Workshop programme

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Schedule

Time	Sun 3	Mon 4	Tue 5	Wed 6	Thu 7	Fri 8	Sat 9
			8:30 Boat trip		*8:00 Waipoua*		
09:00		Garoni	*Paihia wharf*	Kleijn	*forest*	Wright	*9:00 Bus*
09:40		Newman		Kosygina	*trip*	Ruszel	*departs*
10:20		Stein		Hambly		Camia	*Paihia*
11:00		*Snacks*		*Snacks*		*Snacks*	
11:30	*Bus*	Drewitz	*Packed Lunch*	Sakai	*Packed*	Barbour	
12:10	*departs*	Khanin	*Provided*	Valesin	*Lunch*	Khmaladze	
12:50	*airport*	*Lunch*	*Finishes 12:30*	*Lunch*	*Provided*	*Lunch*	
14:00	*midday*	Guttmann		Ramirez		de Gier	
14:40		***Open problems***		***Open problems***		Brydges	
15:20		*Snacks*		*Snacks*	*4pm return*	*Snacks*	
			Russell Ferry 5:30	*5:30 Chuck's*			
	Dinner 6:15	*Dinner 6:15*	*Duke Dinner 6:30*	*70th birthday*			
			last return 10:00	*bash*		*Dinner 6:45*	

Titles and Abstracts

- **Andrew Barbour: Multivariate approximation in total variation**
  When approximating the distribution of an integer valued random variable, it is natural to try to measure the error with respect to total variation distance. For Poisson approximation, this has been extraordinarily successful. In recent years, it has also been shown that approximation in total variation can be established in many circumstances in which a normal approximation is good, strengthening the usual results based on Kolmogorov distance. In our talks, we present a recipe for establishing total variation approximation in 2 or more dimensions, using the discrete normal family, and illustrate it in the context of Stein's method of exchangeable pairs, using a simple graph colouring problem as illustration.

  Joint work with Malwina Luczak and Aihua Xia

- **David Brydges: The Coulomb gas in two dimensions**
  This will be an introduction to the statistical mechanics of an infinite assembly of positively and negatively charged particles on a lattice $\mathbb{Z}^2$. There are some surprising theorems: for example, at large values of the parameter that represents temperature, empirical charge densities in disjoint regions are almost independent, even though the Coulomb interaction is very long range. This is called Debye screening. At small values of the temperature there is the famous Kosterlitz-Thouless phase where all charges are bound into neutral configurations; the system has the same long distance correlations as a random field known as the massless free field. Recent progress by Pierluigi Falco has made it possible to understand the transition into this phase in detail, but this subject will never cease to generate interesting new questions, and I will point out some of those along the way.

- **Federico Camia: Non-backtracking Loops and Statistical Mechanics on Spin Networks**
  I will discuss the edge-occupation field generated by a "gas" of non-backtracking loops (a loop "soup") characterized by a spatial Markov property such that, conditionally on the value of the edge-occupation field on a boundary set, the distributions of the loops and arcs on either side of the boundary are independent of each other. The field configurations generated by the loop soup satisfy the definition of spin-network introduced by Roger Penrose (1971) in an attempt to formulate a discrete, combinatorial theory of space-time, and later used in work on non-perturbative quantum gravity. The distribution of the edge-occupation field, induced by the (Poisson = ideal gas) distribution of loops, can be shown to be a Gibbs distribution with a nearest-neighbor (vertex) Hamiltonian that depends on the fugacities associated to the edges. The homogeneous model with fugacity x is exactly solvable in any dimension, in the sense that the free energy density of the occupation field can be calculated explicitly as a function of x, in the thermodynamic limit, for the d-dimensional torus for any dimension d. The model has a critical fugacity marked by the divergence of a derivative of the free energy density, where the order of the derivative and the type of divergence depend on the dimension d. (Based on joint work with Marcin Lis: arXiv:1507.05065.)

- **Alex Drewitz: The maximal particle of branching random walk in random environment**
  One-dimensional branching Brownian motion has been the subject of intensive research, in particular during the last decade. We consider the discrete space version of branching random walk and investigate the setting of a spatially random branching environment; in particular we are interested in the position of the maximal particle. Via the Feynman-Kac formula this is connected to fluctuations of the solutions to the parabolic Anderson model (i.e., the heat equation with a random potential)

as well as to a randomized version of the Fisher-KPP equation. The Fisher-KPP equation is a fundamental reaction-diffusion partial differential equation which had originally been introduced in order to model the spread of an advantageous alelle in a population of a one-dimensional habitat.

- **Tim Garoni: The worm process for the Ising model is rapidly mixing**
  We consider the worm process for the zero-field ferromagnetic Ising model, introduced by Prokofiev and Svistunov. We prove the process is rapidly mixing on all finite graphs and at all temperatures. As a corollary, we construct fully-polynomial randomized approximation schemes for the Ising susceptibility and two-point correlation function.

  Coauthors: A. Collevecchio, T. Hyndman and D. Tokarev

- **Jan de Gier: The stochastic multi-species asymmetric exclusion process and Macdonald polynomials**

  The stationary state of the multi-species asymmetric exclusion process can be calculated analytically due to the underlying Yang-Baxter integrability of the model. I will show how this stationary state is related to the theory of Macdonald polynomials which are generalisations of Schur polynomials, characters of polynomial representations of the symmetric group. Through this connection, matrix product techniques from stochastic processes lead to novel expressions for Macdonald and related polynomials.

- **Tony Guttmann: Compressed random walks, self-avoiding walks and bridges**
  Our aim originally was to study two-dimensional self-avoiding walks and bridges constrained to lie in a strip, with a force f squeezing the strip together. A walk ω reaching height $h(\omega)$ has associated with it a weight $e^{-f\cdot h(\omega)}$. The *partition function* for walks of length n is just $\sum_{|\omega|=n} e^{-f\cdot h(\omega)}$. This models polymers squeezed between impenetrable surfaces. In order to understand our (numerical) results, we simplified the model to that of random walks. Even here the results surprised us, in that the partition function, which for $f = 0$ is well known to be $\sim \frac{2}{\sqrt{n\pi}} 4^n$, is for $f > 0$

  $$\sim A(f) \cdot 4^n \cdot \mu_1(f)^{n^{1/3}} \cdot n^{-1/6}.$$

  Similar behaviour (with different growth constants and exponents) is observed for SAWs and bridges, and the observed behaviour will be explained by SLE arguments. This is joint work with Nick Beaton, Iwan Jensen and Greg Lawler.

- **Ben Hambly: A simple model for interfaces in a martensitic phase transition**
  A martensitic phase-transformation for a material is a first-order diffusionless transition involving a change of shape of the underlying crystal lattice. In the transition there is a symmetry breaking leading to the formation of different variants with interfaces between them and the original phase. We will consider a simple fragmentation model for the patterns that arise from this phase transition. By encoding the model using a general branching random walk we determine the growth rates for the proportion of interfaces which are of a certain size after a certain time. We calculate explicit descriptions of the interface asymptotics and determine a power law exponent.

- **Kostya Khanin: On statistics of action-minimizing paths for random walks in product-type random environments**
  We shall discuss a model of action-minimizing paths in dimension 1 where the action is defined by

a time-dependent random potential. The setting is similar to the KPZ case. However we consider a potential of the product-type. Namely, it corresponds to a product of two independent terms reflecting the time and the space disorder respectively. It turn out that in this case one can calculate the limiting law for the optimal action and for the end point of the path in the limit when it length tends to infinity.

- **Bas Kleijn: The frequentist consistency of posterior distributions**
  Schwartz theorem on posterior consistency is generalized in several respects. First, while testability remains a requirement, it is shown to be equivalent to Doobs Bayesian form of posterior concentration. Second, to strengthen Bayesian testability to Frequentist testability, we replace Schwartzs Kullback-Leibler condition with a property akin to (but weaker than) contiguity. The Kullback-Leibler condition for the prior remains sufficient, as well as Freedmans tailfreeness property. Le Cams First Lemma is extended, implying sufficiency of the Ghosh-Ghosal-vdVaart construction (2000) as well as more recent criteria based on Hellinger transforms (Kleijn and Zhao, 201x)). Some preliminary results are presented regarding the testing of hypotheses on network structure in the stochastic block model.

- **Estate Khmaladze: Empirical Processes and Unitary Operators - application to the Statistical Problems with Covariates**
  Many of us know about function-parametric empirical processes: if P_n is an empirical distribution based on i.i.d. $\xi_i \in \mathbb{R}^d, i = 1, \dots, n$, and P is the distribution of each ξ_i, then the random measure

$$v_n = \sqrt{n}[P_n - P],$$

  is the (set-parametric) empirical process, and the integral

$$v_n(\phi) = \int \phi(z) v_n(dz), \quad \phi \in L_2(P), \tag{1}$$

  is the function-parametric empirical process. We propose to investigate the construction

$$v_n(U\phi), \quad U - \text{a unitary operator on } L_2(P).$$

  If we do not count a couple of papers in the references, this is completely unexplored construction. However, it offers surprisingly useful results.

  We exemplify it with the following class of statistical problems. Suppose there is a "response" random variable Y and its covariate, or "explanatory variable" X. The object of interest is the conditional distribution $F(y|X)$ and one usually formulates hypothesis about its particular form. There are zillions of problems like this in all sorts of applications, but there is no goodness of fit test, which is asymptotically distribution free, like, say χ^2 or Kolmogorov-Smirnov tests in simple problems.

  We hope to show that (1) leads to the whole class of such tests.

  References:

  E. Khmaladze (2013), Note on distribution free testing for discrete distributions, *The Annals of Statistics*, 41, 6, 2979-2993

E. Khmaladze (2015), Unitary transformations, empirical processes and distribution free testing, *Bernoulli J.*,

- **Elena Kosygina: Excited random walks in Markovian cookie environments on $\mathbb{Z}$.**
  We consider a nearest-neighbor random walk on $\mathbb{Z}$ whose probability $\omega(x,n)$ to jump to the right from site x depends not only on x but also on the number of prior visits n to x. The collection $(\omega(x,n))_{x\in\mathbb{Z},n\geq 0}$ is sometimes called the "cookie environment" due to the following informal interpretation. Upon each visit to a site the walker eats a cookie from the cookie stack at that site and chooses the transition probability according to the "flavour" of the cookie eaten.

  Assume that the cookie stacks are i.i.d. and that the cookie "flavours" at each stack $(\omega(x,n))_{n\geq 0}$ follow a finite state Markov chain in n. Thus, the environment at each site is dynamic, but it evolves according to the local time of the walk at each site rather than the random walk time. We discuss recurrence/transience, ballisticity, and limit theorems for such walks.

  This is a joint work with Jonathon Peterson, Purdue University.

- **Chuck Newman: Minimal Spanning Tree on a Slab**
  In joint work with Vincent Tassion and Wei Wu, we have studied the minimal spanning forests on the nearest neighbor slabs with vertex sets such as $\mathbb{Z}^2 \times 0, 1, ..., k^{d-2}$. For $\mathbb{Z}^d$ itself, it is known that the forest is a single tree for $d =$ (1 and) 2 but nothing is known for $d > 2$ except it is conjectured that the $d = 2$ behavior continues until some d_c (probably 6 or 8) above which there are infinitely many trees in the forest. Our result is that in slabs, there is only a single tree. The work is related to that of Duminil-Copin, Sidoravicius and Tassion who proved that there is no infinite cluster in critical Bernoulli percolation in such slabs. We also get new results for that critical percolation setting.

- **Alejandro Ramírez: Random walk in random environment at low disorder and non-vanishing velocity**
  We consider a random walk whose jump probabilities are i.i.d. perturbations of those of the simple symmetric random walk on the hypercubic lattice $\mathbb{Z}^d$ for dimensions $d \geq 2$. This walk can exhibit a variety of behaviors, ranging from recurrent to transient regimes with zero or non-zero velocity. In the non-zero velocity regime, we establish an asymptotic expansion of the invariant measure of the environmental process. Furthermore, we derive an upper bound for the velocity, corresponding to its asymptotic expansion, when the local drift is very small. This talk is based on joint works with Clément Laurent, Christophe Sabot and David Campos.

- **Wioletta Ruszel: Synchronization for mean-feld XY models in a random external field**
  Synchronization processes are ubiquitous in nature and play a very important role in many different contexts as biology, ecology, climatology or sociology. It is known that synchrony is rooted in human life from the metabolic processes in our cells to the highest cognitive tasks we perform as a group of individuals. The milestone here was to consider biological oscillators as phase oscillators, neglecting the amplitude. Such a population of non-identical oscillators can exhibit a remarkable cooperative phenomenon, the temporal analog of a phase transition. When the strength of the coupling some oscillator spontaneously synchronize to a common frequency, overcoming the disorder of the natural frequencies. This Kuramoto model has been studied extensively under different aspects, e.g. including noise terms. In our contribution we are looking at a XY model (of which the Kuramoto model is a special case) in a random external field and study its synchronization behaviour. This is work in progress in collaboration with F. Collet (University of Bologna).

- **Akira Sakai: Self-avoiding walk on random conductors**
  Self-avoiding walk (SAW) on the d-dimensional integer lattice $\mathbb{Z}^d$ is often used as a model for linear polymers in d dimensions. It is known that the susceptibility (= the sum of the two-point function) diverges as the energy-cost h for each bond goes down to the critical point $-\log \mu$, where μ is the connective constant. Suppose that the energy-cost is not homogeneous, but it is perturbed by i.i.d. random variables, so that the susceptibility becomes random and loses translation invariance. A natural question would be to ask how the randomness affects the phase transition and critical behavior of SAW.

  In the talk, I will explain the recent results on the quenched critical point and future problems.

- **Dan Stein: Thermodynamic Identities and Symmetry Breaking in Short-Range Spin Glasses**
  We present a technique to generate relations connecting pure state weights, overlaps, and correlation functions in short-range spin glasses. These are obtained directly from the unperturbed Hamiltonian and hold for general coupling distributions. All are satisfied in phases with simple thermodynamic structure, such as the droplet-scaling and chaotic pairs pictures. If instead nontrivial mixed-state pictures hold, the relations suggest that replica symmetry is broken as described by a Derrida-Ruelle cascade, with pure state weights distributed as a Poisson-Dirichlet process.

- **Daniel Valesin: Percolation on the stationary distribution of the voter model on $\mathbb{Z}^d$**
  The voter model on $\mathbb{Z}^d$ is a particle system that serves as a rough model for changes of opinions among social agents or, alternatively, competition between biological species occupying space. When the model is considered in dimension 3 or higher, its set of (extremal) stationary distributions is equal to a family of measures μ_α, for α between 0 and 1. A configuration sampled from $\mu_a lpha$ is a field of 0's and 1's on $\mathbb{Z}^d$ in which the density of 1's is α. We consider such a configuration from the point of view of site percolation on $\mathbb{Z}^d$. We prove that in dimensions 5 and higher, the probability of existence of an infinite percolation cluster exhibits a phase transition in α. If the voter model is allowed to have long range, we prove the same result for dimensions 3 and higher. Joint work with Balázs Ráth.

- **A. Larry Wright: Dependence in stock and option model pricing**
  We begin by reviewing some of the more well-known pricing models, such as the Black-Scholes, Cox-Ross-Rubenstein, and Kou Double Exponential models.

  Violations of independence are studied, with an example indicating Markov dependence. Possible improvements in the models using associated processes are explored.

  A discussion of what is really used by traders is given.