell parents in situations of contraceptive use, and/or pregnancy.

There were 221 male respondents and 174 female respondents.

Situation	**Response**			**Sample size**
	Agree %	**Disagree %**	**Unsure %**	
Contraception				
males	33	52	15	221
females	13	79	8	174
Pregnancy				
males	41	43	16	221
females	15	74	11	174

Table: Adolescents' Attitudes Towards Confidentiality

Let p_{agree} be the proportion of all Australian **male** secondary school students (aged 14–18 years) who agree that a counsellor should tell parents in situations of pregnancy and $p_{disagree}$ be the corresponding proportion who disagree.

The results from the study are used to conduct a 2-tailed test for no difference between p_{agree} and $p_{disagree}$.

7. An estimate of the difference between p_{agree} and $p_{disagree}$ is:

   (1) −1.9

   (2) −0.02

   (3) −0.2

   (4) −0.59

   (5) −0.19

8. For the purpose of calculating $se(\hat{p}_{agree} - \hat{p}_{disagree})$, the sampling situation can be described as:

(1) one sample of size 395, several response categories.

(2) one sample of size 395, many yes/no items.

(3) two independent samples of sizes 221 and 174.

(4) one sample of size 221, several response categories.

(5) one sample of size 221, many yes/no items.

9. The expression for evaluating the test statistic for the null hypothesis, H_0: $p_{agree} - p_{disagree} = 0$, is:

(1) $\dfrac{\hat{p}_{agree} - \hat{p}_{disagree}}{se(\hat{p}_{agree} - \hat{p}_{disagree})}$

(2) $\dfrac{\hat{p}_{agree} - \hat{p}_{disagree}}{\sqrt{se(\hat{p}_{agree})^2 - se(\hat{p}_{disagree})^2}}$

(3) $\dfrac{p_{agree} - p_{disagree}}{se(\hat{p}_{agree}) + se(\hat{p}_{disagree})}$

(4) $\dfrac{p_{agree} - p_{disagree}}{se(\hat{p}_{agree} - \hat{p}_{disagree})}$

(5) $\dfrac{\hat{p}_{agree} - \hat{p}_{disagree}}{se(\hat{p}_{agree}) + se(\hat{p}_{disagree})}$

10. Let p_{contra} and p_{preg} be the proportions of Australian **female** students (aged 14–18 years) who **disagree** that a counsellor should tell parents in situations of contraceptive use, and pregnancy, respectively. Information from Table 4 is used to construct a 95% confidence interval for the difference $p_{contra} - p_{preg}$.

The formula for the standard error of the estimate, $se(\hat{p}_{contra} - \hat{p}_{preg})$, is:

(1) $\sqrt{\dfrac{\hat{p}_{contra}(1 - \hat{p}_{contra})}{174} + \dfrac{\hat{p}_{preg}(1 - \hat{p}_{preg})}{174}}$

(2) $\sqrt{\dfrac{\hat{p}^2_{contra}}{174} - \dfrac{\hat{p}^2_{preg}}{174}}$

(3) $\sqrt{\dfrac{(1 - \hat{p}_{contra}) + (1 - \hat{p}_{preg}) - (\hat{p}_{contra} - \hat{p}_{preg})^2}{174}}$

(4) $\sqrt{\dfrac{(\hat{p}_{contra} + \hat{p}_{preg}) - (\hat{p}_{contra} - \hat{p}_{preg})^2}{174}}$

(5) $\sqrt{\dfrac{\hat{p}^2_{contra}}{174} + \dfrac{\hat{p}^2_{preg}}{174}}$

11. The general formula for a confidence interval for the difference between two proportions is:

$$\hat{p}_1 - \hat{p}_2 \pm z \times se(\hat{p}_1 - \hat{p}_2)$$

Which **one** of the following statements about confidence intervals for the difference between two proportions is **false**?

(1) The value of the z-multiplier depends on the confidence level.
(2) The confidence interval method for proportions works only if the sample size is sufficiently large.
(3) The confidence interval is centred on $\hat{p}_1 - \hat{p}_2$.
(4) The value of the z-multiplier depends on the sample size.
(5) The size of the standard error depends on the sampling situation.

Questions 12 to **16** refer to the following information.

A survey of 2171 men and 2412 women in Auckland in the early 1990s found that 10% of men abstained from drinking alcohol compared with 16% of women.

We wish to compare the proportion of female abstainers, p_{female}, with the proportion of male abstainers, p_{male}.

12. The sampling situation is **best** described as:

(1) two independent samples.
(2) one sample, several response categories.
(3) one sample, many yes/no items.
(4) two samples, several response categories.
(5) two samples, many yes/no items.

13. Based on the data, a 95% confidence interval for $p_{\text{female}} - p_{\text{male}}$ is (0.041, 0.079). Which **one** of the following statements is **false**?

(1) Based on the data, a 99% confidence interval would be wider than 0.038.
(2) The point estimate of $p_{\text{female}} - p_{\text{male}}$ is 0.06.
(3) We are confident that the proportion of female abstainers is larger than the proportion of male abstainers.
(4) Zero is a plausible value for $p_{\text{female}} - p_{\text{male}}$.
(5) Based on the data, a 95% confidence interval for $p_{\text{male}} - p_{\text{female}}$ is (−0.079, −0.041).

14. Consider the *P-value* associated with a two-tailed test for no difference between p_{female} and p_{male}. Based on the confidence interval in Question **13**, which **one** of the following statements is **true**?

(1) The *P-value* is much less than 5%.
(2) The *P-value* is around 10%.
(3) We do not have enough information to determine the approximate *P-value*.
(4) The *P-value* is greater than 5%.
(5) The *P-value* is just below 5%.

Questions 15 and **16** refer to the following **additional** information.

Overall, 13% of the 4583 people surveyed abstained from alcohol. We are interested in p_{abstain}, the proportion of people who abstain from alcohol.

A *t*-test of the hypotheses:

$$H_0 : p_{\text{abstain}} = 0.1$$
$$H_1 : p_{\text{abstain}} \neq 0.1$$

gives a test statistic of 6.04 and a *P-value* of 0.000.

15. Which **one** of the following statements is **false**?

(1) The test is significant at the 1% level of significance.
(2) If the null hypothesis is true, it is extremely unlikely that sampling variability would give values further away from the hypothesised value, 0.1, than our sample estimate.
(3) The sample estimate, $\hat{p}_{abstain}$, is approximately 6 standard errors above the hypothesised value, 0.1.
(4) If the null hypothesis is true, sampling variability could never give values further away from the hypothesised value, 0.1, than our sample estimate.
(5) The hypothesised value, 0.1, would be outside a 99% confidence interval for p_{abstain}.

16. Which **one** of the following statements gives the **best** interpretation of the hypothesis test result?

(1) There is some evidence that the true population proportion, p_{abstain}, is **not** 0.1.
(2) There is very strong evidence that the true population proportion, p_{abstain}, is **not** 0.1.
(3) There is no evidence that the true population proportion, p_{abstain}, is **not** 0.1.
(4) There is very strong evidence that the true sample proportion, $\hat{p}_{abstain}$, is **not** 0.1.
(5) There is very strong evidence that the true population proportion, p_{abstain}, is 0.1.

A genetic researcher is interested in the hair colour of children with blue eyes. She uses some data collected on children with blue eyes in Caithness, and with blue eyes from Aberdeen, Scotland. The data are given in the table below.

	Hair					
City	Fair	Red	Medium	Dark	Black	**Totals**
Caithness	326	38	241	110	3	718
Aberdeen	1368	170	1041	398	1	2978
Totals	1694	208	1282	508	4	3696

A table of counts for the Caithness and Aberdeen data.

17. Let $\hat{p}_C$ be the sample proportion of blue-eyed children with fair hair in Caithness, and $\hat{p}_A$ be the sample proportion of blue-eyed children with fair hair in Aberdeen. The formula for the standard error of the difference between these proportions, $se(\hat{p}_C - \hat{p}_A)$, is:

(1) $\sqrt{\dfrac{\hat{p}_C(1-\hat{p}_C)}{3696} + \dfrac{\hat{p}_A(1-\hat{p}_A)}{3696}}$

(2) $\sqrt{\dfrac{Min(\hat{p}_C + \hat{p}_A, \hat{q}_C + \hat{q}_A) - (\hat{p}_C - \hat{p}_A)^2}{3696}}$

(3) $\sqrt{\dfrac{\hat{p}_C + \hat{p}_A - (\hat{p}_C - \hat{p}_A)^2}{3696}}$

(4) $\sqrt{\dfrac{\hat{p}_C(1-\hat{p}_C)}{718} + \dfrac{\hat{p}_A(1-\hat{p}_A)}{2978}}$

(5) $\sqrt{\dfrac{\hat{p}_C(1-\hat{p}_C)}{718} - \dfrac{\hat{p}_A(1-\hat{p}_A)}{2978}}$

18. Which one of the following statements about a confidence interval for a parameter p is **false**?

(1) A two-standard-error interval will always capture the true value of p.

(2) Large samples tend to yield narrower 95% confidence intervals than small samples.

(3) In the long run, if we repeatedly take samples and calculate a 95% confidence interval from each sample, we expect that 95% of the intervals will contain the true value of p.

(4) If I plan to do a study in the future in which I will take a random sample and calculate a 90% confidence interval, there is a 90% chance that I will catch the true value of p in my interval.

(5) If a large number of researchers independently perform studies to estimate p, about 95% of them will catch the true value of p in their 95% confidence intervals.

19. A recent study in the USA investigated the drinking habits of a large sample of pregnant women. It found that 16% of the subjects drank alcohol frequently (seven or more drinks a week).

    Let p be the population proportion of pregnant women in the USA who drink alcohol frequently.

    Which **one** of the following statements is **false**?

    (1) The variability of the sample proportion, $\hat{p}$, increases as we increase the size of the sample on which it is based.

    (2) If a second sample were taken, the population proportion p would not change.

    (3) p is the probability that a randomly chosen pregnant woman in the USA drinks alcohol frequently.

    (4) The variability of the sample proportion, $\hat{p}$, decreases as we increase the size of the sample on which it is based.

    (5) The sample proportion $\hat{p} = 0.16$ is an estimate of p.

20. An article in the New Zealand Herald dated 18 February 1999 quotes results of a CNN poll of a random sample of 543 Americans taken shortly after US and British air strikes against Iraq. 74% of those polled supported the air strikes, whereas 13% were opposed. 30% saw the military strikes as an attempt to divert attention from Clinton's impeachment vote, whereas 66% rejected this idea.

    In a separate poll of a random sample of 510 Americans taken by ABC news around the same time, 73% reported support for the air strikes and 62% rejected the idea that the military strikes were an attempt to divert attention from Clinton's impeachment vote.

    Now we wish to construct a confidence interval to compare the proportion of those who rejected the idea that the military strikes were planned as an impeachment vote diversion in the CNN poll, with the corresponding proportion in the ABC News poll.

    The standard error of the difference between these two proportions is:

    (1) 0.0261

    (2) 0.0485

    (3) 0.0009

    (4) 0.0296

    (5) 0.0501

Questions 21 to **23** refer to the following information.

In an April 1995 survey 655 New Zealanders were asked to choose their favourite advertisement. Listed below are the five most popular advertisements for that month together with the percentage of the sample that chose that advertisement.

Air NZ - the birds	9%
Telecom-Spot	7%
Bluebird-chips/penguins	6%
Telecom-animals	5%
Anchor-family series	5%
Some other advertisement	31%
No favourite advertisement	37%

A two-sided *t*-test is conducted to see if there is any evidence of a difference between p_{birds} and p_{animals}. Let p_{birds} be the proportion who have "Air NZ - the birds" as their favourite and p_{animals} be the proportion who have "Telecom - animals" as their favourite. (This is the *t*-test referred to in the questions below.)

21. Which **one** of the following gives the **correct** null and alternative hypotheses?

(1) $H_0 : p_{\text{animals}} = .05$
$H_1 : p_{\text{birds}} > .05$

(2) $H_0 : p_{\text{animals}} \leq p_{\text{birds}}$
$H_1 : p_{\text{animals}} > p_{\text{birds}}$

(3) $H_0 : p_{\text{birds}} - p_{\text{animals}} = 0$
$H_1 : p_{\text{birds}} - p_{\text{animals}} \neq 0$

(4) $H_0 : p_{\text{birds}} - p_{\text{animals}} = .04$
$H_1 : p_{\text{birds}} - p_{\text{animals}} \neq .04$

(5) $H_0 : \hat{p}_{birds} - \hat{p}_{animals} = 0$
$H_1 : \hat{p}_{birds} - \hat{p}_{animals} \neq 0$

22. The expression used to calculate the *t*-test statistic, t_0, is:

(1) $-4 / se\,(\hat{p}_{birds} - \hat{p}_{animals})$

(2) $-0.04 / se\,(\hat{p}_{\text{birds}} - \hat{p}_{\text{animals}})$

(3) $0.04 \pm t \times se\,(\hat{p}_{\text{birds}} - \hat{p}_{\text{animals}})$

(4) $0.04 / se\,(\hat{p}_{\text{birds}} - \hat{p}_{\text{animals}})$

(5) $4 / se\,(\hat{p}_{\text{birds}} - \hat{p}_{\text{animals}})$

23. The *P-value* of the *t*-test is 0.006. Which **one** of the following statements is **true**?

(1) There is no evidence of a difference between p_{birds} and p_{animals}.

(2) The test is significant at the 5% level.

(3) Pr($p_{\text{birds}} > p_{\text{animals}}$) = 0.006

(4) Since the *P-value* is so small the null hypothesis must be false.

(5) Since the *P-value* is so small the null hypothesis must be true.

24. During January 1998, *The New Zealand Herald's* "Summer of Polls" featured the results of a series of public opinion surveys conducted by *Digipoll*.

A total of 650 New Zealanders were asked "Should New Zealand join Australia under a common government?" The responses were:

Yes	14.3%
No	81.5%
Not Sure	4.2%

A 95% confidence interval for the proportion of all New Zealanders who would have answered "Yes" is approximately:

(1) (0.120, 0.166)

(2) (0.101, 0.185)

(3) (0.116, 0.170)

(4) (0.129, 0.157)

(5) (0.134, 0.152)

ANSWERS

1.	**(1)**	2.	**(4)**	3.	**(1)**	4.	**(4)**	5.	**(3)**	6.	**(3)**
7.	**(2)**	8.	**(4)**	9.	**(1)**	10.	**(3)**	11.	**(4)**	12.	**(1)**
13.	**(4)**	14.	**(1)**	15.	**(4)**	16.	**(2)**	17.	**(4)**	18.	**(1)**
19.	**(1)**	20.	**(4)**	21.	**(3)**	22.	**(4)**	23.	**(2)**	24.	**(3)**