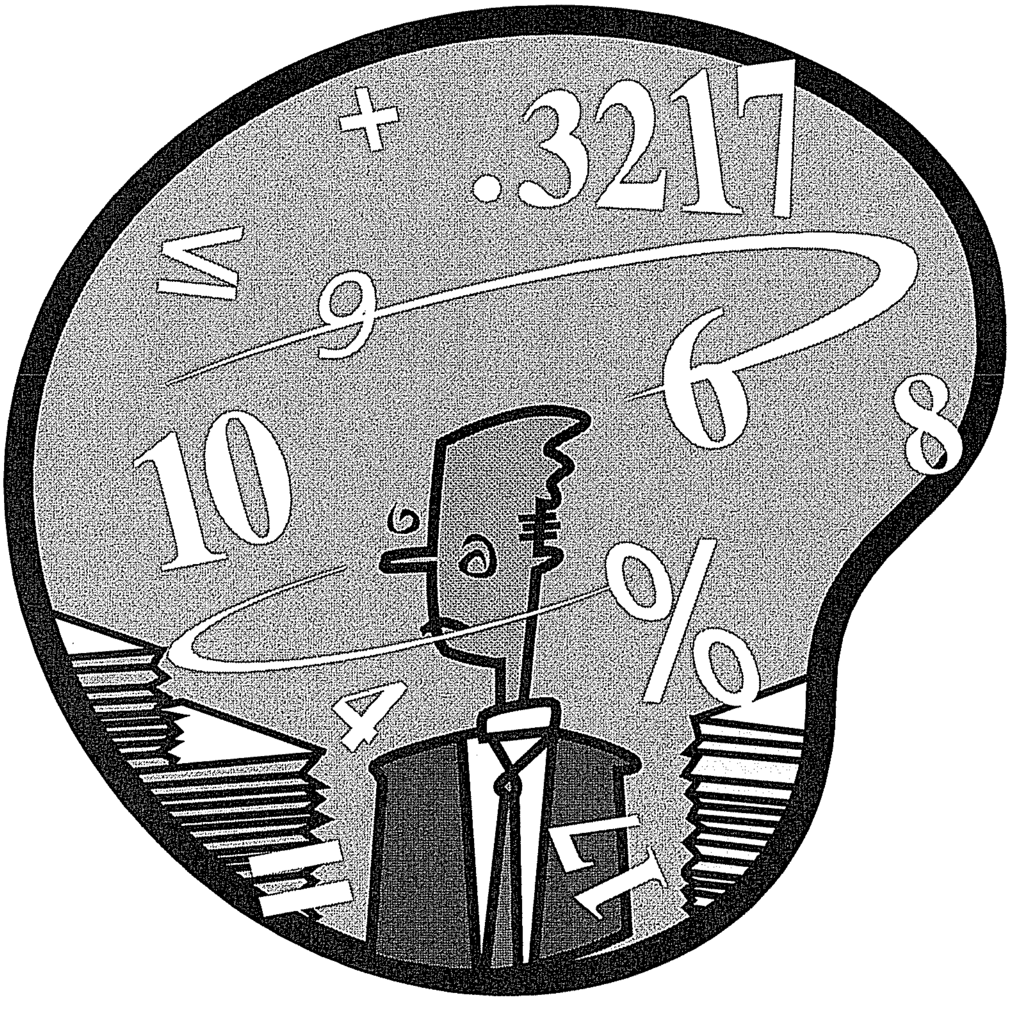
6-10 Qs (25-40%)

Stats 101/108 Workshop

Test Prep 3: Chapters 7 and 8

Friday 10 September, 2010



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Revision Notes 2-4Qs

Chapter 7 – Sampling Distributions of Estimates

Look at blue pages for good notes, 26 statements (2 of which are false!) and test/exam questions for practice

- Statistics is concerned with finding out about the real world and aspects of it specific to our area of interest. Statistical tools allow us to deal with the **uncertainty** present in all samples due to **sampling variation** which occurs because we are unable to survey the entire population of interest.

- We are usually unable to survey the entire population (take a census) as it is too large and/or there are:
  - ✓ budget constraints
  - ✓ time limits
  - ✓ logistical barriers

- This means we are unable to establish the **parameters** of interest within our population, such as:
  - ✓ Population mean, μ
  - ✓ Population standard deviation, σ σ_n $x\sigma_n$

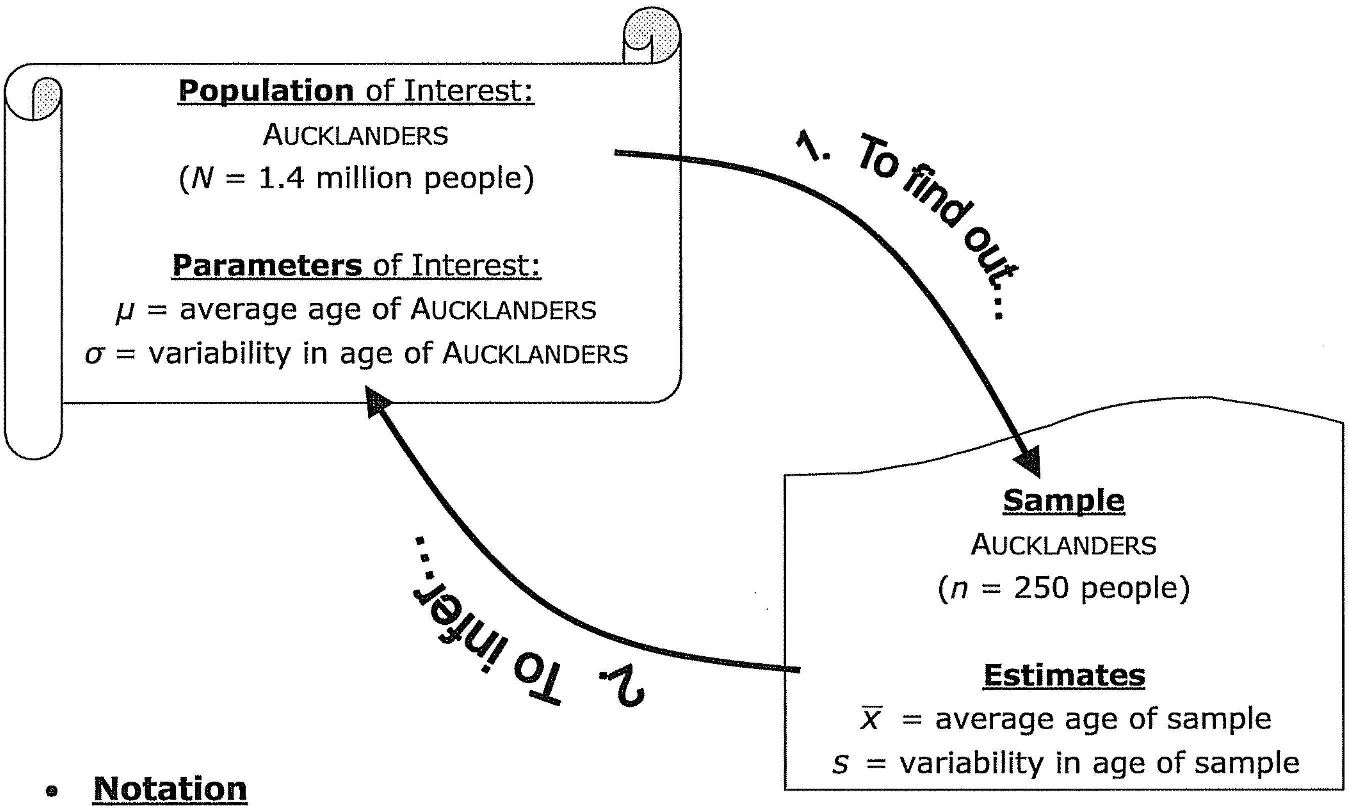
- This means that the **parameter** of interest is an ***unknown* numerical characteristic** for that particular population.

- To estimate an ***unknown* numerical characteristic (parameter)** for our population of interest, we take a sample and find a sample **estimate** from it (that is, we make a **statistical inference**). The **sample estimates** of the above **population parameters** are:
  - ✓ Sample mean, $\bar{x}$
  - ✓ Sample standard deviation, $sd(\bar{x})$ or σ_{n-1} or s $x\sigma_{n-1}$

- Usually ^HATS or ‾BARS are used to distinguish between **sample estimates** and **population parameters**.

- Random variables $X_1, X_2, \ldots X_n$, form a random sample from a distribution if:
  - ✓ they all have the same distribution; and → same mean, μ
  - ✓ they are independent of one another. → same std dev, σ

- The big question which we will answer in Chapter 7 is "But how can we trust the sample estimates ($\bar{x}$ and s) from our single sample of size n?"



- **Notation**

  In statistics we use **CAPITAL letters** to refer to the **variable of interest** for the population and **small letters** to specify the **actual "number" observed** for that variable in our particular sample. a particular # of interest

"distributed as" (in sufficiently large)

Variable of Interest	"Actual "number"
CAPITAL LETTER	**small** LETTER
Ch6: $X \sim$ Normal(μ, σ)	x
Ch7: $\bar{X} \sim$ exactly/approximately Normal$(\mu, \sigma/\sqrt{n})$	$\bar{x}$

a particular sample mean of interest

- **The sample mean – $\bar{X}$**

  If $X_1, X_2, \ldots X_n$, form a random sample from a distribution where $E(X_i) = \mu$ and $sd(X_i) = \sigma$, then,

  mean, average

  ✓ The **expected value** of the sample mean, $E(\bar{X})$ is calculated by:

  $$\mu_{\bar{X}} = \boxed{E(\bar{X}) = \mu}$$

  on formula sheet

  ✓ The **standard deviation** of the sample mean, $sd(\bar{X})$ is calculated by:

  $$\sigma_{\bar{X}} = \boxed{sd(\bar{X}) = \frac{\sigma}{\sqrt{n}}}$$

- **The Central Limit Theorem**

  ✓ The Central Limit Theorem tells us that the larger the sample size, the closer the distribution of $\overline{X}$ comes to a Normal distribution. Even if the distribution of X is non-Normal, the distribution of $\overline{X}$ will be **approximately** Normal for a sufficiently large sample size n.

  ✓ If X is from a "well-behaved" distribution (i.e. symmetric, no outliers) the Central Limit Theorem works reasonably fast. n = 10 may be sufficient!

  ✓ In **general**, n = 30 works well for most distributions *except* distributions that are severely skewed or have large outliers.

  ✓ If the distribution is **severely skewed**, n = 50 should be sufficient.

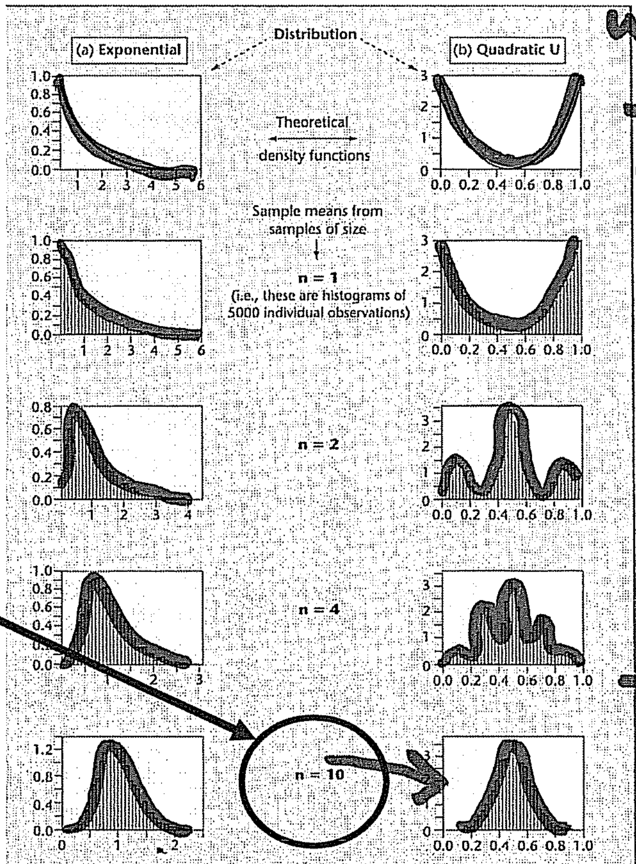


FIGURE 7.2.4 Histograms of means calculated from random samples from the Exponential and Quadratic U distributions.

Chance Encounters, C.J. Wild & G.A.F. Seber, p286

  ✓ If X is from a Normal distribution, then $\overline{X}$ is **exactly** Normally distributed.

- $\overline{X}$ is an **unbiased** estimator of μ because $E(\overline{X}) = \mu$.

theory

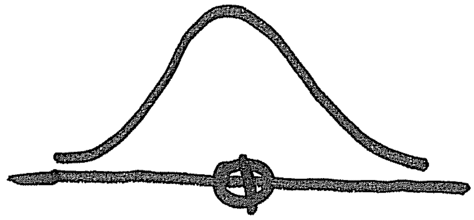
real world

- **Standard errors**

However, as $sd(\overline{X}) = \frac{\sigma}{\sqrt{n}}$, it is not a useful measure of the precision of $\bar{x}$, because we do not know the value of σ. Therefore, we have to use the ***standard error*** of the sample mean to estimate the precision of $\bar{x}$ as an estimate of μ:

$$\text{The standard error of } \bar{x} = \frac{\text{sample standard deviation}}{\sqrt{\text{sample size}}} = se(\bar{x}) = \frac{s}{\sqrt{n}}$$

- **Student's *t*-distribution**



  - ✓ Parameter: Degrees of Freedom (*df*).
  - ✓ Bell shaped and centred at 0 like the (Standard) Normal (0,1) distribution but it's more variable.
  - ✓ As *df* becomes larger, the Student (*df*) distribution becomes more and more like the Standard Normal distribution. → n gets bigger
  - ✓ Student's *t*-distribution ($df = \infty$) and Normal (0,1) are the same distribution. ↳ n > 150 for means
  - ✓ The random sample from a Normal distribution?: $T = \dfrac{\overline{X} - \mu}{se(\overline{X})}$ is exactly distributed as Student($df = n - 1$)
  - ✓ Methods based on this distribution works very well even for small samples that are from very non-Normal distributions. → conf. intervals
  - ✓ By t_{df} (*prob*), we mean the number *t* such that when $T \sim$ Student(*df*), pr($T \geq t$) = *prob*; that is, the tail area above *t* (that is to the right of *t* on the graph) is *prob*:

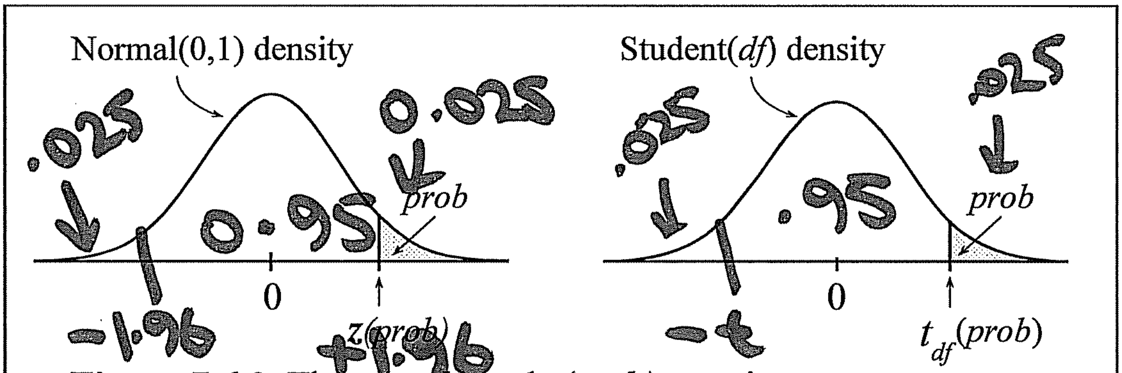


Figure 7.6.2 The *z(prob)* and *t(prob)* notations.

Ch6: $z = \dfrac{X - \mu}{\sigma}$

Ch7: $z = \dfrac{\overline{X} - \mu}{\sigma / \sqrt{n}}$

$T = \dfrac{\overline{X} - \mu}{s / \sqrt{n}}$

Chapter 8 – Confidence Intervals 3-8Q5

Look at blue pages for good notes, 26 statements (2 of which are false!) and test/exam questions for practice

- A **confidence interval** gives a range of plausible values for the parameter of interest that is consistent with the data (at the specified level of confidence). It determines the **size** of the effect or difference.
- You can do all kind of CI's, 90%, 95%, 99%...
- Increasing the confidence level will **increase** the width of the interval.

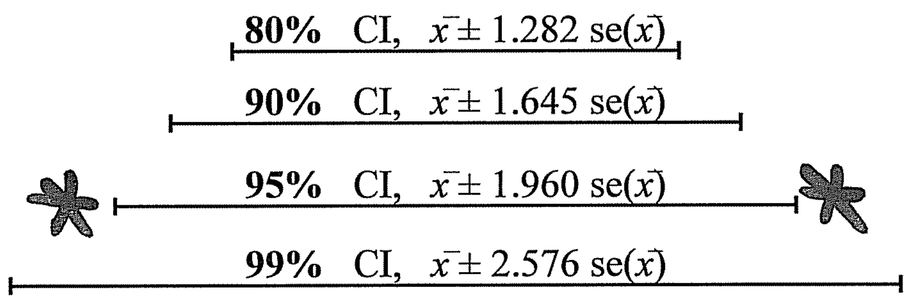


Figure 8.1.3 The greater the confidence level, the wider the interval

From *Chance Encounters* by C.J. Wild and G.A.F. Seber, © John Wiley & Sons, 2000.

narrower

- Increasing the sample size will make the confidence interval more precise.
- To double the accuracy of the confidence interval we **need 4 times** as many observations.

  halve the width

- To triple the accuracy of the confidence interval we **need 9 times** as many observations.

  third the width

- 95% confidence interval
  - ✓ Range of plausible values for the parameter of interest that contains the **true value** of our parameter of interest for 95% of samples taken.
  - ✓ 5% of samples taken will not have the parameter within the calculated confidence interval.
  - ✓ We do not know if the sample we have taken is one of the 95% that contains the true unknown parameter. All we can say is that 95% of the time it will.

✓ If you take 1000 samples, based on the same sampling protocol, then you can expect approximately 950 of these samples will contain the true value (e.g. true mean) of the population.

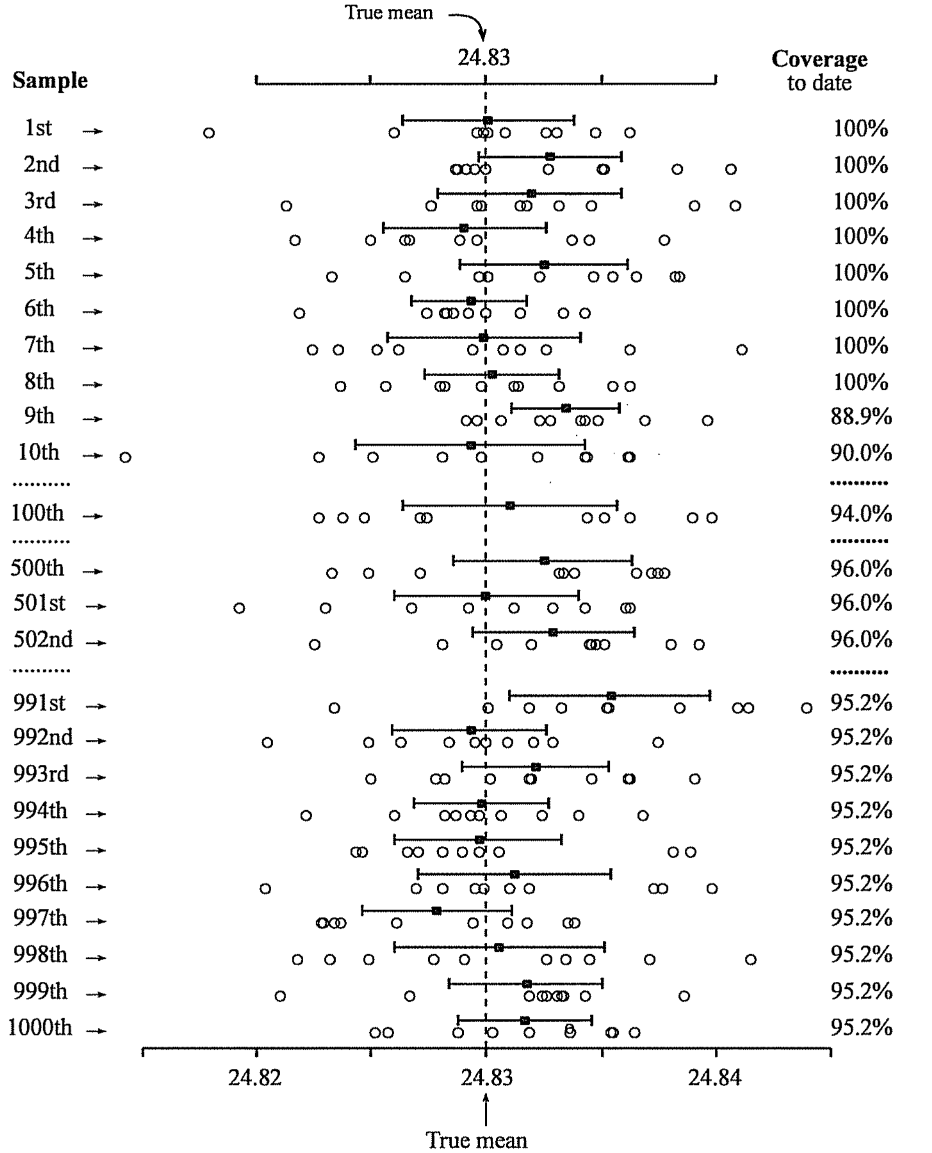


Figure 8.1.2 Samples of size 10 from a Normal(μ=24.83, s=.005) distribution and their 95% confidence intervals for μ..

Step-by-Step Guide to Producing a Confidence Interval by Hand

1. State the **parameter** to be estimated (symbol and words).
   Is it μ, p, $\mu_1 - \mu_2$, or $p_1 - p_2$?
   1. 2. 3. 4.
2. State the **estimate** and its value.
3. Write down the **formula** for a CI, from the Formula Sheet
   $$\boxed{\textit{estimate} \pm \overbrace{t \times \text{se}(\textit{estimate})}^{\text{margin of error}}},$$
4. Find the appropriate **standard error** from the Formula Sheet.
5. Find the appropriate ***df***.
6. Find the appropriate ***t*-value.** This is called the 't-multiplier'.
   Need to know the confidence level and *df*.
7. Calculate the **confidence limits**, the end points of the confidence interval.
8. Interpret the interval using plain English.
   Use the confidence limits to construct an answer to the original question in plain English.

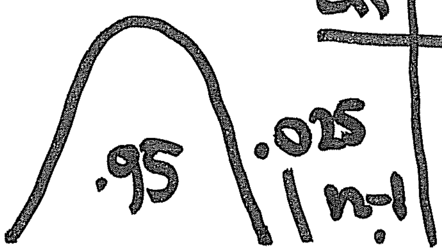
- There are four different types of problem covered in Chapter 8:
  1. Single mean 2. Single proportion 3. Difference between two means
  4. Difference between two proportions:
     Situation (a) ***Proportions from two independent samples***
     Situation (b) ***One sample of size n, several response categories***
     Situation (c) ***One sample of size n, many yes/no items***

- The **estimate** is based on the **parameter** of interest we are investigating: Step 2

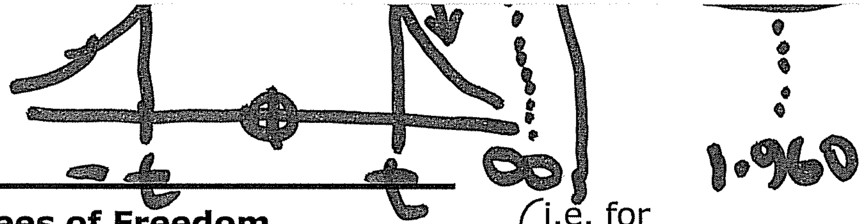
Parameter Step 1	**Estimate** Step 2
1. Single mean $\boldsymbol{\mu}$:	*estimate* = $\bar{x}$
2. Single proportion $\boldsymbol{p}$:	*estimate* = $\hat{p}$
3. Difference between two means $\boldsymbol{\mu_1 - \mu_2}$: (independent samples)	*estimate* = $\bar{x}_1 - \bar{x}_2$
4. Difference between two proportions $\boldsymbol{p_1 - p_2}$:	*estimate* = $\hat{p}_1 - \hat{p}_2$

Steps 5 & 6

- The ***t*-multiplier** (means) / ***z*-multiplier** (proportions) is based on:
  - ✓ Whether we are investigating means or proportions
  - ✓ The desired level of confidence
  - ✓ The degrees of freedom



df	0.025
	⋮
	t



Estimate	Degrees of Freedom
1. $estimate = \bar{x}$	$df = n - 1$
2. $estimate = \hat{p}$	$df = \infty$ 95% CI: 1.96
3. $estimate = \bar{x}_1 - \bar{x}_2$	$df = minimum(n_1 - 1, n_2 - 1)$
4. $estimate = \hat{p}_1 - \hat{p}_2$	$df = \infty$ use 1.96

i.e. for proportions, assume the degrees of freedom is infinity, hence replace t with z score (i.e. the standard Normal).

- The **standard error** is based on the estimate, the number of samples and sample size(s). step 4

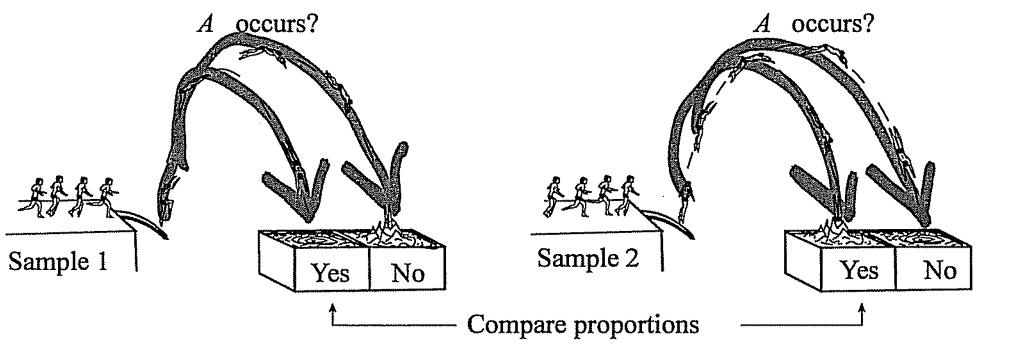
Estimate	se(estimate)
1. $estimate = \bar{x}$	$se(\bar{x}) = \frac{s}{\sqrt{n}}$
2. $estimate = \hat{p}$	$se(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
3. $estimate = \bar{x}_1 - \bar{x}_2$	$se(\bar{x}_1 - \bar{x}_2) = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$
4. $estimate = \hat{p}_1 - \hat{p}_2$	**Situation (a)** ***Proportions from two independent samples*** $se(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$ **Situation (b)** ***One sample of size n, several response categories*** $se(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{\hat{p}_1 + \hat{p}_2 - (\hat{p}_1 - \hat{p}_2)^2}{n}}$ **Situation (c)** ***One sample of size n, many yes / no items*** $se(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{minimum(\hat{p}_1 + \hat{p}_2, \hat{q}_1 + \hat{q}_2) - (\hat{p}_1 - \hat{p}_2)^2}{n}}$ where $\hat{q}_1 = 1 - \hat{p}_1$ and $\hat{q}_2 = 1 - \hat{p}_2$

- $\hat{p}_1 + \hat{p}_2$
- $\hat{q}_1 + \hat{q}_2$

- CIs for the difference between two means/proportions:
  - ✓ If the CI contains 0 (i.e. one negative and one positive number), there may be no difference between the two means/proportions. $(-.05, +.01)$
  - ✓ If CI is positive, then μ_1/p_1 is higher/larger than μ_2/p_2. $(.01, .05)$
  - ✓ If CI is negative, then μ_1/p_1 is lower/smaller than μ_2/p_2. $(-.05, -.01)$

- **3 sampling situations for the difference between two proportions**

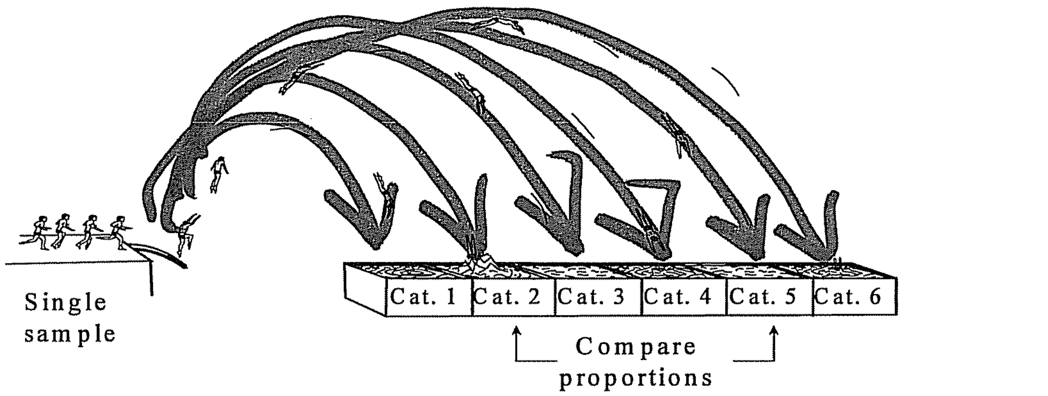
Situation (a) ***Proportions from two independent samples***



From *Chance Encounters* by C.J. Wild and G.A.F. Seber, © John Wiley & Sons, 1999.

1Q only, 1 answer only

Situation (b) ***One sample of size n, several response categories***



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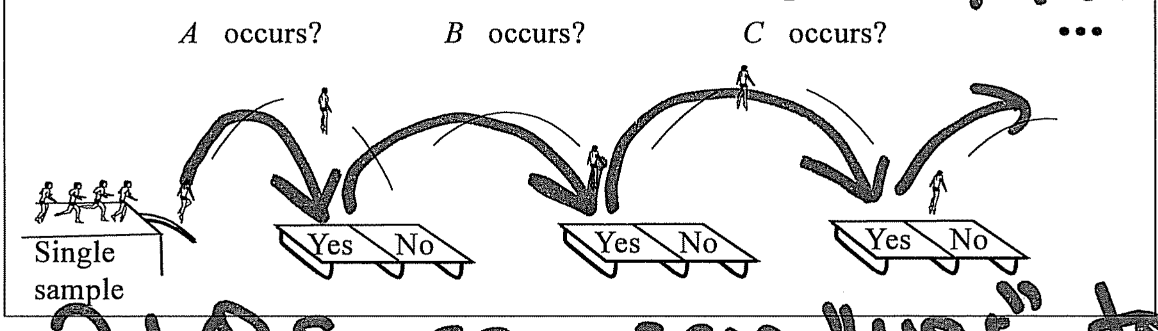
1Q only, can give more than 1 answer, if so wish.

Situation (c) ***One sample of size n, many yes/no items***



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2+Qs, can say "yes" to each, if so wish.

Chapters 7 & 8 – Questions n

1. Suppose $X_1, X_2, \ldots, X_{11}$ are the weights of 11 randomly selected packets of M & M's which come from a distribution with mean $\mu = 54$ and standard deviation $\sigma = 2$. Then the distribution of the sample mean $\bar{X} = (X_1 + X_2 + \cdots + X_{11})/11$ has mean $\mu_{\bar{X}}$ and standard deviation $\sigma_{\bar{X}}$ given by:

   ~~(1)~~ $\cancel{\mu_{\bar{X}} = 4.91}$, $\cancel{\sigma_{\bar{X}} = 0.182}$
   ~~(2)~~ $\cancel{\mu_{\bar{X}} = 594}$, $\cancel{\sigma_{\bar{X}} = 2.000}$
   (3) $\mu_{\bar{X}} = 54$, $\sigma_{\bar{X}} = 0.603$
   ~~(4)~~ $\mu_{\bar{X}} = 54$, $\cancel{\sigma_{\bar{X}} = 2.000}$
   ~~(5)~~ $\cancel{\mu_{\bar{X}} = 4.91}$, $\sigma_{\bar{X}} = 0.603$

$\mu_{\bar{X}} = E(\bar{X}) = \mu = 54$

$\sigma_{\bar{X}} = sd(\bar{X}) = \sigma/\sqrt{n} = 2/\sqrt{11} = .603$

2. Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with mean μ and standard deviation σ. Which **one** of the following statements is **false**?

   T (1) $E(X_1) = E(X_2) = \ldots = E(X_n)$.
   T (2) $X_1, X_2, \ldots, X_n$ all have the same distribution.
   T (3) $X_1, X_2, \ldots, X_n$ are independent of each other.
   T (4) $sd(X_1) = sd(X_2) = \ldots = sd(X_n)$.
   (5) $nX_1 = X_1 + X_2 + \ldots + X_n$.

$3 \times 1 \neq 1 + 2 + 3$

$3 \times 2 \neq 2 + 6 + 1$

3. A random variable T has a Student's t-distribution with df degrees of freedom. Which **one** of the following statements is **false**?

   T (1) The t-distribution for small degrees is flatter with wider tails than the Normal distribution.
   T (2) The mean of T is zero.
   T (3) The t-distribution gets closer to the Normal(0,1) distribution as df gets larger.
   (4) The t-distribution gets closer to the Normal(0,1) distribution as df gets smaller.
   T (5) The distribution of T is symmetric.

4. Let $X_1, \ldots, X_{10}$ be a random sample of size 10 n from a distribution with $\mu = 12$ and $\sigma = 2$, then the distribution of $\bar{X}$ has expected value $E[\bar{X}]$ and standard deviation $sd[\bar{X}]$ where:

   ~~(1)~~ $\cancel{E[\bar{X}] = 1.2}$, $\cancel{sd[\bar{X}] = 2}$
   ~~(2)~~ $E[\bar{X}] = 12$, $\cancel{sd[\bar{X}] = 0.2}$
   ~~(3)~~ $\cancel{E[\bar{X}] = 1.2}$, $sd[\bar{X}] = 0.63$
   (4) $E[\bar{X}] = 12$, $sd[\bar{X}] = 0.63$
   ~~(5)~~ $\cancel{E[\bar{X}] = 120}$, $\cancel{sd[\bar{X}] = 6.32}$

$E(\bar{X}) = \mu = 12$

$sd(\bar{X}) = \sigma/\sqrt{n} = 2/\sqrt{10} = .63$

5. According to Auckland Regional Transport, the mean distance an Aucklander travels to work is 12.6 km. Assume that the distance travelled by an Aucklander to work is well modelled by a Normal distribution with a mean of 12.6 km and a standard deviation of 4.3 km. Let Z be a random variable which has the standard Normal distribution. The probability that the mean distance travelled by a random sample of 10 Aucklanders is greater than 14 km is:

μ σ $\bar{X}$ n

(1) $\Pr\left(Z > \dfrac{10 \times 14 - 10 \times 12.6}{4.3/\sqrt{10}}\right)$

(2) $\Pr\left(Z > \dfrac{14 - 12.6}{\sqrt{10} \times 4.3}\right)$

(3) $\Pr\left(Z > \dfrac{14 - 12.6}{4.3}\right)$

(4) $\Pr\left(Z > \dfrac{14 - 12.6}{\sqrt{10}/4.3}\right)$

(5) $\Pr\left(Z > \dfrac{14 - 12.6}{4.3/\sqrt{10}}\right)$

$Z \sim N(0,1)$

$\bar{x}$ – a particular sample mean

$z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$\Pr\left(Z > \dfrac{14 - 12.6}{4.3/\sqrt{10}}\right)$

6. Which one of the following statements is **false?**

T (1) For large random samples, the true value of μ lies inside the interval $\bar{x} \pm 2se(\bar{x})$ for a little more than 95% of all samples taken.

T (2) For random samples for a Normal distribution, $T = (\bar{X} - \mu)/se(\bar{X})$ is exactly distributed as Student's t-distribution ($df = n - 1$).

T (3) The precision of an estimate refers to its variability – one estimate is less precise than another if it has more variability.

(4) The Student's t-distribution ($df = \infty$) distribution has ~~'fatter' or 'heavier'~~ the same tails than the Normal($\mu = 0$, $\sigma = 1$) distribution.

T (5) For large random samples, $T = (\bar{X} - \mu)/se(\bar{X})$ is distributed as approximately Normal(μ_X=0, σ_X=1).

7. Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with mean μ and standard deviation σ. Let $\bar{X}$ be the sample mean. Which **one** of the following statements is **false**?

T (1) $\bar{X}$ may not be approximately Normally distributed for small samples.

T (2) $\bar{X}$ is a random variable.

(3) sd($\bar{X}$) = sd(X) $/\sqrt{n}$

T (4) E($\bar{X}$) = E(X).

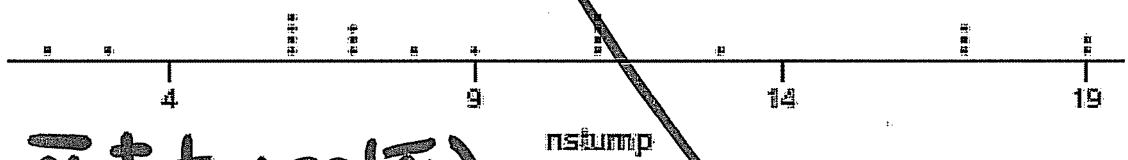
T (5) $\bar{X}$ is at least approximately Normally distributed for large samples.

Questions 11 and **12** refer to the following information.

Data were collected on the number of cricket test match innings in which the batsman was stumped by the wicket keeper. The number of stumping dismissals per year (nstump) were recorded for the years 1970 to 1990 (inclusive). Summary statistics and a dot plot of these data are given below:

	n	$\bar{x}$	s	se($\bar{x}$)
nstump	21	10.14	5.15	1.1238

$= s/\sqrt{n}$



$\bar{x} \pm t \times se(\bar{x})$

11. A 95% confidence interval for the cricket data was calculated to be (7.80, 12.49). Which one of the following statements is **false**?

T (1) If many such samples are taken and a 95% confidence interval for μ_{nstump} is calculated from each sample, then statements such as "μ_{nstump} is somewhere between the two confidence limits" are true, on average, 19 times out of 20. 95 times out of 100

T (2) The technique used to calculate the confidence interval generates an interval which contains the true population mean approximately 95% of the time, in the long run.

(3) F With 95% confidence, the ~~sample~~ mean is somewhere between 7.8 and 12.5. true

T (4) In light of the data, the plausible values of μ_{nstump} are between 7.8 and 12.5. $12.5 - 2.3 = 10.2$

T (5) With 95% confidence, the value of μ_{nstump} is estimated to be 10.1, with a margin of error of 2.3.

(7.8, 12.5) $12.49 - 7.8 = 4.69$
moe $= 4.69 \div 2 = 2.345$

12. Based on the years 1960 to 1969, the mean of nstump is 7.4 and the standard deviation of nstump is 4.35. Which one of the following statements about a new 95% confidence interval for μ_{nstump} based on these data compared to the original confidence interval given above is **true**?

$n = 10$ $\bar{x} = 7.4$ $s = 4.35$

T (1) The t-multiplier used in the new confidence interval is larger. $4.35/\sqrt{10} = 1.3756$

F (2) The standard error used in the new confidence interval is ~~smaller~~. larger

(3) F The new confidence interval is centred around a ~~higher~~ value. lower

F (4) Both confidence intervals would have the same width because they are both 95% confidence intervals for μ_{nstump}.

F (5) The new confidence interval is ~~narrower~~ than the original confidence interval. wider

13. The Central Limit Theorem says that the distribution of the mean, $\overline{X}$, of a random sample:

Ch7

(1) is a Student's t-distribution with $n-1$ degrees of freedom.
(2) is *approximately* Normally distributed when the mean is large.
(3) is an F–distribution with degrees of freedom, which can be calculated from the sample data.
(4) can be approximated by the Normal distribution for small samples.
(5) is *approximately* Normal in large samples.

Questions 14 and 15 refer to the following situation:

The weight gain of women during pregnancy has an important effect on the birth weight of their children. In a study conducted in three countries, weight gains (in kg) of women during the last three months of pregnancy were measured. The results are summarised in the following table:

Country		n	$\overline{x}$	s_X
Egypt	(E)	11	4.55	1.83
Kenya	(K)	11	3.29	0.851
Mexico	(M)	11	2.9	1.8

14. We wish to determine the size of the difference in average weight gain between Egyptian and Kenyan women. Then the value of the standard error of $\overline{x}_E - \overline{x}_K$, $se(\overline{x}_E - \overline{x}_K)$, is (select **one** only):

(1) 0.5990
(2) 0.6085
(3) 0.3703
(4) 0.7739
(5) 0.9288

$$= \sqrt{\frac{1.83^2}{11} + \frac{.851^2}{11}} = .6085$$

15. When calculating a 90% confidence interval for $\mu_E - \mu_K$, the value of the t-multiplier obtained from a *Student's* t-table is 2.228. Assuming $se(\overline{x}_E - \overline{x}_K) = 0.7$ (it is not), then the **margin of error** for the 90% confidence interval is:

(1) $(4.55 - 3.29) \pm 2.228 \times \frac{0.7}{\sqrt{11}}$

(2) $\pm 2.228 \times \frac{0.7}{\sqrt{11}}$

(3) $(3.29 - 4.55) \pm 2.228 \times 0.7$

(4) $(3.29 - 4.55) \pm 2.228 \times \frac{0.7}{\sqrt{11}}$

(5) $\pm 2.228 \times 0.7$

$t = 2.228$

$se(\overline{x}_E - \overline{x}_K) = 0.7$

$\pm t \times se(est)$

$\pm 2.228 \times 0.7$

Questions 16 & 17 refer to the TIME MORGAN POLL "*Living Well Beats Being Green, Say Most Voters*" of July 1994. 662 voters were interviewed by telephone throughout New Zealand and asked whether *developing the economy* or *protecting the environment* would be more important in the short and in the longer term. Below are the numbers of people who gave each response for the short term.

Short term	National	Labour
Economy	136	80
Environment	66	55
Undecided	36	27

There were 238 National and 162 Labour voters in the poll.

Let p_N be the true proportion of National supporters and let p_L be the true proportion of Labour supporters who think that *protecting the environment* is more important in the *short* term.

16. We estimate the difference in proportions $p_N - p_L$ to be (select **one** only):

(1) −0.62
(2) 6.20
(3) 0.062
(4) −0.062
(5) 0.62

$\hat{p}_N - \hat{p}_L = 66/238 - 55/162 = -.062$

17. The 95% confidence interval for the difference between the proportions $p_N - p_L$ is `(-0.154673, 0.0302824)`. Which **one** of the following interpretations is **true**?

$(-.1547, .0303)$

F (1) With a probability of 0.95, the true difference of proportions $p_N - p_L$ lies between −0.1547 and 0.0303.

F (2) In repeated sampling the 95% confidence interval [−0.1547, 0.0303] will contain the true difference in proportions in 95% of the samples taken. with 95% conf

F (3) In repeated sampling the true proportion p_N will be somewhere between 0.1547 larger and 0.0303 smaller than p_L.

(4) With 95% confidence the true proportion p_N is somewhere between 0.1547 smaller and 0.0303 larger than p_L.

(5) With 95% confidence the true proportion p_N is 0.1850 larger than p_L.

with a margin of error of .185/2 = .0925

−.062
.062 smaller

18. The data below are taken from a study by M.C.Gilly (1988), 'Sex roles in advertising: a comparison of television advertisements in Australia, Mexico, and the United States', *Journal of Marketing* 52, pp 75-85. In this study, a random sample of TV commercials was cross-classified, as shown below. The following table shows the different settings that women in commercials are portrayed in.

Setting	Australia	United States	Mexico	Totals
Home	15	57	31	103
Retail outlet	3	15	19	37
Occupational	0	6	1	7
Outdoors	3	19	14	36
Other	31	72	55	158
Totals	52	169	120	341

The sample estimate of the difference between the proportion of **Mexican** and **Australian** women being portrayed **outdoors** in TV commercials, $\hat{p}_{MEX} - \hat{p}_{AUS}$, is:

(1) 11
(2) 0.306
(3) 0.021
(4) 0.059
(5) 0.03

Questions 19 to **21** refer to the following information.

In 1990 *CNN/Time* sought information on how young American adults viewed their parents' marriage. In a telephone poll, one of the questions they asked of six hundred and two (602) 18-29 year old Americans was "Would you like to have a marriage like the one your parents have?". Forty-four percent (44%) responded "Yes".

n $\hat{p}$

19. *CNN/Time* were interested in determining what proportion p of the 18-29 year old American population would answer "Yes" to this question. Which **one** of the following statements is **false**?

T (1) The value of the parameter of interest p is an unknown quantity.

T (2) In this context, 0.44 is an estimate for the parameter of interest. $\hat{p}$

(3) The ~~parameter of interest~~ sample estimate depends on the sample and hence is a random quantity.

T (4) A confidence interval for the parameter of interest will give a range of possible values for this parameter.

T (5) The p parameter of interest is the proportion of 18-29 year old Americans who would have answered "Yes" in 1990.

20. An approximate 95% confidence interval for the proportion of the 18-29 year old American population who would have answered "Yes" to this question in 1990 is:

(1) [0.415,0.465]

(2) [0.439,0.441]

(3) [0.400,0.480]

(4) [0.420,0.460]

(5) [0.407,0.473]

95% CI for p:

est ± t × se(est)

$\Rightarrow \hat{p} \pm z \times \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$\Rightarrow .44 \pm 1.96 \times \sqrt{\frac{.44(1-.44)}{602}}$

21. If two thousand four hundred (2400) 18-29 year old Americans had been sampled instead of six hundred and two (602) 18-29 year old Americans, then the new 95% confidence interval would be approximately:

(1) twice as wide.

(2) one-quarter as wide.

(3) half as wide.

(4) four times as wide.

(5) equally as wide.

$\Rightarrow (.400, .480)$

$602 \xrightarrow{4\times} 2400$

Questions 22 to **24** refer to the following information:

The *Listener/Heylen* poll from August 6, 1994 reported the following results on what New Zealanders think about the "Ten Commandments" from a sample of 1,000 randomly chosen New Zealanders. For each of three commandments, the percentage of people agreeing that the commandment "fully applies to me" is given. Also reported were the results of a 1985 poll, which asked 1,000 New Zealanders the same questions.

n_{1994} n_{1985}

- I am the Lord your God; worship no god but me. (*One God*)
- Do not commit murder. (*Do Not Murder*)
- Do not desire another person's goods. (*Do Not Envy*)

Percentage Agreeing

Year	*One God*	*Do Not Murder*	*Do Not Envy*
1985	39%	85%	53% p_2
1994	32% p_N	89% p_M	62% p_1

sit(c) $\hat{p}_M + \hat{p}_N = .89 + .32 = 1.21$

$\hat{q}_M + \hat{p}_N = .11 + .68 = .79$

22. Consider the 1994 poll. Let p_M denote the proportion that think that *Do Not Murder* fully applies to them and let p_N denote the proportion that think that *One God* fully applies to them. The sample estimate of $\hat{p}_M - \hat{p}_N$ and its associated standard error, $se(\hat{p}_M - \hat{p}_N)$, are:

~~(1) 0.57~~ and 0.0298

~~(2) 0.57~~ and 0.0178

(3) 0.57 and 0.0216

(4) 0.57 and 00298

(5) 0.57 and 0.0178

$\hat{p}_M - \hat{p}_N = .89 - .32 = .57$

$se(\hat{p}_M - \hat{p}_N) = \sqrt{\left(\frac{\left[.79 - .57^2\right]}{1000}\right)}$

$= .0216$ (4dp)

sit(a)

23. Let p_1 denote the proportion of New Zealanders that in 1994 thought that *Do Not Envy* fully applied to them. Let p_2 denote the proportion of New Zealanders that in 1985 thought that *Do Not Envy* fully applied to them. A 95% confidence interval for $p_1 - p_2$ is given by:

(1) (0.047,0.133)

(2) (0.035,0.145)

(3) (–0.126,–0.054)

(4) (0.054,0.126)

(5) (–0.133,–0.047)

est ± t × se(est)

$\Rightarrow \hat{p}_1 - \hat{p}_2 \pm z \times \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$

$\Rightarrow .62 - .53 \pm 1.96 \sqrt{\frac{.62(1-.62)}{1000} + \frac{.53(1-}{1000}}$

$\Rightarrow (.047, \;)$

24. A 99% confidence interval for the proportion of New Zealanders who believe that *One God* fully applies to them, p_G, is given by (0.282, 0.358). Which one of the following statements is **true**?

F (1) The interval (0.282, 0.358) will cover the true, but unknown parameter p_G for 99% of samples taken.

F (2) Between 28.2 and 35.8 per cent of New Zealanders believe that *One God* fully applies to them 99% of the time.

F (3) A 95% confidence interval for p_G would be ~~wider~~ narrower than this interval.

F (4) The probability that the interval (0.282, 0.358) covers the sample proportion is ~~0.99~~ 1

(5) The probability that another interval calculated in the same way from a new sample of 1000 New Zealanders covers p_G is 0.99.