The Margin of Error

Two different snap polls commissioned by two Sunday newspapers immediately preceding the nation's September 2005 general election gave two apparently conflicting sets of results:

Herald on Sunday:
Labour 42%
National 38.5%
NZ First 5.5%
Margin of error: 4.9%
($n = 400$)

Sunday Star Times:
National 44%
Labour 37.2%
NZ First 4.7%
Margin of error: 4.4%
($n = 540$)

The margins of error remind us that these reported levels of party support are just the results for the respective samples and that there is uncertainty about the (unknown) true corresponding population percentages.

The margin of error and its associated confidence interval

From the 170 or so respondents in the *Herald's* sample of 400 (42%) who indicated support for Labour, we can conclude that the true population percentage would have been about 42%. Only *about* 42% because with another random sample, a different 400 respondents, the number supporting Labour would have been slightly different, giving a slightly different point estimate for the true population percentage. Not surprisingly, this phenomenon is termed 'sampling variability'. The margin of error enables us to quantify the word 'about', i.e., it enables us to take account of this sampling variability. When the margin of error (0.049) is added to and subtracted from the point estimate (0.42) we get an interval estimate (0.42 ± 0.049 = from 0.37 to 0.47) for the true population value. This is a range of possible values for the unknown population proportion, i.e., an approximate 95% confidence interval, with half-width equal to the margin of error.

> Confidence Interval:
>
> estimate ± *margin of error*

(Assume 95% level of confidence unless otherwise stated.)

Hence from the *Herald on Sunday's* snap poll results, we conclude that somewhere between 37% and 47% of the NZ voting population would have indicated support for Labour at that time. We would also want to remind ourselves that conclusions such as this one are correct, in the long-run, for only approximately 19 out of every 20 (95%) surveys taken. We are unable to determine whether this is one of the 5% 'rogue' surveys but take comfort in the fact that this 95%-confidence-interval-building process works most (approximately 95%) of the time and, as such, rogue surveys are very few and far between.

The sampling error, nonsampling errors and the margin of error

It is also important to recall at this point, that the confidence interval, and hence the margin of error, takes into account only the effect of sampling variability, i.e., the sampling error. The actual difference between a sample estimate and its true population value comprises two types of error – sampling error and nonsampling error. The sampling error is caused by the act of sampling; it has the potential to be bigger in smaller samples; and, provided some form of

random sampling has been employed, we can determine how large it can be (the margin of error). Sampling error is unavoidable, it is part of the cost of sampling.

In surveys, the margin of error does **not** take into account other potential sources of error such as bias due to:

- people refusing to respond or excluding groups who could not be contacted or who had not yet made up their minds (nonresponse bias)
- people lying (response bias)
- sampling from a subsection of the target population by deliberately or even inadvertently excluding specific groups within the population from the sampling process (selection bias)

These types of errors are called nonsampling errors. Nonsampling errors can be much larger than the sampling error; they are impossible to correct for after the completion of the survey; and, we cannot determine how badly they affect the results of the survey. Care must be taken in the design and implementation of the survey in an attempt to avoid, or at the very least minimise the effects of these nonsampling errors.

Calculating the margin of error for a single proportion

The formula for an approximate 95% confidence interval for a population proportion, p, is

$\hat{p} \pm 1.96 \times \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$ where $\hat{p} = $ *the sample proportion* and $n = $ *the sample size.*

This formula (and the associated margin of error formula below) is based on a large sample Normal approximation. This means that the sample size, n, has to be sufficiently large for the formula to be valid. Just how large n has to be depends on the value of the sample proportion, $\hat{p}$. Wild & Seber give a table (based on published research by Samuels and Lu, 1992) which shows how large n must be for various values of $\hat{p}$, e.g., for $\hat{p} = 0.5$, n must be at least 10; for $\hat{p} = 0.35$, $n \geq 23$ and for $\hat{p} = 0.05$, $n \geq 960$. (These minimum sample sizes are generally larger than the ones allowed for under the more familiar $n\hat{p} > 5$ and $n(1-\hat{p}) > 5$ or $n\hat{p}(1-\hat{p}) > 10$ conditions.) Based on values suggested in Wild & Seber's table, we would have no concerns about the validity of using the sample proportions from the above polls to calculate the confidence intervals (and margins of error for the corresponding estimates) for the proportions in the population giving support to Labour and to National. We would however be well justified in expressing some concern about calculating the same for the NZ First party which has sample proportions around 0.05 from samples of 400 and 540 in size.

From the 95% confidence interval formula, we see:

$$\text{margin of error for a single proportion} = 1.96 \times \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$$

which means that the margin of error changes as the value of the sample proportion, $\hat{p}$, changes.

When $\hat{p} = 0.5$, $1.96 \times \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$ is very close to $1/\sqrt{n}$, and only a little less than $1/\sqrt{n}$ for any other value of $\hat{p}$ between about 0.3 and 0.7. When $\hat{p}$ is much smaller than 0.3 or much bigger than 0.7, then $1/\sqrt{n}$ is appreciably bigger than the margin of error:

$$
\begin{array}{l}
\text{margin of error} \\
\text{for a single proportion}
\end{array}
\begin{cases}
\approx 1/\sqrt{n} & \text{for } 0.3 \le \hat{p} \le 0.7 \\
\ll 1/\sqrt{n} & \text{for } \hat{p} < 0.3 \text{ or } \hat{p} > 0.7
\end{cases}
$$

How the media usually report the margin of error

When poll results are reported in the media, there is usually only one margin of error quoted to cover all estimated proportions – we see this in both survey reports above. A media-reported margin of error for a single proportion is almost always calculated using the '$1/\sqrt{n}$' formula (or some close form of it). The calculation does not involve the value(s) of the sample proportion(s) to which it refers. This media-reported margin of error is a *conservative* margin of error in the sense that none of the *true* margins of error for the individual sample proportions would be larger than it.

In the *Herald* survey, the media-reported margin of error (0.049) is a good approximation for the true margins of error for $\hat{p}_{Labour} = 0.42$ and $\hat{p}_{National} = 0.385$ (both estimates are between 0.3 and 0.7) but is not a good approximation for $\hat{p}_{NZ First}$. The true margin of error for

$\hat{p}_{NZ First} = 0.055$ ($1.96 \times \sqrt{\dfrac{0.055 \times (1-0.055)}{400}}$ = 0.022) is less than half the media-reported one.

Similar comments apply to the results of the *Times* survey.

Comments such as 'Support for minor Party *X* has fallen within the margin of error . . .' are often made in media reports on political polls. In these situations the estimated level of support would always be much less than 0.3 and consequently the media-reported margin of error will always be much greater than the true margin of error. In almost all these cases, the estimated level of support for Party *X* will not be less than its true margin of error. And even if it were, what are the media inviting us to infer from their 'fallen-within-the-margin-of-error' comment – Party *X* may have zero level of support, or even, a negative level of support?

Comparing poll results

The main questions we look to answer from poll results are of a comparison-type: "Which party is in the lead?". The *Times* survey shows that National was 6.8% ahead of Labour. This difference is an estimate and, as such, is subject to sampling error (sampling variability). The margin of error attempts to take this sampling error into account. A media-reported margin of error is not a good approximation for the true margin of error *for a difference* – it will always be too small. In the *Times* survey, the difference in levels of support between Labour and National (0.068) has a true margin of error of 0.076. With a margin of error of 7.6%, we would conclude that the 6.8% difference is not significant. Using the media-reported margin of error (4.4%), we would have said the opposite, i.e, the 6.8% difference is significant and the support for National in the population was higher than that for Labour. Formulae for calculating the true margin of

error for the difference between two proportions depend on whether the two proportions come from the same single sample or from two independent samples. These formulae are given in Wild & Seber.

Wild & Seber also give 2 rules-of-thumb for using a media-reported margin of error to quickly (but roughly) approximate the true margin of error for the difference between two sample proportions, $\hat{p}_1$ and $\hat{p}_2$:

$\hat{p}_1$ and $\hat{p}_2$ from the same single sample	$\hat{p}_1$ and $\hat{p}_2$ from two independent samples
True margin of error ≈ 2 × media-reported margin of error	True margin of error ≈ 1.5 × media-reported margin of error

---

Summary:

- Confidence intervals and margins of error take into account only the effect of the sampling error and not the nonsampling errors
- The margin of error:
  - is the amount which is added to and subtracted from an estimate when constructing a confidence interval
  - is the half-width of the associated confidence interval
  - for a single proportion depends on the value of the proportion
- The media usually just give one margin of error for all estimates reported
- A 'media-reported' margin of error for a single proportion is a:
  - reasonable approximation for the true margin of error for a sample proportion between about 0.3 and 0.7
  - poor approximation for the true margin of error for a difference between two proportions

References:

De Veaux, R. D., Velleman, P. F., and Bock, D. E., (2004). *Intro Stats*, 1st ed. Pearson.

Shaughnessy, J. M., and Chance, B. L., (2005). *Statistical Questions from the Classroom,* National Council of Teachers of Mathematics.

Utts, J. M., and Heckard, F.H. (2006). *Statistical Ideas and Methods,* 1st ed. Duxbury.

Wild, C. J., and Seber, G. A. F., (2000). *Chance Encounters: A First Course in Data Analysis and Inference,* 1st ed. Wiley.

Wikipedia, The Free Encyclopedia: http://en.wikipedia.org/wiki/Margin_of_error

Matt Regan
Department of Statistics
The University of Auckland