Test: 6-10Qs (25%-40%)

STATS 101/108 WORKSHOP

SATURDAY 3: CHAPTERS 7 AND 8

SATURDAY 4 SEPTEMBER, 2010

Please get a handout from the back!

Students **MUST REGISTER** for <u>all workshops</u> with
The Student Learning Centre, 3rd Floor, Information Commons

Student Learning Centre

Topics we teach and can provide advice on include:

- ✓ Essay writing
- ✓ Computer skills
- ✓ Reading and notetaking
- ✓ Memory and concentration
- ✓ Report writing
- ✓ Test and examination skills
- ✓ Thesis and dissertation writing
- ✓ Tutorial skills
- ✓ Research skills
- ✓ Time and stress management
- ✓ Mathematics
- ✓ **Statistics**
- ✓ Oral presentation and seminar skills
- ✓ Language learning
- ✓ Specific learning disabilities
- ✓ Motivation and goal setting
- ✓ Survival skills (in the University system)

Programmes within SLC include:

- Te Puni Wananga
  Maori university tutors committed to enhancing Maori students' success

- Fale Pasifika
  Pacific Island tutors committed to enhancing success for Pacific Island students

- Students with impairments
  Learning assessments are available for students with specific learning disabilities; academic assistance is available for these students and those with mental health impairments.
  If you have any special learning requirements, please feel free to discuss this with Leila in person or via email.

- Academic English Conversation Groups
  Improve your academic English; develop communication skills including critical/creative thinking and clear expression of ideas and opinions. Weekly class held at the SLC on Thursdays, 3-5pm (during semester)

Statistical help available at the SLC

The Student Learning Centre (SLC) offers help for STATS 101/108 by offering:

- one-on-one tutoring help, and
- a number of workshops

One-on-one help

The SLC employs tutors specifically to help students with one-on-one assistance for STATS 101/108. One-on-one tutoring must be booked at SLC reception on the third floor of the Information Commons in person or by calling 373-7599 X 88850. Enquire at the SLC reception for available times.

Note: SLC tutors are not allowed to help students complete their assignments.

SLC STATS 101/108 Workshops

Any questions regarding STATS 101/108 workshops should be forwarded to:

Leila Boyle
SLC Statistics Co-ordinator
l.boyle@auckland.ac.nz

Workshops are run in a relaxed environment, typically set at a pace for those students that find the Statistics Department's tutorials too fast. All workshops allow plenty of time for questions. In fact, this is encouraged ☺

1) Saturday Workshops x 5

These five 3-hour workshops are held on Saturdays throughout the semester to help students with different sections of the course.

2) Computer Workshops: Excel / PASW (SPSS) x 3

These three computer-based workshops introduce students to the skills needed for Excel and PASW (SPSS) use in STATS 101/108 assignments.

3) Pre-test Workshops x 3

These three workshops will cover the basics that you need for the test.

4) Pre-exam Workshops x 6 each repeated twice

These six workshops will cover the basics that you need for the exam.

Note: All workshops concentrate on questions reviewing the basic concepts, rather than questions on finer details. They are designed to assist students to achieve a pass; they are not designed to cover all material.

The timetable for these workshops is available with this handout. Currently the SLC website is still partly down so online enrolments are not available until further notice. Please enrol in each of your preferred classes at the Student Learning Centre by:

- **Going to the SLC in person**
- **Emailing slc@auckland.ac.nz with your name, ID number and the workshop/s you wish to attend.**
- **Calling the SLC reception on 373-7599 ext. 88850 and enrol over the phone. Make sure you know which workshop/s you want to enrol in and have your ID number handy.**

Useful Websites

- SLC webpage: **www.slc.auckland.ac.nz** (The SLC website currently has all functionality except online enrolment! Download an undergraduate brochure and enrol in workshops in person or by emailing/phoning the SLC Reception as per above instructions).
- Cecil: **https://cecil.auckland.ac.nz/**
- **Leila's website for STATS 101/108 SLC workshop handouts & information: www.stat.auckland.ac.nz/~leila**

2-4Qs

Revision Notes

Chapter 7 – Sampling Distributions of Estimates

Look at blue pages for good notes, 26 statements (2 of which are false!) and test/exam questions for practice

- Statistics is concerned with finding out about the real world and aspects of it specific to our area of interest. Statistical tools allow us to deal with the **uncertainty** present in all samples due to **sampling variation** which occurs because we are unable to survey the entire population of interest.

- We are usually unable to survey the entire population (take a census) as it is too large and/or there are:
  - ✓ budget constraints
  - ✓ time limits
  - ✓ logistical barriers

- This means we are unable to establish the **parameters** of interest within our population, such as:
  - ✓ Population mean, μ
  - ✓ Population standard deviation, σ

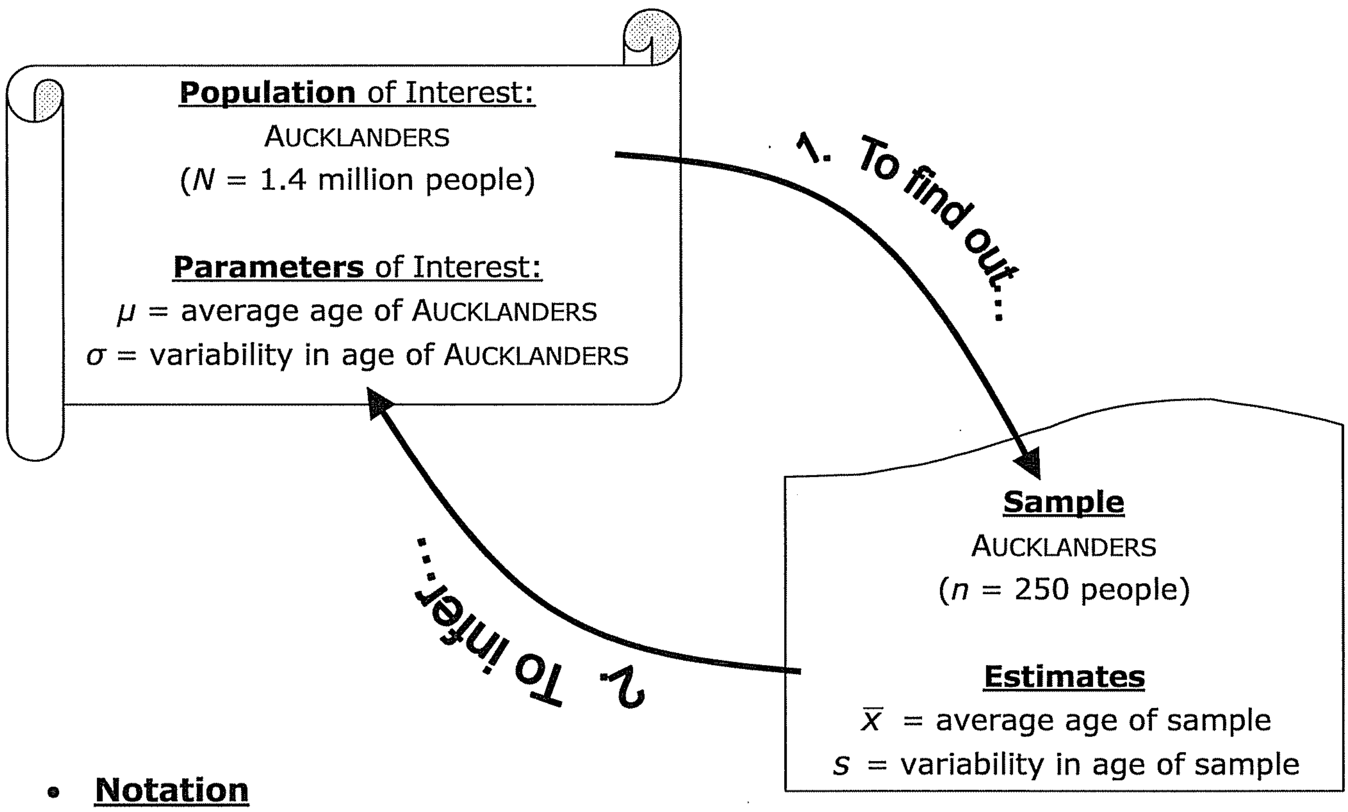
- This means that the **parameter** of interest is an ***unknown* numerical characteristic** for that particular population.

- To estimate an ***unknown* numerical characteristic (parameter)** for our population of interest, we take a sample and find a sample **estimate** from it (that is, we make a **statistical inference**). The **sample estimates** of the above **population parameters** are:
  - ✓ Sample mean, $\bar{x}$
  - ✓ Sample standard deviation, $sd(\bar{x})$ or σ_{n-1} or s

- Usually ^HATS or ‾BARS are used to distinguish between **sample estimates** and **population parameters**.

- Random variables X_1, X_2, ... X_n, form a random sample from a distribution if:
  - ✓ they all have the same distribution; ~~and~~ → same mean, μ same std dev, σ
  - ✓ they are independent of one another.

- The big question which we will answer in Chapter 7 is "But how can we trust the sample estimates ($\bar{x}$ and s) from a single sample of size n?"



- **Notation**

  In statistics we use **CAPITAL letters** to refer to the **variable of interest** for the population and **small letters** to specify the **actual "number" observed** for that variable in our particular sample.

"distributed as"   a particular number of interest

Variable of Interest **CAPITAL** LETTER	Actual "number" **small** LETTER
Ch6: $X \sim$ Normal(μ, σ)	x
Ch7: $\bar{X} \sim$ exactly/approx. Normal$(\mu, \sigma/\sqrt{n})$ (n sufficiently "large")	$\bar{x}$ a particular sample mean of interest

- **The sample mean – $\bar{X}$**

  If $X_1, X_2, \ldots X_n$, form a random sample from a distribution where $E(X_i) = \mu$ and $sd(X_i) = \sigma$, then,

  mean, average

  ✓ The **expected value** of the sample mean, $E(\bar{X})$ is calculated by:

  $$\mu_{\bar{X}} = \boxed{E(\bar{X}) = \mu}$$

  on formulae sheet

  ✓ The **standard deviation** of the sample mean, $sd(\bar{X})$ is calculated by:

  $$\sigma_{\bar{X}} = \boxed{sd(\bar{X}) = \frac{\sigma}{\sqrt{n}}}$$

- **The Central Limit Theorem**

  ✓ The Central Limit Theorem tells us that the larger the sample size, the closer the distribution of $\overline{X}$ comes to a Normal distribution. Even if the distribution of X is non-Normal, the distribution of $\overline{X}$ will be **approximately** Normal for a sufficiently large sample size n.

  ✓ If X is from a "well-behaved" distribution (i.e. symmetric, no outliers) the Central Limit Theorem works reasonably fast. n = 10 may be sufficient!

  ✓ In **general**, n = 30 works well for most distributions *except* distributions that are severely skewed or have large outliers.

  ✓ If the distribution is **severely skewed**, n = 50 should be sufficient.

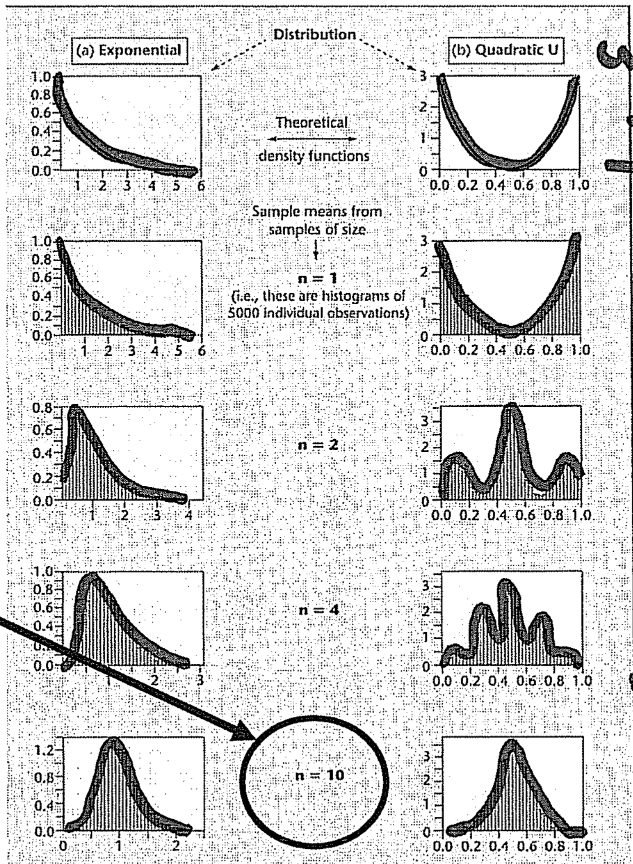


FIGURE 7.2.4 Histograms of means calculated from random samples from the Exponential and Quadratic U distributions.

Chance Encounters, C.J. Wild & G.A.F. Seber, p286

uniform

  ✓ If X is from a Normal distribution, then $\overline{X}$ is **exactly** Normally distributed.

theory

- $\overline{X}$ is an **unbiased** estimator of μ because $E(\overline{X}) = \mu$.

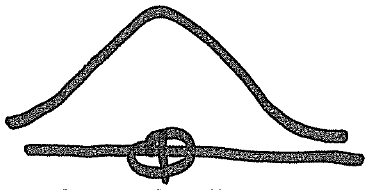
- **Standard errors**

real world

However, as $sd(\overline{X}) = \dfrac{\sigma}{\sqrt{n}}$, it is not a useful measure of the precision of $\overline{x}$, because we do not know the value of σ. Therefore, we have to use the ***standard error*** of the sample mean to estimate the precision of $\overline{x}$ as an estimate of μ:

$$\text{The standard error of } \overline{x} = \frac{\text{sample standard deviation}}{\sqrt{\text{sample size}}} = \text{se}(\overline{x}) = \frac{s}{\sqrt{n}}$$

- **Student's *t*-distribution**



  ✓ Parameter: Degrees of Freedom (*df*).

  ✓ Bell shaped and centred at 0 like the (Standard) Normal (0,1) distribution but it's more variable.

  ✓ As *df* becomes larger, the Student (*df*) distribution becomes more and more like the Standard Normal distribution.

  ✓ Student's *t*-distribution ($df = \infty$) and Normal (0,1) are the same distribution.
  ↳ proportions, means

  ✓ The random sample from a Normal distribution $\boxed{T = \frac{\bar{X} - \mu}{se(\bar{X})}}$ is exactly distributed as Student($df = n - 1$) $n > 150$

  → conf. intervals

  ✓ Methods based on this distribution works very well even for small samples that are from very non-Normal distributions.

  ✓ By t_{df} (*prob*), we mean the number t such that when $T \sim$ Student(*df*), pr($T \geq t$) = *prob*; that is, the tail area above t (that is to the right of t on the graph) is *prob*:

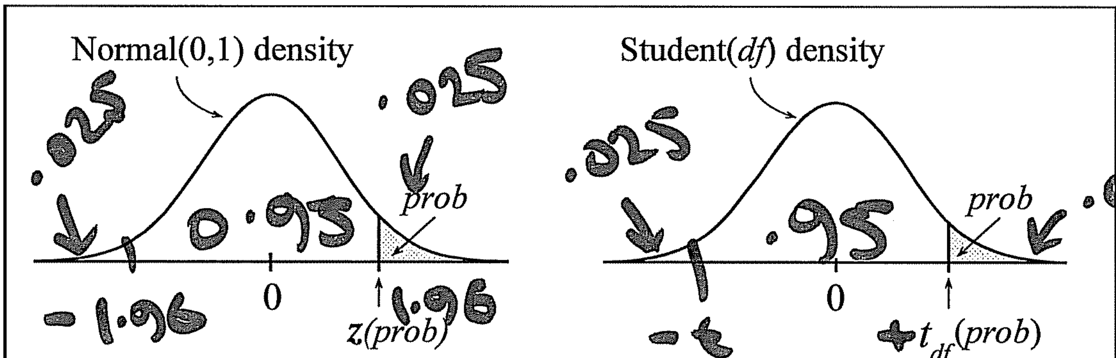


Figure 7.6.2 The *z(prob)* and *t(prob)* notations.

Ch6: $z = \frac{X - \mu}{\sigma}$

Ch7: $z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$

$T = \frac{\bar{X} - \mu}{s / \sqrt{n}}$

5-8 Qs on test

Chapter 8 – Confidence Intervals

Look at blue pages for good notes, 26 statements (2 of which are false!) and test/exam questions for practice

- A **confidence interval** gives a range of plausible values for the parameter of interest that is consistent with the data (at the specified level of confidence). It determines the **size** of the effect or difference.

- You can do all kind of CI's, 90%, 95%, 99%...

- Increasing the confidence level will **increase** the width of the interval.

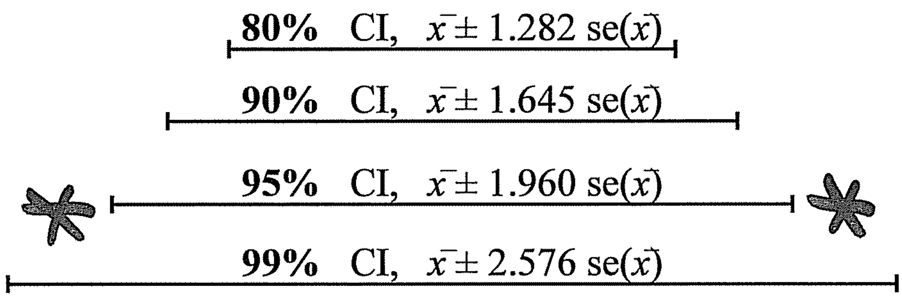


Figure 8.1.3 The greater the confidence level, the wider the interval

From *Chance Encounters* by C.J. Wild and G.A.F. Seber, © John Wiley & Sons, 2000.

- Increasing the sample size will make the confidence interval more precise. narrower

- To double the accuracy of the confidence interval we **need 4 times** as many observations. → halve the width

- To triple the accuracy of the confidence interval we **need 9 times** as many observations. → third the width

- 95% confidence interval

  ✓ Range of plausible values for the parameter of interest that contains the **true value** of our parameter of interest for 95% of samples taken.

  ✓ 5% of samples taken will not have the parameter within the calculated confidence interval.

  ✓ We do not know if the sample we have taken is one of the 95% that contains the true unknown parameter. All we can say is that 95% of the time it will.

✓ If you take 1000 samples, based on the same sampling protocol, then you can expect approximately 950 of these samples will contain the true value (e.g. true mean) of the population.

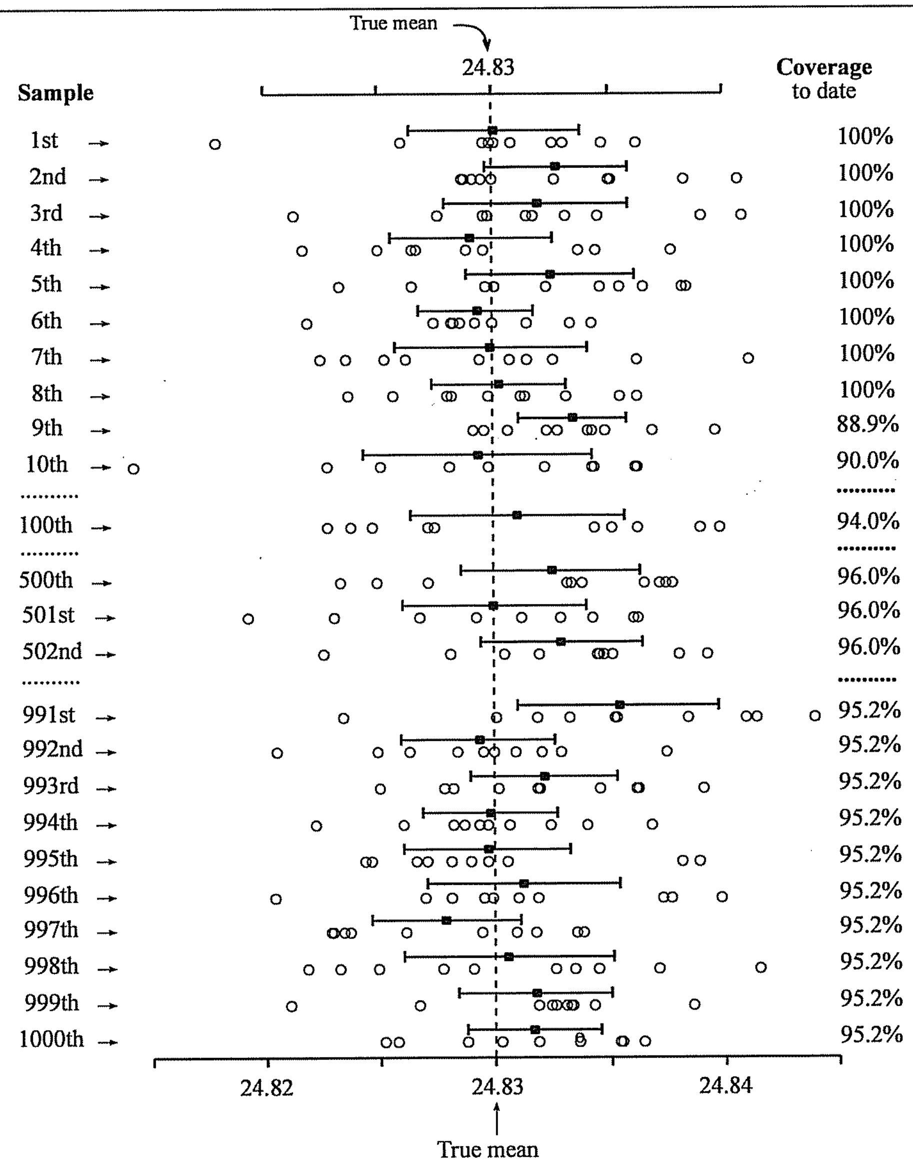


Figure 8.1.2 Samples of size 10 from a Normal(μ=24.83, s=.005) distribution and their 95% confidence intervals for μ..

From *Chance Encounters* by C.J. Wild and G.A.F. Seber, © John Wiley & Sons, 1999.

Step-by-Step Guide to Producing a Confidence Interval by Hand

1. State the **parameter** to be estimated (symbol and words).
   Is it μ, p, $\mu_1 - \mu_2$, or $p_1 - p_2$?
   1. 2. 3. 4.
2. State the **estimate** and its value.
3. Write down the **formula** for a CI, from the Formula Sheet — $\boxed{\textit{estimate} \pm \overbrace{t \times \text{se}(\textit{estimate})}^{\text{margin of error}}}$
4. Find the appropriate **standard error** from the Formula Sheet.
5. Find the appropriate ***df***.
   step 2 step 5 step 4
6. Find the appropriate ***t*-value.** This is called the '*t*-multiplier'. Need to know the confidence level and *df*.
7. Calculate the **confidence limits**, the end points of the confidence interval.
8. Interpret the interval using plain English. Use the confidence limits to construct an answer to the original question in plain English.

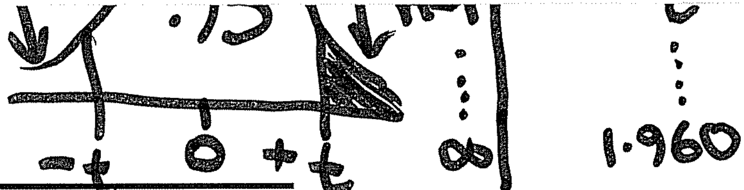
- ~~There are four different types of problem covered in Chapter 8:~~
  1. Single mean 2. Single proportion. 3. Difference between two means
  4. Difference between two proportions:
     Situation (a) ***Proportions from two independent samples***
     Situation (b) ***One sample of size n, several response categories***
     Situation (c) ***One sample of size n, many yes/no items***

- The **estimate** (step 2) is based on the **parameter** (step 1) of interest we are investigating:

Parameter	**Estimate**
1. Single mean $\boldsymbol{\mu}$:	$\textit{estimate} = \bar{x}$
2. Single proportion $\boldsymbol{p}$:	$\textit{estimate} = \hat{p}$
3. Difference between two means $\boldsymbol{\mu_1 - \mu_2}$: (independent samples)	$\textit{estimate} = \bar{x}_1 - \bar{x}_2$
4. Difference between two proportions $\boldsymbol{p_1 - p_2}$:	$\textit{estimate} = \hat{p}_1 - \hat{p}_2$

- The ***t*-multiplier** (means) / ***z*-multiplier** (proportions) is based on:
  - ✓ Whether we are investigating means or proportions
  - ✓ The desired level of confidence 95%
  - ✓ The degrees of freedom

df	0.025
3	⋮
⋮	⋮
n−1	+



Estimate	Degrees of Freedom
1. $estimate = \overline{x}$	$df = n - 1$
2. $estimate = \hat{p}$	$df = \infty$ ← z=1.96(95% CI)
3. $estimate = \overline{x}_1 - \overline{x}_2$	$df = minimum(n_1 - 1, n_2 - 1)$
4. $estimate = \hat{p}_1 - \hat{p}_2$	$df = \infty$ z=1.96(95% CI)

i.e. for proportions, assume the degrees of freedom is infinity, hence replace t with z score (i.e. the standard normal).

- The **standard error** is based on the estimate, the number of samples and sample size(s). step 4

Estimate	se(estimate)
1. $estimate = \overline{x}$	$se(\overline{x}) = \frac{s}{\sqrt{n}}$
2. $estimate = \hat{p}$	$se(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
3. $estimate = \overline{x}_1 - \overline{x}_2$	$se(\overline{x}_1 - \overline{x}_2) = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$
4. $estimate = \hat{p}_1 - \hat{p}_2$	**Situation (a)** ***Proportions from two independent samples*** $se(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$ **Situation (b)** ***One sample of size n, several response categories*** $se(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{\hat{p}_1 + \hat{p}_2 - (\hat{p}_1 - \hat{p}_2)^2}{n}}$ **Situation (c)** ***One sample of size n, many yes / no items*** $se(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{minimum(\hat{p}_1 + \hat{p}_2, \hat{q}_1 + \hat{q}_2) - (\hat{p}_1 - \hat{p}_2)^2}{n}}$ where $\hat{q}_1 = 1 - \hat{p}_1$ and $\hat{q}_2 = 1 - \hat{p}_2$

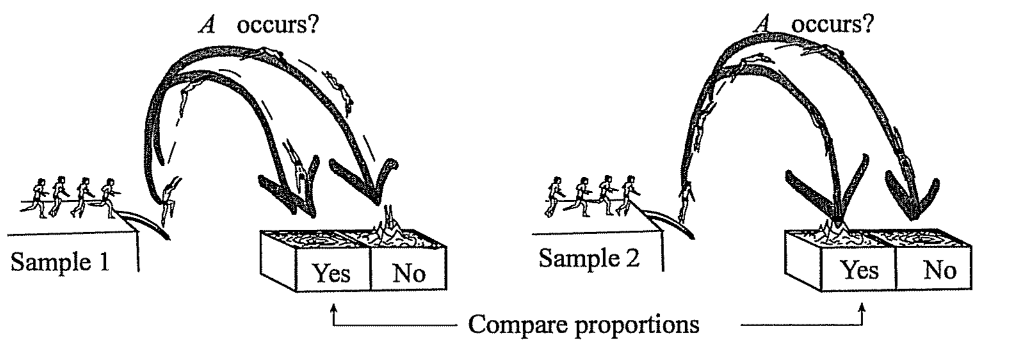
$\hat{p}_1 + \hat{p}_2$

$\hat{q}_1 + \hat{q}_2$

- CIs for the difference between two means/proportions:
  - (−6, +3) If the CI contains 0 (i.e. one negative and one positive number), there may be no difference between the two means/proportions.
  - (+3, +6) If CI is positive, then μ_1/p_1 is higher/larger than μ_2/p_2.
  - (−6, −3) If CI is negative, then μ_1/p_1 is lower/smaller than μ_2/p_2.

- **3 sampling situations for the difference between two proportions**

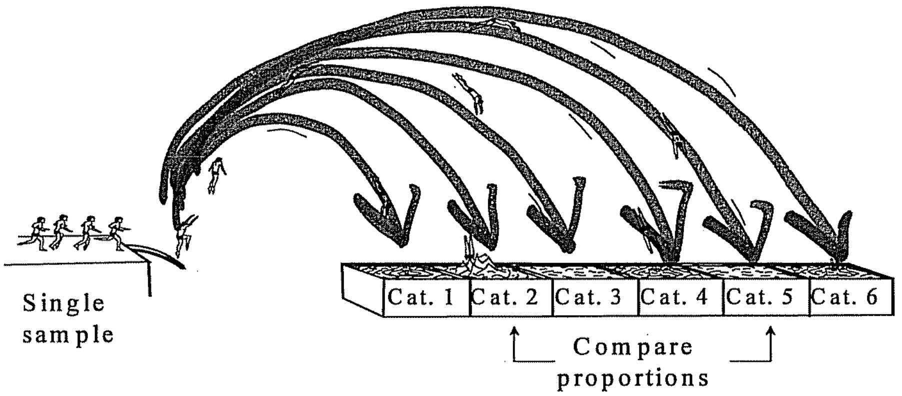
Situation (a) ***Proportions from two independent samples***



From *Chance Encounters* by C.J. Wild and G.A.F. Seber, © John Wiley & Sons, 1999.

1Q only, 1 answer only

Situation (b) ***One sample of size n, several response categories***



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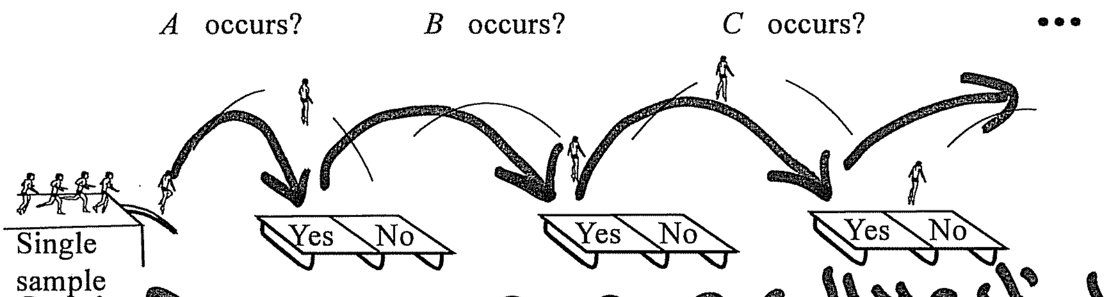
1Q only, more than 1 answer if so wish

Situation (c) ***One sample of size n, many yes/no items***



2+Qs, can answer "yes" to each if so wish

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Chapters 7 & 8 – Questions

1. Which one of the following statements is **false?**
   (1) $se(\overline{x})$ is an estimate of $sd(\overline{X})$.
   (2) $\overline{x}$ is an unbiased estimate of μ since $E(\overline{X}) = \mu$.
   (3) An estimate is a known quantity computed from data.
   (4) A parameter is a numerical characteristic of a population or distribution.
   (5) If the distribution of a random variable, X, is exactly Normal, then, according to the Central Limit Theorem, the distribution of $\overline{X}$ will be approximately Normal.

2. Which one of the following statements is **true?**
   (1) A lack of symmetry in the sample usually means that a large sample size will be required for the Central Limit Theorem to hold.
   (2) The smaller the sample size, the better the Normal approximation for $\overline{X}$ will be.
   (3) The Central Limit Theorem does not hold for discrete distributions.
   (4) The Central Limit Theorem only applies to samples from a normal distribution.
   (5) The Central Limit Theorem says that the distribution of $\overline{X}$ is approximately Normal for small samples.

3. Which one of the following is **false**?
   (1) The standard error of the sample mean is an estimate of the standard deviation of the sample mean.
   (2) For large samples, a random sample mean falls within two standard errors of the true population mean approximately 95% of the time.
   (3) We use $\overline{X}$ to talk about the random behaviour of sample means in repeated sampling and $\overline{x}$ to refer to the observed sample mean of a particular sample.
   (4) A parameter is always a known quantity calculated from the sample data.
   (5) The mean of the distribution of the sample mean, $\mu_{\overline{X}}$, is equal to the population mean, μ.

4. The Central Limit Theorem says that the distribution of the mean, $\overline{X}$, of a random sample from any distribution is:
   (1) exactly Binomial in large samples.
   (2) approximately Normal in large samples.
   (3) exactly Student's t-distribution with $n - 1$ degrees of freedom in large samples.
   (4) approximately Binomial in large samples.
   (5) exactly Normal in large samples.

5. Given a simple random sample, which one of the following statements is **false?**

(1) The mean of the distribution of the sample mean, $\mu_{\bar{X}}$, is equal to the population mean, μ.

(2) The sample mean is an unbiased estimate of the population mean since $E(\bar{X}) = \mu$.

(3) The distribution of the sample mean, $\bar{X}$, is approximately Normal for very large samples only if the distribution from which the sample has been drawn is not skewed.

(4) The standard error of the sample mean is an estimate of the standard deviation of the sample mean.

(5) Increasing the sample size by a factor of 4 will double the accuracy of the sample mean as an estimate of the population mean.

6. Let $X_1, X_2, X_3, \ldots, X_9$ be a random sample of size 9 (n) from a distribution with $\mu = 27$ and $\sigma = 6$. Then the distribution of the sample mean $\bar{X} = (X_1 + X_2 + \ldots X_9)/9$ has a mean $\mu_{\bar{X}}$ and a standard deviation $\sigma_{\bar{X}}$, where:

~~(1)~~ $\mu_{\bar{X}} = 27$ $\cancel{\sigma_{\bar{X}} = 0.67}$

~~(2)~~ $\mu_{\bar{X}} = 27$ $\cancel{\sigma_{\bar{X}} = 6}$

~~(3)~~ $\cancel{\mu_{\bar{X}} = 3}$ $\cancel{\sigma_{\bar{X}} = 0.67}$

(4) $\mu_{\bar{X}} = 27$ $\sigma_{\bar{X}} = 2$

~~(5)~~ $\cancel{\mu_{\bar{X}} = 3}$ $\sigma_{\bar{X}} = 2$

$\mu_{\bar{X}} = E(\bar{X}) = \mu = 27$

$\sigma_{\bar{X}} = sd(\bar{X}) = \sigma/\sqrt{n} = \frac{6}{\sqrt{9}} = 2$

7. Statistics New Zealand runs a quarterly income survey on 26,000 businesses involving 921,900 employees. The November 1995 survey determined that the weekly wage, to the nearest dollar, had a mean of μ \$661 for male full-time employees (more than 30 hours work) and a standard deviation of \$82 (σ). Suppose 10 (n) male full-time employees are chosen at random. Then the distribution of their mean weekly wage in dollars, $\bar{X}$, is approximately:

~~(1) Student~~ (df = 9)

~~(2)~~ Normal (μ = 661, ~~σ = 8.2~~)

~~(3)~~ Normal (μ = 661, ~~σ = 82~~)

(4) Normal (μ = 661, σ = 25.93)

~~(5) Binomial~~ (n = 10, p = 66.1)

$\mu_{\bar{X}} = \mu = 661$

$\sigma_{\bar{X}} = \sigma/\sqrt{n} = \frac{82}{\sqrt{10}} = \$25.93$

8. Suppose that $X_1, \ldots, X_n$ is a random sample from a distribution with a mean, μ, and standard deviation, σ. Which of the following is **false?**

(1) $X_1, \ldots, X_n$ are independent with the same distribution.

(2) As the sample size, n, increases the distribution of $\overline{X}$ becomes closer to the normal distribution.

(3) If the distribution of $X_1, \ldots, X_n$ is normal, the distribution of $\overline{X}$ is normal even for small sample sizes, n.

(4) The sample mean, $\overline{X}$, always has a normal distribution.

(5) For large sample sizes, n, $\overline{x} \pm 2\frac{s_x}{\sqrt{n}}$ is an approximate 95% confidence interval for μ.

9. The number of junk e-mails received each week, from a simple random sample of 5 weeks, is: 13, 22, 7, 25, 9. From this sample, David wanted to calculate the 2 standard error interval for the mean number of junk e-mails received each week. The appropriate formulae to use is:

(1) $\overline{X} \pm 2\frac{s_x}{\sqrt{n}}$

(2) $\hat{p} \pm 2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

(3) $\hat{p} \pm 2\sqrt{\frac{p(1-p)}{n}}$

(4) $\overline{x} \pm 2\frac{s_x}{\sqrt{n}}$

(5) $np \pm 2\sqrt{np(1-p)}$

10. Let μ be the population mean and let σ be the population standard deviation of each X_i from an exponential distribution. Let $\overline{x}$ be the sample mean and let s be the sample standard deviation of the observations $x_1, x_2, \ldots, x_{30}$. Which one of the following statements is **false?**

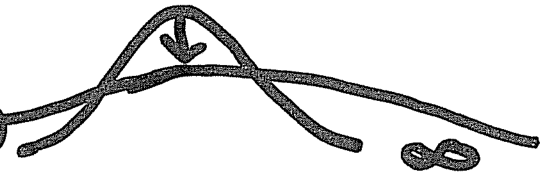
(1) $\dfrac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{30}}}$ has approximately a Standard Normal distribution.

(2) s is an estimate of σ.

(3) The Central Limit Theorem cannot be applied to the distribution of $\overline{X}$, because $\overline{X}$ is the mean of Exponential random variables.

(4) $\dfrac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{30}}}$ has a Student's t-distribution with parameter $df = 29$.

(5) $\overline{x}$ is an unbiased estimate of μ.



11. Which one of the following statements is **false?**

T (1) For very large values of the degrees of freedom, the distribution of $T = \dfrac{\overline{X} - \mu}{\text{se}(\overline{X})}$ is almost identical to the standard Normal distribution. $S/\sqrt{n}$

T (2) $T = \dfrac{\overline{X} - \mu}{\text{se}(\overline{X})}$ measures the difference between $\overline{X}$ and μ in terms of the number of standard errors of the sample mean.

F (3) The graph of the Student's t-distribution with 20 degrees of freedom ($df = 20$) has fatter tails than the graph of the Student's t-distribution with 10 degrees of freedom ($df = 10$).

T (4) The graph of the Student's t-distribution with 50 degrees of freedom ($df = 50$) has fatter tails than the graph of the standard Normal distribution. ($df = \infty$)

T (5) Student's t-distribution describes a family of distributions indexed by a parameter called the degrees of freedom.

12. Which one of the following statements is **false**?

T (1) The standard error of a sample mean is the sample standard deviation divided by the square root of the sample size. $se(\bar{x}) = S/\sqrt{n}$

T (2) For large samples a sample mean falls within two standard errors of the true population mean approximately 95% of the time.

T (3) The standard error of an estimate is a measure of its precision.

T (4) The standard error of an estimate is an estimate of its standard deviation.

F (5) The standard deviation of the sample mean is the population standard deviation divided by the sample size. square root of the $sd(\overline{X}) = \sigma/\sqrt{n}$

13. Which one of the following statements is true?

(1) A point estimate is preferred to a confidence interval because the interval summarises the uncertainty due to sampling variation.

(2) The standard error used to construct the interval will be identical for all samples of the same size.

(3) If I plan to do a study in the future in which I will take a random sample and calculate a 90% confidence interval, there is a 90% chance that I will catch the true value of μ in my interval.

(4) The size of the multiplier t depends on only the sample size and not the desired confidence level.

(5) The process of using a population parameter to construct an interval for the data estimate is an example of statistical inference.

14. A manufacturer of fire sprinkler systems claims that its sprinklers are activated at a mean temperature of 54 degrees Celsius. Assume that the activation times are Normally distributed with standard deviation of 1.50 degrees Celsius. A sample of 9 systems was tested. If the manufacturer is correct, the sampling distribution of the mean of this sample, $\overline{X}$, is:

(1) exactly Normal with mean 54 and standard deviation 0.17

(2) approximately Normal with mean 54 and standard deviation 0.50

(3) exactly Normal with mean 54 and standard deviation 1.50

(4) approximately Normal with mean 54 and standard deviation 1.50

(5) exactly Normal with mean 54 and standard deviation 0.50

15. Which one of the following statements is **true**?

(1) In a poll, all estimates of population proportions, including estimates for subgroups of the population, will have the same standard error.

(2) The Student's t-distribution is used instead of the standard Normal distribution for the distribution of $(\overline{X} - \mu)/(\sigma_X/\sqrt{n})$ when the population standard deviation is replaced by the sample standard deviation.

(3) The Central Limit Effect can only be detected for sample sizes that are greater than 30.

(4) When sampling, taking a large sample guarantees an accurate estimate of the parameter of interest.

(5) For small samples, the shape of the distribution of the sample mean, $\overline{X}$, is always Normal regardless of the shape of the distribution of the random variable *X*.

16. When using a *t*-procedure to construct a confidence interval for a population mean, the confidence interval is constructed using the formula:

$$estimate \pm t \times se(estimate)$$

Which one of the following statements is **false**?

(1) The margin of error is the quantity added to and subtracted from the estimate to construct the interval.

(2) The standard error used to construct the interval will be identical for all samples of the same size.

(3) A confidence interval is preferred to a point estimate because the interval summarises the uncertainty due to sampling variation.

(4) The size of the multiplier, *t*, depends on both the sample size and the desired confidence level.

(5) Large samples tend to yield narrower 95% confidence intervals than small samples.

17. Cyclozocine was an alternative to methadone for treating heroin addiction. The following data came from 14 males who were chronic heroin addicts. After cyclozocine had removed the addicts' physical dependence on heroin, they were asked a list of questions designed to assess their psychological dependence. The test scores are called Q-scores and high values represent less psychological dependence.

Sample Size	Mean	Standard Deviation
14	39.93	11.96

A 95% confidence interval for the true mean (with a t-multiplier = 2.16) Q-score of psychological dependence is:

(1) [36.73, 43.13]

(2) [33.03 46.83]

(3) [38.09, 41.78]

(4) [39.24, 40.62]

(5) [47.32, 49.68]

$\rightarrow n$

Questions 18 and **19** refer to the following information.

Printed on every packet of "Yummo" corn chips is a weight of 150g. A consumer collects 48 packets of "Yummo" corn chips and finds a mean weight of 148.5g and a standard deviation of 2.1g. $\rightarrow s$

$\bar{x}$

18. The standard error of the sample mean, se($\bar{x}$), is given by (select one only):

(1) 0.044

(2) 3.09

(3) 0.303

(4) 21.43

(5) 0.0919

$se(\bar{x}) = \frac{s}{\sqrt{n}} = \frac{2.1}{\sqrt{48}} = .303$ (3dp)

19. Assume the standard error of $\bar{x}$, se($\bar{x}$), to be 0.7 and that the t-multiplier is 2.025. Then a 95% confidence interval for the true mean weight of "Yummo" packets is given by (select one only):

(1) [147.08, 149.92]

(2) [148.30, 148.70]

(3) [147.89, 149.11]

(4) [147.13, 149.87]

(5) [147.32, 149.68]

95% CI for μ:

$\Rightarrow$ est $\pm$ t × se(est)

$\Rightarrow \bar{x} \pm t \times se(\bar{x})$

$\Rightarrow 148.5 \pm 2.025 \times 0.7$

$\Rightarrow (147.08, 149.92)$

20. Identify which one of the following statements about confidence intervals is false?

(1) If a large number of researchers independently perform studies to estimate μ, about 95% of them will catch the true value of μ in their 95% confidence intervals.

(2) Large samples tend to yield narrower 95% confidence intervals than small samples.

(3) A two-standard error interval will always capture the true value of μ.

(4) In the long run, if we repeatedly take samples and calculate a 95% confidence interval from each sample, we expect that 95% of the intervals will contain the true value of μ.

(5) The margin of error is the quantity added to and subtracted from, the estimate to construct the interval.

Questions 21 and **22** refer to the following information.

n

A survey of 1,146 New Zealanders was published in the 23 March 1992 issue of Time magazine. In response to the question "Is it a good time to buy a major household item?" 585 respondents replied "Yes", 332 replied "no" and 229 replied "don't know".

21. The proportion of the sample who think that it is a good time to buy a major household item is:

(1) 0.20

(2) 0.29

(3) 0.49

(4) 0.51

(5) 0.95

$\hat{p} = \frac{585}{1146} = 0.51$ (2dp)

22. A 95% confidence interval for the proportion of New Zealanders who think that it is a good time to buy a major household item is:

p

(1) [0.43, 0.50]

(2) [0.45, 0.57]

(3) [0.48, 0.54]

(4) [0.50, 0.52]

(5) [0.61, 0.67]

95% CI for p:

$\Rightarrow$ est $\pm$ t $\times$ se(est)

$\Rightarrow \hat{p} \pm z \times se(\hat{p})$

$\Rightarrow .51 \pm 1.96 \times \sqrt{\frac{.51(1-.51)}{1146}}$

$\Rightarrow (0.48, 0.54)$

Questions 23 to **25** refer to the following information.

Urea formaldehyde foam insulation (UFFI) is used to insulate homes that are already built. It has good insulation features and the desirable property that it can be pumped into cavities as a liquid where it solidifies, maintaining the same volume. At one time, it was a popular material. However, problems arose because in some homes in which UFFI was installed, some people had an allergic reaction. It was not clear whether the reaction was due to UFFI.

In some applications, UFFI gives off formaldehyde, a gas to which some people have a strong reaction. It was suspected that this might be the cause of the allergic reactions. However the situation was confused by the fact that there are many other sources of formaldehyde in the home. A study was carried out to compare the concentration of formaldehyde in the air in homes with and without UFFI.

Measurements of formaldehyde concentration were made on a random sample of 445 homes with UFFI, and on a random sample of 243 homes without UFFI. Summary statistics (units = parts per million) are given below.

	Homes with UFFI	Homes without UFFI
Sample Size	$n_1 =$ 445	$n_2 =$ 243
Sample Mean	$\bar{x}_1 =$ 56.8	$\bar{x}_2 =$ 48.1
Sample Standard Deviation	$s_1 =$ 12.3	$s_2 =$ 11.0

We wish to construct a 95% confidence interval for the difference between the average formaldehyde concentration in homes with UFFI and in homes without UFFI.

mean

23. The value of the estimate is:
    (1) 48.1
    (2) 104.9
    (3) 8.7
    (4) 56.8
    (5) 1.3

$\bar{x}_1 - \bar{x}_2$
$= 56.8 - 48.1$
$= 8.7$

24. The value for the $se(\bar{x}_1 - \bar{x}_2)$ is:
    (1) 0.84
    (2) 0.92
    (3) 0.07
    (4) 0.27
    (5) 16.50

$= \sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}$
$= \sqrt{\left(\frac{12.3^2}{445} + \frac{11^2}{243}\right)}$
$= 0.92$ (2dp)

25. The value for *df* in the above formula is:
    (1) 444
    (2) 445
    (3) 688
    (4) 242
    (5) 243

$df = \min(n_1 - 1, n_2 - 1)$
$= \min(445-1, 243-1)$
$= 242$

Question 26 and **27** refer to the following information.
The table below represents the beer market for the New Zealand brewing companies DB group and Lion Nathan Ltd taken from a random sample of 2400 beer drinkers. (Reported by Metro magazine, July 1993.) n

Estimated Brand Market Shares (%)
(as of February 1993)

Lion Nathan		**DB Group**	
Lion Red	25.9	DB Draught	12.1
Lion Brown	6.3	DB Bitter	4.5
Speight's	6.2	DB Export	3.2
Australian brands	6.2	DB Export Dry	1.9
Steinlager (green)	4.9	Double Brown	1.8
Steinlager (blue)	4.1	Imported Beer	1.5
Waikato	4.1	Tui	1.2
Canterbury Draught	4.0		
TOTAL	**61.7**	**TOTAL**	**26.2**

Note: Home brew, independent brands and other imported brands make up the remaining 12.1 per cent of the market. Each drinker was classified according to the brand of beer they most commonly consumed. Sit(b)

A 99% confidence interval for *p*, the proportion of beer drinkers who drink Lion Red (as a proportion of the total market share) is (0.236, 0.282).

26. Which of the following statements is **true**?

F (1) 99% of all beer drinkers have between a 23.6% and 28.2% chance of drinking Lion Red.

F (2) The probability that *p* is between 0.236 and 0.282 is 0.99.

F (3) A 95% confidence interval for *p* will be wider than this interval. narrower

(4) 99% of all such samples would give an interval that contains the true proportion *p*.

F (5) The interval (0.236, 0.282) will cover the true, but unknown parameter *p* for 99% of samples taken.

27. Let p_G denote the proportion of people who drink Steinlager Green, and let p_B denote the proportion of people who drink Steinlager Blue. Thus an estimate for the difference in these two proportions, $p_G - p_B$, is given by $\hat{p}_G - \hat{p}_B$. The standard error for $\hat{p}_G - \hat{p}_B$ is given by: sit(b)

~~(1)~~ $\sqrt{\frac{\hat{p}_G(1-\hat{p}_G)}{2400} + \frac{\hat{p}_B(1-\hat{p}_B)}{2400}}$ sit(a) $\hat{p}_G = .049$

~~(2)~~ $\sqrt{\frac{\text{Min}(\hat{p}_G + \hat{p}_B, \hat{q}_G + \hat{q}_B) - (\hat{p}_G - \hat{p}_B)^2}{2400}}$ sit(c) $\hat{p}_B = .041$ where $\hat{q}_G = 1 - \hat{p}_G$ and $\hat{q}_B = 1 - \hat{p}_B$

(3) $\sqrt{\frac{(\hat{p}_G - \hat{p}_B)(1 - \hat{p}_G - \hat{p}_B)}{2400}}$

~~(4)~~ $\sqrt{\frac{\hat{p}_G(1-\hat{p}_G)}{2400} + \frac{\hat{p}_B(1-\hat{p}_B)}{2400}}$

(5) $\sqrt{\frac{\hat{p}_G + \hat{p}_B - (\hat{p}_G - \hat{p}_B)^2}{2400}}$ sit(b) $= \sqrt{\frac{.049 + .041 - (.049 - .041)^2}{2400}}$

$= 6.1215 \times 10^{-03}$

$= 0.0061215$

Questions 28 and **29** refer to the following information.

Death Penalty Survey Results

"Should convicted murderers be put to death?"		
	Australia	N.Z.
Yes	46%	42%
No	39%	41%
Can't Say	15%	17%

"Should an Australian or New Zealander convicted of drug trafficking in Malaysia or Sri Lanka be put to death?"		
	Australia	N.Z.
Yes	76%	64%
No	19%	26%
Can't Say	5%	10%

[Polls of 1307 Australians & 1010 New Zealanders]

28. The standard error associated with the sample estimate of the difference between the proportion of Australians supporting the **death penalty for N.Z. and Australian drug traffickers in South East Asia** and the proportion of Australians supporting the **death penalty for convicted murderers** is given by: p_1 p_2

$\hat{p}_1 - \hat{p}_2 = .76 - .46 = .3$ sit(c)

(1) $\sqrt{\dfrac{0.76\times 0.46}{1307}+\dfrac{0.64\times 0.42}{1010}}$ s.e.(a)

(2) $\sqrt{\dfrac{0.76\times 0.24}{1307}+\dfrac{0.46\times 0.54}{1307}}$ s.e.(c)

(3) $\sqrt{\dfrac{0.64+0.42-0.22^2}{1010}}$

(4) $\sqrt{\dfrac{0.24+0.54-0.30^2}{1307}}$

(5) $\sqrt{\dfrac{0.76+0.46-0.30^2}{1307}}$ s.e.(b)

$\hat{p}_1 = .76$ $\quad \hat{p}_1+\hat{p}_2 = 1.22$
$\hat{p}_2 = .46$

$\hat{q}_1 = .24$ $\quad \hat{q}_1+\hat{q}_2 = .78$
$\hat{q}_2 = .54$

$\sqrt{\dfrac{[.78-(.76-.46)^2]}{1307}} = .0230$ (4dp)

29. The standard error associated with the estimate of the difference between *the proportion of New Zealanders who said 'Yes'* (p_Y) and *the proportion of New Zealanders who said 'No'* (p_N) to supporting the death penalty for N.Z. and Australian drug traffickers in South East Asia is given by:

$\hat{p}_Y = .64$ $\quad \hat{p}_N = .26$
$n = 1010$

(1) $\sqrt{\dfrac{0.38-(0.64-0.26)^2}{1010}}$

(2) $\sqrt{\dfrac{0.76\times 0.19}{1307}+\dfrac{0.64\times 0.26}{1010}}$

(3) $\sqrt{\dfrac{0.64+0.26-0.64-0.26^2}{1010}}$

(4) $\sqrt{\dfrac{0.9-(0.64-0.26)^2}{1010}}$

(5) $\sqrt{\dfrac{0.24+0.36-0.12^2}{1307}}$

$\sqrt{\dfrac{[.64+.26-(.64-.26)^2]}{1010}} = .0274$ (4 dp)

ANSWERS

1.	**(5)**	2.	**(1)**	3.	**(4)**	4.	**(2)**	5.	**(3)**
6.	**(4)**	7.	**(4)**	8.	**(4)**	9.	**(4)**	10.	**(3)**
11.	**(3)**	12.	**(5)**	13.	**(3)**	14.	**(5)**	15.	**(2)**
16.	**(2)**	17.	**(2)**	18.	**(3)**	19.	**(1)**	20.	**(3)**
21.	**(4)**	22.	**(3)**	23.	**(3)**	24.	**(2)**	25.	**(4)**
26.	**(4)**	27.	**(5)**	28.	**(4)**	29.	**(4)**		