STATS 10x WORKSHOP

EXAM PREP 5: CHAPTER 11

MON 25 OCT & MON 1 NOV 2010

Students **MUST REGISTER** for all workshops with
The Student Learning Centre, 3rd Floor, Information Commons

Statistical help available at the SLC

The Student Learning Centre (SLC) offers help for STATS 10x by offering:

- one-on-one tutoring help, and
- a number of workshops

One-on-one help over S2 2010 including exam period

One-on-one assistance for STATS 10x is available at the SLC. Check appointment availability and book at SLC reception in person (third floor, Information Commons building) or by calling 373-7599 ext. 88850.

Note: SLC tutors are not allowed to help students complete their assignments.

SLC STATS 10x Exam Prep Workshops

Any questions regarding STATS 10x workshops should be forwarded to:

Leila Boyle; SLC Statistics Co-ordinator: l.boyle@auckland.ac.nz

These twelve workshops (six different sessions, each repeated twice) are held prior to the exam, from Saturday 2 October until Monday 1 November 2010 (inclusive).

These workshops concentrate on questions reviewing the **basic concepts**, rather than questions on finer details. They are designed to assist students to achieve a pass and **don't cover all material**.

The timetable for these workshops is available at this workshop, at SLC Reception and on Leila's website. Please enrol in each of your preferred workshops by EITHER:

- ***Dropping by* the SLC Reception to enrol in person (Room 320, Level 3, Information Commons Building, 11 Symonds Street) OR**
- ***Emailing* slc@auckland.ac.nz with your name, ID number, and the name, date and time of the workshop/s you wish to attend OR**
- ***Calling* the SLC Reception on 373-7599 ext. 88850 and book over the phone.**

Useful Websites

- SLC webpage: **www.slc.auckland.ac.nz**
- Cecil: **https://cecil.auckland.ac.nz**
- **Leila's website for STATS 10x SLC workshop handouts & information: www.stat.auckland.ac.nz/~leila**

Ch 11: 7-10 Qs in exam

Revision Notes

1 or 2 qual. var.s

Chapter 11 Review – Chi-Square Tests

Look at blue pages for good notes and test/exam questions for practice

- We use **Chi-Square** tests to test **proportions** from **tables of counts**.
- There are 2 kinds of Chi-Square tests:
  1. For **one-way tables of counts**: Test for **goodness of fit** 1 qual var
  2. For **two-way tables of counts**: Test for **independence** 2 qual var.s
- We determine which Chi-Square test to use by the number of samples taken and the number of qualitative variables / factors to be tested.
- **Assumptions** for Chi-Square tests to be **valid** are:
  - **At least 80%** of the **expected counts** must be **5 or more**, and
  - **Every expected count** must be **greater than 1**.
- The **test-statistic** for Chi-Square tests is the **sum** of all the cell contributions from the table:

$$\boxed{x_0^2 = \sum_{\text{all cells in the table}} \frac{(\text{observed - expected})^2}{\text{expected}}} = \sum\frac{(O-E)^2}{E}$$

On formulae sheet

1. One-Way Tables of Counts – Chi-square test for Goodness of Fit

- Use when you have **1 sample** & **1 factor** of interest.
- **Hypotheses** 1 qual var.

  H_0: The data come from the specified distribution.

  H_1: The data do not come from the specified distribution.

- **Expected Count**: For one-way tables: Expected count in j^{th} cell $= p_j \times n$ where p_j is the specified cell's probability
- **Chi-square test-statistic**: $x_0^2 = \sum_{\text{all cells in the table}} \frac{(\text{observed - expected})^2}{\text{expected}}$
- **Degrees of freedom** For one-way tables: $df = J - 1$ where J is the number of categories.

2. Two-Way Tables of Counts – Chi-square test for **Independence** cross-classify

- Use when you have **1 sample** & **2 factors** of interest.

  2 qual var.s **OR** **2 or more independent random samples** & **1 factor** of interest

- **Hypotheses**

  H_0: the two factors are **independent**/not associated/not related

  H_1: the two factors are **not independent**/dependent/associated/related

- **Expected Count**

  For two-way tables:

  Expected count in cell $(i, j) = \frac{R_i C_j}{n}$

  where R_i is the row total
  and C_j is the column total
  and n is the table total

- **Chi-square test-statistic:**

$$x_0^2 = \sum_{\text{all cells in the table}} \frac{(\text{observed - expected})^2}{\text{expected}}$$

- **Degrees of freedom**

  For two -way tables:
  $df = (I - 1)(J - 1)$

  where I is the number of rows
  and J is the number of columns

Another way of looking at it – Chi-square test for **Homogeneity** same family or distn.

The null hypothesis is often written as a statement of **homogeneity** (sameness).

H_0: the underlying distribution of variable 1 is **the same** for each level of variable 2.

H_1: the underlying distribution of variable 1 is **not the same** at all levels of variable 2.

The sampling situation determines which one of the two variables is variable 1 and which one is variable 2. There are two possibilities:

- If you have **2 or more independent samples** taken from **different populations** (variable 2) and **each sample** is then **divided** up by a **factor/qualitative variable** (variable 1) then the null hypothesis can be a statement of homogeneity among the populations from which the samples have been taken:

  H_0: The distribution of variable 1 is the same for each population (variable 2)

- If a **single random sample** has been **cross classified** by variable 1 and variable 2 then the null hypothesis can be either a statement of homogeneity (sameness) on the **rows** or on the **columns,** i.e. it doesn't matter which variable is variable 1 and which variable is variable 2, the hypotheses above are completely interchangeable.

Steps in performing a **Chi-square test**

1. Identify which **situation** you have (either 1 sample or 2 or more independent samples)

2. State the null hypothesis – $\boldsymbol{H_0}$

3. State the alternate hypothesis – $\boldsymbol{H_1}$

4. Calculate the **expected count** for each cell one-way: $\boxed{p_j \times n}$ two-way: $\boxed{\frac{R_i C_j}{n}}$

5. Calculate the **cell contribution** for each cell

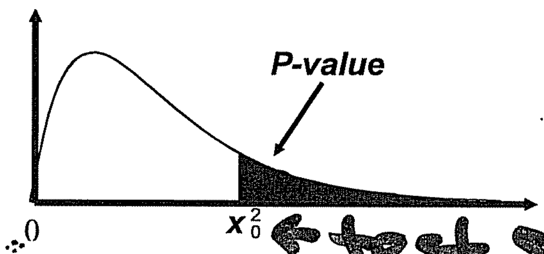
6. Find the Chi-square **test statistic** $\boxed{\sum \frac{(O-E)^2}{E}}$

7. Determine the **degrees of freedom** one-way: $\boxed{J-1}$ two-way: $\boxed{(I-1)(J-1)}$

8. Find the ***P*-value**.

9. **Interpret** the *P*-value and answer the original question.

***P*-value**



← test stat

- *P-value* = pr($\boldsymbol{X}^2 \geq \boldsymbol{x}_0^2$), where $\boldsymbol{X}^2 \sim$ Chi-square(*df*)

The **bigger** the test statistic is, the **stronger** the evidence **against** H_0.

- As with the *t*-test and the *F*-test, the *P-value* is the conditional probability of observing a test statistic as extreme as that observed or more so, **given that** the null hypothesis is true.

- If you have a **small** *p-value* – note which **cells contributed the most** to the **large** Chi-square statistic, i.e. look for cells where there are **large differences** between the **observed** counts and the **expected** counts.

- Look at the size of the differences: create confidence intervals for differences between two proportions (work out the correct standard error formula). Ch 8

Chapter 11 – Questions

Questions 1 to **6** are about the following information.

The data in the following table came from a study of predictors of social distress among 245 American third and fifth grade children by Crick and Ladd reported in *Developmental Psychology*, 1993. One aim of the study was to determine whether there was a relationship between the level of social distress and the peer status of the child.

n

dependence

Level of Social Distress by Peer Status

Level of Social Distress	**Peer Status** Popular	Average	Neglected	Rejected	Controversial	Total
High	8	19	10	26	2	65
Low	41	57	32	33	17	180
Total	49	76	42	59	19	245

n

1. The most appropriate test to use is a:
   - (1) One-way analysis of variance.
   - (2) Two independent sample t-test.
   - (3) Chi-square test of independence.
   - (4) Chi-square test of goodness-of-fit.
   - (5) One sample t-test.

2. Suppose it is appropriate to conduct a Chi-square test for independence (**Note**: This may **not** be correct). The null and alternative hypotheses for this test are:
   - (1) H_0: The level of social distress for the child is not independent of the peer status of the child.
     H_1: The level of social distress for the child is independent of the peer status of the child.
   - (2) H_0: The level of social distress for the child is independent of the peer status of the child.
     H_1: The level of social distress for the child is not independent of the peer status of the child.
   - (3) H_0: The means of the social distress levels for the child are the same for each peer status factor.
     H_1: The means of the social distress levels for the child are not the same for each peer status factor.

F-test

(4) H_0: The level of social distress for the child is related to the peer status of the child.

H_1: The level of social distress for the child is not related to the peer status of the child.

(5) H_0: The distribution of the social distress levels for the child are not the same for each peer status factor.

H_1: The distribution of the social distress levels for the child are the same for each peer status factor.

The Chi-square test for independence is conducted and PASW (SPSS) output is shown below. Some values have been replaced with **.

Level of Social Distress * Peer Status Crosstabulation

			Peer Status					Total
			Popular	Average	Neglected	Rejected	Controversial	
Level of Social Distress	High	Count	8	19	10	26	2	65
		Expected Count	13.0	20.2	11.1	**	5.0	65.0
	Low	Count	41	57	32	33	17	180
		Expected Count	36.0	55.8	30.9	43.3	14.0	180.0
Total		Count	49	76	42	59	19	245
		Expected Count	49.0	76.0	42.0	59.0	19.0	245.0

test stat χ_0^2

$df=(2-1)(5-1)$
$=1\times 4$
$=4$ Q5

$\frac{65\times 59}{245}$ Q3
$=15.7$ (1dp)

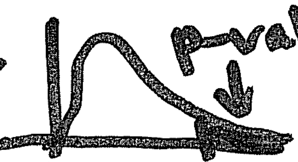
Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)
Pearson Chi-Square	14.674(a)	**	.005
~~Likelihood Ratio~~	~~14.541~~	~~4~~	~~.006~~
~~Linear-by-Linear Association~~	~~3.177~~	~~1~~	~~.075~~
N of Valid Cases	245		

a 0 cells (.0%) have expected count less than 5. The minimum expected count is 5.04.

n

p-value



The cell contributions to the Chi-square test statistic are shown in the table below:

$\frac{(57-55.8)^2}{55.8} = .026$ (3dp) Q4

Level of Social Distress	Peer Status				
	Popular	Average	Neglected	Rejected	Controversial
High	1.923	0.071	0.109	6.757	1.800
Low	0.694	***	0.039	2.450	0.643

Cell contributions to the Chi-square test statistic

3. The **expected count** for those children who had a **high** level of social distress in relation to a **rejected** peer status is:

(1) 14.0

(2) 15.7

(3) 5.0

(4) 11.1

(5) 43.3

4. The **cell contribution** for those children who had a **low** level of social distress in relation to an **average** peer status is:

(1) 0.109

(2) 0.694

(3) 0.039

(4) 0.026

(5) 0.071

5. The **degrees of freedom** for this test are:

(1) 5

(2) 10

(3) 8

(4) 4

(5) 2

6. The *P-value* for the test described in question 5 is 0.005. Which one of the following statements gives the **best** interpretation of this *P-value*?

(1) There is strong evidence that a child's level of social distress is not independent of their peer status.

(2) There is strong evidence that a child's level of social distress is independent of their peer status.

(3) There is evidence that a child's level of social distress is not independent of their peer status.

(4) There is weak evidence that a child's level of social distress is not independent of their peer status.

(5) There is no evidence that a child's level of social distress is not independent of their peer status.

Questions 7 to **9** are about the following information.

An experiment in chicken breeding results in offspring having very curly, slightly curly, or normal feathers. These are called phenotypes 1, 2 and 3 respectively. In one such experiment, 93 chickens were born, giving the following data:

n $2.7\times10^{3} = .0027$

Phenotype	Observed Frequency	Expected Frequency $p_j \times n$	cell contribution $\frac{(O-E)^2}{E}$
1: very curly	23	$.25 \times 93 = 23.25$	$(23-23.25)^2/23.25$
2: slightly curly	50	$.5 \times 93 = 46.5$	$(50-46.5)^2/46.5 = .2634$
3: normal	20	$.25 \times 93 = 23.25$	$(20-23.25)^2/23.25 = .4543$
Total	93 n		test stat, $\chi_0^2 = .7204$

We wish to test the null hypothesis that $p_1 = 0.25$, $p_2 = 0.50$ and $p_3 = 0.25$, where p_j is the probability a chicken is born with phenotype j.

$H_0: p_1 = .25; p_2 = .5; p_3 = .25$

H_1: at least one p_j is not that specified

7. The **expected counts** for Phenotypes 1, 2, and 3 are:

(1) 5.75; 25.00; 5.00

(2) 23.25; 23.25; 23.25

(3) 23.25; 46.50; 23.25

(4) 5.00; 25.00; 5.75

(5) 23.25; 25.00; 23.25

8. The value of the **Chi-square statistic** is:

$\frac{(O-E)^2}{E}$

(1) $\frac{(23-23.25)}{23.25} + \frac{(50-46.50)}{46.50} + \frac{(20-23.25)}{23.25}$

(2) $\frac{(23-23.25)^2}{23.25} + \frac{(50-46.50)^2}{46.50} + \frac{(20-23.25)^2}{23.25} + \frac{(73-69.75)^2}{69.75} + \frac{(70-69.75)^2}{69.75}$

(3) $\frac{(23-23.25)^2}{23.25} + \frac{(50-46.50)^2}{46.50} + \frac{(20-23.25)^2}{23.25}$

(4) $\frac{(23-23.25)^2}{23} + \frac{(50-46.50)^2}{50} + \frac{(20-23.25)^2}{20}$

(5) $\frac{(23-23.25)^2}{93} + \frac{(50-46.50)^2}{93} + \frac{(20-23.25)^2}{93}$

9. The Chi-square test statistic is 0.7204, to four decimal places. The P-value for this test is calculated by: $Pr(X^2 \geq .7204)$

(1) $Pr(0 \leq X^2 \leq 0.7204)$ where $X^2 \sim$ Chi-square(df)

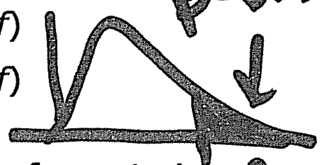
(2) $2 \times Pr(X^2 \geq 0.7204)$ where $X^2 \sim$ Chi-square(df)

(3) $Pr(X^2 \geq 0.7204)$ where $X^2 \sim$ Chi-square(df)

(4) $Pr(X^2 \leq 0.7204)$ where $X^2 \sim$ Chi-square(df)

(5) $2 \times Pr(X^2 \leq 0.7204)$ where $X^2 \sim$ Chi-square(df)

p-val.



χ^2_0

10. Which one of the following statements about data in tables of counts is **false**?

(1) A Chi-square test of homogeneity on the column distributions can be used on a single random sample cross-classified by two response factors.

(2) A Chi-square test of goodness-of-fit can be used on a single random sample classified into categories by the response factor.

(3) A Chi-square test of goodness-of-fit can be used on several random samples each classified into the same categories of the response factor.

(4) A Chi-square test of homogeneity on the row distributions can be used on a single random sample cross-classified by two response factors.

(5) A Chi-square test of independence can be used on a single random sample cross-classified by two response factors.

11. The **most appropriate** plot to use for analysing proportions that are to be tested using Chi-square tests is:

(1) Scatterplot

(2) Side-by-side dot plot

(3) Side-by-side box plot

(4) Table of counts

(5) Histogram

12. When is it **not** appropriate to conduct a Chi-square test?

(1) 1-sample cross-classified by two factors of interest.

(2) Only 20% of expected counts are greater than 5. $\geq 20\%$ Or equal to

(3) 1-sample with 1 factor of interest.

(4) No expected counts are less than 1.

(5) Testing proportions from tables of counts.

Questions 13 to **17** are about the following information.

For the following questions we will consider all the people represented in the table below as being a random sample taken from the U.S. population and **then** cross-classified by *Job Satisfaction* and *Income*.

	Job Satisfaction				
Income (US$)	Very Dissatisfied	Little Dissatisfied	Moderately Satisfied	Very Satisfied	Totals
< 6,000	20	~~24~~	80	82	206
6,000 – 15,000	22	38	104	125	289
15,000 – 25,000	13	28	81	113	235
> 25,000	7	18	54	92	171
Totals	62	108	319	412	901

Using only part of the table, researchers believed that there would be no difference in the level of job satisfaction for people earning less than US$6,000 per year.

13. What **type of test** should be applied:

(1) Chi-square test of independence

(2) Chi-square test of goodness-of-fit

(3) Chi-square test of homogeneity

(4) Chi-square test of independence & Chi-square test of homogeneity

(5) Chi-square test of goodness-of-fit & Chi-square test of homogeneity

14. If the researchers wanted to test that there would be no difference in the 4 levels of job satisfaction for people earning less than US$6,000 per year, what **proportion** would they use?

(1) 0.40

(2) 0.04

(3) 4

(4) 0.25

(5) 0.025

15. The Chi-square χ^2 statistic for the goodness-of-fit test is 67.7864. The **best** interpretation of the *P-value* of 0.000 is:

(1) We have ~~weak~~ evidence against the null hypothesis that there would be ~~no~~ difference in the level of job satisfaction for people earning less than US$6,000 per year.

(2) We have very strong evidence against the null hypothesis that there would be no difference in the level of job satisfaction for people earning less than US$6,000 per year, which implies that the null hypothesis is ~~true~~. false

(3) We have ~~no~~ evidence against the null hypothesis that there would be no difference in the level of job satisfaction for people earning less than US$6,000 per year.

(4) We have ~~strong~~ evidence against the null hypothesis that there would be ~~no~~ difference in the level of job satisfaction for people earning less than US$6,000 per year.

(5) We have very strong evidence against the null hypothesis that there would be no difference in the level of job satisfaction for people earning less than US$6,000 per year.

Questions 16 and **17** are about the following additional information.

A chi-square test was then conducted on the $\boxed{\text{entire data set}}$ and the PASW (SPSS) output is given below.

Level of Income (US$) * Job Satisfaction Crosstabulation

			Job Satisfaction				Total
			Very Dissatisfied	Little Dissatisfied	Moderately Satisfied	Very Satisfied	
Income (US$)	< 6,000	Count	20	24	80	82	65
		Expected Count	14.2	24.7	72.9	94.2	206.0
	6,000 – 15,000	Count	22	38	104	125	289
		Expected Count	19.9	34.6	102.3	132.2	289.0
	15,000 – 25,000	Count	13	28	81	113	235
		Expected Count	16.2	28.2	83.2	107.5	235.0
	> 25,000	Count	7	18	54	92	171
		Expected Count	11.8	20.5	60.5	78.2	171.0
Total		Count	62	108	319	412	901
		Expected Count	62.0	108.0	319.0	412.0	901.0

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)
Pearson Chi-Square	11.989 (a)	**	.214
~~Likelihood Ratio~~	~~14.541~~	~~**~~	~~.220~~
~~Linear-by-Linear Association~~	~~3.177~~	~~**~~	~~.886~~
N of Valid Cases	901		

a 0 cells (.0%) have expected count less than 5. The minimum expected count is 11.77.

df = (4−1)(4−1)
= 3×3 = 9

p-value

The cell contributions to the Chi-square test statistic are shown in the table below:

Income (US$)	Job Satisfaction: Very Dissatisfied	Little Dissatisfied	Moderately Satisfied	Very Satisfied
< 6,000	2.393	0.019	0.684	1.579
6,000 – 15,000	0.225	0.326	0.028	0.387
15,000 – 25,000	0.622	0.001	0.058	0.286
> 25,000	1.931	0.304	0.707	2.438

Cell contributions to the Chi-square test statistic

16. Which one of the following is **false**?

T (1) The Chi-square test can be used to test for independence between *Job Satisfaction* and *Income*. $P_i \times n$ $R_i C_j / n$

T (2) In general, the expected frequencies to be used in the Chi-square test statistic depend upon the hypothesis one wishes to test.

T (3) When testing for independence between *Job Satisfaction* and *Income*, the null hypothesis tested is that *Income* is independent of *Job Satisfaction*.

T (4) For Chi-square tests, a large value for the Chi-square statistic provides evidence against the null hypothesis.

(5) When testing for independence between *Job Satisfaction* and *Income*, a ~~large~~ Small *P*-value provides evidence against independence.

17. On the basis of the information from the Chi-square test, which one of the following is **false**?

T (1) Small departures from independence between *Income* and *Job Satisfaction* cannot reliably be detected with small samples.

(2) The Chi-square test works best when there are ~~small~~ large counts in several of the cells.

T (3) The sampling situation for this data is "One sample cross-classified by two factors."

T (4) Large departures from independence between *Income* and *Job Satisfaction* can reliably be detected with large samples.

T (5) This data provides no evidence of a relationship between *Income* and *Job Satisfaction*. p-value = .214

Questions 18 to **22** are about the following information.

A market research company interviewed 299 randomly selected car owners in Auckland. Each car owner was given a questionnaire. From this questionnaire each person was classified as cautious conservative (CC), middle-of-the-roader (MR), or confident explorer (CE). At the same time, each person was asked to give an overall opinion of small cars.

Opinion of Small Cars	**Self-Perception** CC	MR	CE	Totals
Favourable	79	58	49	186
Neutral	10	8	9	27
Unfavourable	10	34	42	86
Totals	99	100	100	299

The market research company was trying to investigate whether a person's opinion of small cars was the same regardless of their self-perception as a driver.

homogeneity

18. The **correct hypotheses** for the market research company's investigation are:

(1) H_0: Self-perception and opinion of small cars are related.
H_1: Self-perception and opinion of small cars are independent.

(2) H_0: For every level of opinion of small cars the distribution of self-perception is the same.
H_1: The distribution of self-perception differs for some levels of opinion of small cars.

(3) H_0: The distribution of self-perception differs for some levels of opinion of small cars.
H_1: For every level of opinion of small cars the distribution of self-perception is the same.

(4) H_0: Self-perception and opinion of small cars are associated.
H_1: Self-perception and opinion of small cars are not associated.

(5) H_0: All treatment means are the same.
H_1: There is at least one treatment mean that differs from the remaining population means.

F-test, Ch10

19. The **expected count** for ***middle-of-the-roaders*** who have an ***unfavourable*** *opinion of small cars* is:

(1) 285.2
(2) 27.4
(3) 0.95
(4) 28.8
(5) 34

$$\frac{86 \times 100}{299}$$

20. Suppose the expected count for *cautious conservatives* who have an *unfavourable opinion of small cars* is 28.5. The **cell contribution** for ***cautious conservatives*** who have an ***unfavourable*** *opinion of small cars* is:

(1) 12.0
(2) 0.42
(3) -0.65
(4) 34.1
(5) 1.14

$$\frac{(10-28.5)^2}{28.5}$$

Indep

Suppose that the market research company was also interested in investigating if there was an association between the opinion of small cars and a driver's self-perception. A Chi-square test was conducted and the following results obtained from PASW (SPSS). Some values have been replaced with **.

Opinion of Small Cars * Self-Perception Crosstabulation

			Self-Perception			Total
			CC	MR	CE	
Opinion of Small Cars	Favourable	Count	79	58	49	186
		Expected Count	61.6	62.2	62.2	186.0
	Neutral	Count	10	8	9	27
		Expected Count	8.9	9.0	9.0	27.0
	Unfavourable	Count	10	34	42	86
		Expected Count	28.5	**	28.8	86.0
Total		Count	99	100	100	299
		Expected Count	99.0	100.0	100.0	299.0

Q19

test stat

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)
Pearson Chi-Square	14.674(a)	**	.000
~~Likelihood Ratio~~	~~14.541~~		~~.000~~
~~Linear-by-Linear~~ Association	~~3.177~~		~~.000~~
N of Valid Cases	299		

a 0 cells (.0%) have expected count less than 5. The minimum expected count is 8.9.

$df = (3-1)(3-1)$
$= 2 \times 2 = 4$

p-value

The cell contributions to the Chi-square test statistic are shown in the table below:

Opinion of Small Cars	**Self-Perception** CC	MR	CE
Favourable	4.924	0.285	2.804
Neutral	0.126	0.118	0.000
Unfavourable	**	0.954	6.092

Q20

Cell contributions to the Chi-square test statistic

21. The **degrees of freedom** for the above test would be:
    (1) 9
    (2) 6
    (3) 4
    (4) 5
    (5) 1

22. From the results of the above test the **best** interpretation would be:
    (1) There is very strong evidence that the distribution of self-perception of a driver and their opinion of a small car are ~~the same~~. diff
    (2) There is no evidence that the distribution of self-perception of a driver and their opinion of a small car are not the same.
    (3) There is very strong evidence that the self-perception of a driver and their opinion of a small car are not ~~related~~. indep.
    (4) There is no evidence that the self-perception of a driver and their opinion of a small car are not related.
    (5) There is very strong evidence that the self-perception of a driver and their opinion of a small car are related.

Questions 23 to **28** refer to the following information.

The University of Otago Injury Prevention Research Unit recently published a report titled *Road traffic practices among a cohort of young adults in New Zealand*. The aim of the study was to describe the road safety practices of young adults in New Zealand. Face-to-face interviews were conducted with 21-year-olds who were born in Dunedin. The report concluded that unsafe road practices, especially among males, were unacceptably high.

One area of the study investigated the wearing of seat belts. Some results are given in the table below, a two-way table of counts for seat belt usage by rear seat passengers.

	Usage				
Gender	Always	Nearly Always	Sometimes	Never	**Total**
Female	138	79	139	107	463
Male	103	66	152	161	482
Total	241	145	291	268	945

Table: Self-reported seat belt usage by rear seat passengers

We used PASW (SPSS) to conduct a Chi-square test to investigate any differences between females and males for the Usage distribution. The output is given below.

Some values have been removed and replaced with an asterisk (∗).

Level of Gender * Usage Crosstabulation

			Usage				Total
			Always	Nearly Always	Sometimes	Never	
Gender	Female	Count	138	79	139	107	463
		Expected Count	118.1	***	142.6	***	***
	Male	Count	103	66	152	161	482
		Expected Count	122.9	***	148.4	***	***
Total		Count	241	145	291	268	945
		Expected Count	241.0	145.0	291.0	268.0	945.0

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)
Pearson Chi-Square	17.337(a)	**	.001
Likelihood Ratio	14.541	**	.001
Linear-by-Linear Association	3.177	**	.001
N of Valid Cases	945		

a 0 cells (.0%) have expected count less than 5. The minimum expected count is 71.0.

The cell contributions to the Chi-square test statistic are shown in the table below:

Gender	Usage Always	 Nearly Always	 Sometimes	 Never
Female	*** 3.4	***	0.091	4.497
Male	3.222	0.865	0.087	4.320

Table: Cell contributions to the Chi-square test statistic

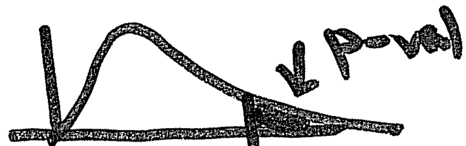
23. For this investigation the null hypothesis is:
   - ~~(1)~~ H_0: The distribution of Usage is different for females and males
   - ~~(2)~~ H_0: The factors Gender and Usage are associated.
   - ~~(3)~~ H_0: $p_1 = p_2 = p_3 = p_4$ where p_i is the proportion of 21-year-olds in each Usage group.
   - (4) H_0: The distribution of Usage is the same for females and males.
   - ~~(5)~~ H_0: The factors Female and Male are dependent.

24. The expected cell count, under the null hypothesis, for those 21-year-old **males** who **never** wear a rear seat belt is:
   - (1) 137.1
   - (2) 130.3
   - (3) 136.8
   - (4) 131.3
   - (5) 136.7

25. The **degrees of freedom** for this Chi-square test is:
   - (1) 6
   - (2) 4
   - (3) 8
   - (4) 3
   - (5) 2

26. Consider the cell for **Female** and **Always**. This cell's **contribution** to the Chisquare test statistic value of 17.337 is:
   - (1) 2.9
   - (2) 0.1
   - (3) 1.6
   - (4) 0.2
   - (5) 3.4

27. Which **one** of the following statements regarding the *P-value* of 0.001 is **true**?
   - F (1) Such a small *P-value* indicates that there must be a big difference between the **Female** and **Male Usage** distributions.
   - F (2) Such a small *P-value* indicates that the alternative hypothesis ~~must~~ is prob. be true.
   - F (3) The probability that the null hypothesis is false is 0.001.
   - (4) If the null hypothesis for this test is true, then the probability of getting a test statistic at least as large as 17.337 is 0.001.
   - F (5) The probability that the null hypothesis is true is 0.001.



28. Which **one** of the following statements is **false**?

T (1) One of the main reasons for such a small *P-value* in this test is because of the relatively small number of **Males** who said that they were **Always** users of rear seat belts.

T (2) If the Chi-square test statistic had been 27.000 instead of 17.337, then the resulting *P-value* would have been smaller than 0.001.

(3) One of the main reasons for such a small *P-value* in this test is because of the relatively large number of **Males** who said that they were **Sometimes** users of rear seat belts.

T (4) If one of the cells had an expected count of less than 1, then it would have been unwise to interpret the output from this test.

T (5) The sum of the expected counts for **Males** is 482 and the sum of the expected count for **Females** is 463.

Questions 28 to **30** refer to the following information.

The paper "Family Planning: Football Style. The Relative Age Effect in Football." investigated the relationship between month of birth and achievement in sports for men. Birth dates were collected on all players in teams competing in the 1990 World Cup soccer games, and they are summarised in Table 9 below.

Birthdays by Quarter	Frequency (obs.)	exp
Quarter 1: Aug–Oct	150	132
Quarter 2: Nov–Jan	138	132
Quarter 3: Feb–April	140	132
Quarter 4: May–July	100	132
Total	528	528

exp frequency
.25×528
= 132.

Table 9: Birth dates, 1990 World Cup Soccer Players

The paper claims that the distribution of players' birth dates is not random and that the number of players is related to the "Quarters of the football year". The claim is based on the results of a Chi-square test for goodness-of-fit.

29. The hypotheses for such a test are:

(1) H_0: Over a year, the greatest proportion of players are born in Quarter 1: Aug–Oct.
H_1: Over a year, the greatest proportion of players are not born in Quarter 1: Aug–Oct.

(2) H_0: 25% of all players are born in each Quarter.
H_1: There are at least two Quarters in which the proportion of all players born is not 25%.

(3) H_0: The proportion of players born in each Quarter is different for each Quarter.
H_1: The proportion of players born in each Quarter is the same for each Quarter.

(4) H_0: The proportion of players born in each Quarter is approximately 0.28, 0.26, 0.27, and 0.19.
H_1: The proportions of players born in each Quarter are not those given in H_0.

PJ's

(5) H_0: 25% of all players are born in each Quarter.
H_1: There is no Quarter in which the proportion of all players born is 25%.

30. Under H_0, the expected count for the number of players born in Quarter 1: Aug–Oct is:

(1) 528
(2) 138
(3) 132
(4) 117
(5) 150

same for Quarters 2 3 & 4

31. The contribution by Quarter 4: May–July to the Chi-square test statistic is approximately:

(1) 2.45
(2) 0.5
(3) 1188
(4) 7.8
(5) 0.3

$$\frac{(100-132)^2}{132} = 7.8$$

ANSWERS

1. **(3)**	2. **(2)**	3. **(2)**	4. **(4)**	5. **(4)**	6. **(1)**
7. **(3)**	8. **(3)**	9. **(3)**	10. **(3)**	11. **(4)**	12. **(2)**
13. **(2)**	14. **(4)**	15. **(5)**	16. **(5)**	17. **(2)**	18. **(2)**
19. **(4)**	20. **(1)**	21. **(3)**	22. **(5)**	23. **(4)**	24. **(5)**
25. **(4)**	26. **(5)**	27. **(4)**	28. **(3)**	29. **(2)**	30. **(3)**
31. **(4)**					