SUPPORTING MATHEMATICS TEACHERS' LEARNING: BUILDING ON CURRENT INSTRUCTIONAL PRACTICES TO ACHIEVE A PROFESSIONAL DEVELOPMENT AGENDA

By

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INTRODUCTION

Professional development programs in which groups of mathematics teachers meet with facilitators on an ongoing basis to work on instructional issues are an important way in which mathematics educators have attempted to support teachers' professional growth. The challenge of making this form of teacher support effective, generative, and sustainable gains in importance when the goal is to support teachers' development of forms of mathematics teaching that are informed by research on mathematical learning. These forms of teaching are complex and demanding, and do not typically develop without substantial external supports.

This dissertation contributes to the efforts to understand how to support teachers' development of effective instructional practices via professional development programs. My primary goal is to contribute to the development of a professional development design for supporting the learning of groups of middle school mathematics teachers. I build on previous research that has recognized the task of designing effective professional development programs not only as important, but also that it is a complex task that cannot draw readily on an established set of theoretical tools that would provide an adequate guidance. Part of my work therefore focuses on reviewing, critiquing, and adapting theoretical and methodological tools that proved useful in prior research to the task of supporting the learning of mathematics teachers.

The analysis I report is based on the last three years of a five-year design experiment[1] conducted with a group of middle-school mathematics teachers who taught in an urban school district with a diverse student population. The district is located in a southeastern state that had instituted a rigorous high-stakes accountability program. The design experiment consisted of 6 whole-day work-sessions and a 3-day summer workshop during each of the final three years. Out of the 12 teachers who participated during year three, 6 were part of the group from the beginning of the design experiment. Six new teachers from the same district joined the group at the beginning of year 3, four at the beginning of year 4, and two more at the beginning of year 5.

[1] My work is informed by analysis reported by Dean (2005) who analyzed developments in the teacher group in first two years of the design experiment.

The new teachers were recruited to replace their leaving colleagues[2], and thus the total of participating teachers over years 3-5 remained 12. In addition, two district mathematics leaders were active members of the group.

My dissertation study aims to advance the research agenda on professional development of mathematics teachers by pursuing two interrelated subgoals. The first subgoal is to produce an account of actual learning of the group of teachers and how this learning was supported over a three-year period. The second subgoal is to analyze the role of classroom-related artifacts (such as student work, classroom videos, and instructional materials) in supporting and organizing the learning of the teacher group. I therefore analyze the different ways in which groups of teachers might participate in designed professional development activities, as well as the means by which the different forms of teachers' participation might be supported. The analysis allows me to propose a set of revisable design principles for ongoing professional development that are grounded in specific design challenges that led to their formulation.

The dissertation is organized into eight chapters. In Chapter II, I describe the two specific goals of my research in more detail and explain their relevance to the overarching purpose of contributing to the development of effective professional development designs. I also provide background to the professional development collaboration with the group of middle-school mathematics teachers and explain how the findings generalize beyond the specific context in which they were developed. In Chapter III, I address the significance of my research goals by discussing how they advance knowledge in the field. I review several influential research programs in mathematics teacher education and professional development, paying particular attention to both the ways in which teacher learning was theorized and the methodological tools that enabled the investigators to develop insights into the process of supporting teachers' learning. In the course of the review, I further clarify my research focus and develop the methodological tools that I use in my analysis. In Chapter IV, I specify the data sources, explain the methodological approaches that are used in the analysis, and address the general trustworthiness of the study. In Chapter V, I summarize the learning of the professional teaching community documented by Dean (2005) that took place in first two years of the design research

[2] The reasons for leaving the group are listed in Appendix A. Most frequent reasons were moving out of the district and family reasons such as giving birth and staying home with the baby.

collaboration. In doing so, I establish the starting points for the subsequent learning of the teacher group.

Chapters VI and VII form the core of this dissertation in that they present two complementary analyses of the collective learning of the teacher group. In Chapter VI, I demonstrate that even though the membership of the teacher group changed during the three years of the study, it is reasonable to view it as a single, evolving professional teaching community that was characterized by joint enterprise, mutual engagement, and a shared repertoire of tools. As I document, the practices of the group were reestablished during the initial professional development sessions that were conducted after new teachers joined the group. Changes in the group membership are therefore best conceptualized as the induction of new members into a single professional teaching community that evolved over time, rather than as the emergence of a new group each time the membership changed. This analysis justifies the approach that I adopt in Chapter VII, in which I examine the actual learning of the single professional teaching community over the three-year period.

In Chapter VII, I discuss realized learning of the teacher community by describing developments in the ways of talking and reasoning that became normative in the group. I foreground shifts in pedagogical reasoning that were in the center of our design and research efforts, and document that the teachers came to view students' reasoning as a resource in their instructional planning by the last, fifth year of our collaboration. The documented means of supporting these crucial developments provide important insights, especially as our initial efforts at supporting the envisioned shifts in teachers' pedagogical reasoning were unsuccessful. In addition to describing the means that eventually proved effective, I also document the revisions that were made to the initial conjectures about supporting learning of a professional teaching community. I argue that the key point in developing design conjectures that proved viable was to take teachers' current instructional practices, concerns, and interests in consideration in ways that allowed us to further the professional development agenda.

Finally, in Chapter VIII, I discus implications of the analyses I have presented for professional development design and research. In doing so, I formulate a revised trajectory for learning of a professional teaching community.

CHAPTER II

SITUATING THE STUDY AND ITS GOALS

There is a consensus in the literature that we have a lot to learn about conducting effective teacher professional development (e.g., Ball & Cohen, 1999; Borko, 2004). One systematic approach to accumulating the needed insights is by formulating and refining domain-specific teacher development theories (P. Cobb, Dean, & Zhao, 2006; cf. Gravemeijer, 2004). As Cobb and colleagues have discussed, such a theory would specify a range of possible learning trajectories for a group of teachers along with the specific means by which this learning would be supported and organized with respect to the school and district institutional setting in which the teachers work. The potential value of such a theory is that it could inform the efforts of other researchers and teacher educators as they adapt the outlined learning trajectories to the contingencies of new institutional settings in a conjecture-driven manner. In this way, the initial theory would be subjected to both testing and further refinement every time it would be used to organize professional development collaboration with new groups of teachers. Linking the research and design in this manner frames mathematics teacher professional development as a design science in which the development of both theoretical insights and designs for supporting teachers' learning are cumulative. This view of professional development is compatible with a vision of educational reform as an ongoing, iterative process of improvement. As I illustrate in Chapter III, domain-specific teacher development theories that are open for further study and refinement are currently rare.

The overarching purpose of the presented analysis is to contribute to the development of a domain-specific teacher-development theory for supporting the learning of middle school mathematics teachers. The goal of the five-year professional development collaboration on which I draw in the analysis was to support middle school mathematics teachers in developing instructional practices where students' reasoning serves as a prime instructional resource. Dean (2005) analyzed the first two years of this professional development study and I analyze years three through five. My analysis yields relevant insights given that (a) the study was situated in an institutional setting in which school and district administrators responded to accountability pressures by attempting to monitor teachers and regulate their classroom practices, and (b) during years 3—5 that are the focus of my analysis, the group of mathematics teachers already collaborated in ways that are conjectured to support productive teacher learning (e.g., Ball & Cohen, 1999; Franke & Kazemi, 2001b) and constituted a professional teaching community[3]. Studying professional collaborations with the first characteristic alone is important given that similar response of administration is typical in many districts in the current era of high-stakes accountability. Dean (2004, 2005) documented the process by which the emergence of a professional teaching community was proactively supported during the initial two years of the study. It is similarly important to keep in mind that once a professional teaching community has been established, its existence alone does not guarantee that significant changes will occur in how students learn mathematics (McLaughlin & Talbert, 2001). Therefore, longitudinal studies of supporting further learning of professional teaching communities are also critical. My position as a design-research team member[4] in a longitudinal study with both these characteristics provided me with an opportunity to make a significant contribution.

I address the overarching purpose of my study by pursuing two related subgoals. The first subgoal is to document the actual learning of the professional teaching community in the last three years of the five-year collaboration along with the means that supported that learning. As I elaborate in Chapter IV, this primarily involves documenting the collective mathematical and pedagogical learning of the community. The second subgoal is to gain insight into the role that classroom-related artifacts (e.g., classroom video) played in supporting teachers' learning, and an extent to which they became tools for the teachers to conduct professional analyses of their own and others' teaching. As in other occupational practices, teachers' effective use of tools constitutes a key aspect of effective practice (e.g., Hutchins, 1993; Ueno, 2000). I therefore specifically document the shifts in the group's participation in professional development activities that involved classroom-related artifacts, and the roles that the artifacts came to play in these activities over time. Together, the two subgoals allow me to propose a set of revisable design principles that are grounded in the specific design challenges that led to their formulation.

The analysis of this particular professional development study is of value to the field for two additional reasons. First, significant developments occurred in the teachers' practices of planning for instruction by the end of the collaboration. At the beginning of the three-year period

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[3] I elaborate on the notion of a professional teaching community when I review related literature in Chapter III.
[4] This study is part of a larger research project. Members of the research team included Paul Cobb, Kay McClain, Chrystal Dean, Teruni Lamberg, Jose Cortina, Qing Zhao, Lori Tyler, Melissa Gresalfi, and myself.

that I analyze, student reasoning was largely seen as irrelevant to planning within the group. In contrast, three years later the teachers routinely considered ways in which their students might reason about mathematics when they critiqued and adapted instructional materials as they planned for instruction. Given that the shifts in the group's practices were aligned with the professional development agenda, that of supporting teachers' development of instructional practices where students' reasoning would serve as a prime instructional resource, the study of the means by which these shifts were supported is important.

Second, some of the classroom-related artifacts used in the collaboration became important tools for supporting teachers' learning during this period. Specifically, while use of classroom videos did not prove to be a viable tool for supporting teachers' focus on issues of students' reasoning during the first two years of the collaboration (Dean, 2005), it became a viable tool for doing so in later years. The collected data provided an opportunity to understand both how and why classroom videos became a useful tool for supporting learning of professional teaching communities. This type of analysis is a significant contribution because the need exists to better understand why similar artifacts promote teacher learning in certain situations while they did not prove viable in others.

The purpose of studying learning of a specific teacher group and how it was supported is to develop conceptual resources that would allow for adjustments of the professional development activities and tools to the needs of new sites in a conjecture-driven manner. It is not my claim that the same professional development activities and tools will necessarily be useful in new settings. Rather, a framework explicating *why* specific means of support promoted learning of the group of teachers is proposed to facilitate other professional developers' design decisions. In this way, the design research findings can be seen to transcend the specific contexts in which they were developed, and generalize to new sites and situations (Steffe & Thompson, 2000).

From a broader perspective, the analysis I present, when combined with the previous work of Dean (2005), produces a systematic account of the collaboration with a group of middle-school mathematics teachers over a period of five years, situating the learning of the group in the institutional context of their schools and district. This account will document the developments that occurred in the group from the inception of a professional teaching community to the point where the teachers drew on envisioned forms of students' reasoning when they planned for instruction in the communal setting. Accompanied by descriptions of the means that supported and organized the group developments and a set of revisable design principles, this account will constitute a domain-specific design theory of supporting middle school mathematics teachers in developing instructional practices where students' reasoning serves as a prime instructional resource. As previously discussed, the value of theories of this kind is that they produce a systematic record of learning in the field by accumulating both theoretical and practical insights.

Designing to Research Teacher Learning

The retrospective analysis I present constitutes the second part in a two-step process of formulating domain-specific teacher development theories via the means of design research (Brown, 1992; P. Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003). In the first part, the five-year professional development collaboration with mathematics teachers was conducted as a design experiment to test and propose modifications to the initial professional development design. To situate the second part, the retrospective analysis, in the larger context of the design-research study, I now discuss some of the major assumptions and tools that are characteristic of design research.

Design Research Methodology

One of the major tools of design research is the construct of a conjectured learning trajectory (Simon, 1995) as a way of thinking about the means of supporting envisioned developments. As adapted by Cobb and McClain (2001) for research aimed at supporting the learning of a professional teaching community, a conjectured learning trajectory encompasses researchers' conjectures about the course of teachers' development as well as the means of supporting it. The value of its formulation lies in the opportunity to test, revise, and modify the conjectures *while* working with teachers. A conjectured learning trajectory guides the researchers' local design decisions about how to support further learning of a teacher group. The viability of these decisions is tested against the actual activity of the teachers and the underlying trajectory is revised and modified accordingly. In this iterative process of design and analysis, competing explanations for documented developments are (a) constantly formulated, (b) critically examined by engaging the teachers in follow-up activities designed for this purpose, and, as a result, (c) corroborated or rejected on the basis of teachers' actual participation. Upon the conclusion of the collaboration, the actual learning trajectory differs from the initial, conjectured one. As Cobb and Gravemeijer

(2008) point out, the deviation of the actual learning trajectory from the envisioned at the outset provides a general summative record of the research team learning while experimenting to support the learning of the participants.

In design research, an actual learning trajectory of the group of teachers and how their learning was supported is documented in a retrospective analysis of the data generated in the course of the experiment. Throughout the retrospective analysis, modifications made to the conjectured learning trajectory are re-analyzed to establish their validity. In this process, additional examination of competing explanations is conducted where the decisions about refuting or accepting explanations are made with hindsight, against the entire dataset. Because my goal is to document the actual learning trajectory of a professional teaching community, the specific methodology of my retrospective analysis aligns with this broad delineation. I discuss the details of used methodology in Chapter IV.

Consistent with this perspective on design, the goal when collaborating with teachers is not simply to assess whether an approach to professional development formulated at the outset works. Instead, the goal is to improve the approach by drawing on analyses of the teachers' participation in the professional development sessions, as well as of instructional practices that they develop. In other words, the focus is on the actual *process* of the teachers' learning and the means by which it was supported and organized. Consequently, the goal when conducting a retrospective analysis is to learn from the differences in the conjectured and actual process of teachers' learning. Understanding why certain conjectures were not viable can help uncover critical aspects of teachers' learning that were previously not on the researchers' horizon, and thus help formulate a more informed *revised* learning trajectory that can be further tested and modified when used as a basis for collaboration with new groups of teachers.

The design research approach to professional development of mathematics teachers orients the review of literature that I present in the Chapter III. I now explain that the proposed analysis is well aligned with the overarching goal of the mathematics education research community – that of improving students' learning of mathematics with understanding. By discussing the demands for teaching mathematics for understanding, I elucidate the complexity of designing effective professional development programs.

Learning Mathematics with Understanding

For the past 20 years, an important goal for mathematics teacher educators has been to change the nature of mathematics teaching and learning in classrooms. Reformers have proposed substantial changes in the content and pedagogy of the K–12 mathematics curriculum, so that all students have the opportunity to learn more intellectually demanding mathematics. In the United States, the National Council of Teachers of Mathematics has taken the lead in promoting this change by publishing a series of influential standards documents (National Council of Teachers of Mathematics, 1989, 1991, 2000). For its part, the National Science Foundation has funded the development of several innovative textbook series and has supported a number of large-scale teacher development efforts. Despite these developments, extensive research indicates that advances have been limited (e.g., Borko, 2004), especially when we take the quality of students' mathematical learning as the basis for assessment[5].

Among the important contributions of the reform efforts to this point is that they have "shed light on the vital role played by teachers in educational change" (Llinares & Krainer, 2006, p. 439). Teachers have been identified as the "final brokers" (McLaughlin, 1987; Spillane, 2000) of instructional reform. As Berk (2005) describes, they are the ones who ultimately decide what mathematics students learn and how they learn it. They play a crucial role in mediating the reform efforts of curriculum designers, policy makers, and school leaders.

Research on curriculum design and implementation highlights the key role of teachers in using new textbooks and curricula in classrooms by drawing the distinction between *designed* and *enacted* curricula (Ball & Cohen, 1996). While designed curricula and textbooks are viewed as important instructional resources, teachers are conceptualized as *designers* of curricula that are enacted in their classrooms (Doyle, 1993; Remillard, 1999). This is in strong contrast to earlier perspectives on instructional improvement where curriculum materials were conceptualized as primary means of impacting instructional changes (Bruner, 1960; Dow, 1991). Initiatives oriented by this perspective led to the design and development of "teacher-proof" curricula in the 50s and 60s that failed to produce desired instructional changes (Ball & Cohen, 1996; Remillard, 1999). This failure, as Ball and Cohen (1996) argue, is often explained as failure to take account of teachers' current knowledge and practices (Fennema, Carpenter, & Peterson, 1989; Sarason,

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[5] It is important to clarify that I am not referring to increase in students' scores on state tests as a measure of quality of professional development. The key to emphasize is that such measures should take into consideration the quality of students' learning mathematics that is worth knowing.

1982; Schwille et al., 1983), and the approach is critiqued for “de-skilling” teaching (Apple, 1990).

Ball & Cohen (1999) proposed that large scale instructional changes in mathematics education can be best supported by adopting a “bottom up” approach – an approach that places students’ learning, rather than educational policies, in the foreground. Drawing on McLaughlin’s (1987) work, they proposed that implementation is a problem of the smallest unit and argue that in bringing policies to life in local classroom settings, we ultimately rely on teachers, their perspectives, and interpretations (see also Schwille et al., 1983). In a review of implementation literature Garn (1999) concluded:

> Social scientists from various disciplines studying an array of social programs acknowledge that policies emanating from higher levels of government are inherently problematic. McLaughlin (1998) identified local capacity and will as two paramount variables that affect the outcomes of the implementation process (URL: http://epaa.asu.edu/epaa/v7n26.html).

As an illustration, the NCTM *Standards* is a policy document that the mathematics education research community interprets as portraying worthwhile mathematics to be taught in classrooms. However, opportunities for specific students to learn mathematics that is worth knowing are profoundly shaped by their teachers’ interpretations of the *Standards* and by these teachers’ motivation and capacity to pursue the vision outlined in the *Standards*.

A similar argument that highlights the role of teachers in promoting educational improvement can be found in the research on school and district leadership (e.g., Coburn, 2005; Elmore, Peterson, & McCarthey, 1996; Gamoran et al., 2003; Rowan, 1990; Spillane et al., 2002). School leaders are increasingly conceptualized as mediators of policies (Spillane et al., 2002) who play a key role in shaping schools’ and districts’ instructional vision, in aligning instructional efforts, and in securing resources to support teachers’ professional work (Elmore et al., 1996; Gamoran et al., 2003; Rowan, 1990). This view acknowledges that the school and district leaders’ “improvement efforts are always ultimately dependent upon teachers’ actions in their classrooms” (Coburn, 2005, p. 35), and charges the leaders with the task of designing institutional contexts that are supportive of teacher learning (P. Cobb & Smith, 2008).

Acknowledgement of the critical role that teachers play in promoting students’ learning turns our attention to instructional practices that would support all students’ learning of mathematics with understanding (Carpenter et al., 2004; Carpenter & Lehrer, 1999; Hiebert &

Carpenter, 1992). The envisioned instructional practices are grounded in research on student learning of significant mathematics and call for teachers to attend to their students' reasoning when they plan for and orchestrate instruction. Specifically, these practices require that teachers build from their students' current reasoning while, at the same time, keeping in mind significant mathematical ideas that are the goal of instruction (Ball, 1993; Carpenter & Fennema, 1992; P. Cobb, 1999; Gravemeijer, 2004; Hiebert et al., 1997; Hiebert & Grouws, 2007; Lampert, 2001; McClain, 2002; Schifter, 1998). This necessitates that teachers develop relatively deep understandings of mathematics (Ball & Cohen, 1999; McClain, 2005) and, more specifically, *mathematical knowledge for teaching* in the domains of mathematics that they teach (Ball & Bass, 2000, 2003; Fennema & Franke, 1992; Silver, Clark, Ghousseini, Charalambous, & Sealy, 2007). Forms of the envisioned instructional practices emphasize students' opportunities to engage in mathematically challenging tasks, maintaining the level of challenge as tasks are enacted in the classroom (Hiebert & Grouws, 2007; Stein & Lane, 1996; Stein, Smith, Henningsen, & Silver, 2000), and students' opportunities to communicate their mathematical thinking in classroom discussions (P. Cobb, Boufi, McClain, & Whitenack, 1997; Hiebert et al., 1997; Lampert, 2001)[6]. These forms of instructional practices are complex, demanding, uncertain, and not reducible to predictable routines (Ball & Cohen, 1999; Clark, 1988; Lampert, 2001; McClain, 2002; Schifter, 1995; Smith, 1996), and differ significantly from those observed in most US classrooms (Hiebert et al., 2005; Prawat, 1992; Stigler & Hiebert, 1999). The goal in the professional development study that I analyze was to support teachers' development of instructional practices of this kind.

The challenge of supporting teachers' development of such instructional practices has been documented by numerous investigations that focused on teacher professional development (e.g., P. Cobb & McClain, 2001; Fennema, Carpenter, Franke, & Carey, 1993; Franke & Kazemi, 2001a; Simon & Tzur, 1999). Prior research documents that even in cases when teachers were willing to collaborate and seemed engaged in the professional development setting, understanding children's reasoning was not always easy (Ball, 2001; Schifter, 2001). In addition, teachers did not always see the use of their new knowledge as immediately relevant to their classroom practice (Fennema et al., 1993).

[6] The research base for these broad recommendations is presented in a research companion volume to the National Council of Mathematics' (2000) *Principles and Standards for School Mathematics* edited by Kilpatrick, Martin, and Schifter (2003).

Part of the challenge resides in the nature of the required teacher learning (Ball, 1997; Simon, 1997). This learning targets what Elmore (1996) called "the core of educational practice" – that is, the nature of mathematics that is beneficial for students to learn, as well as teachers' and students' roles in teaching and learning (cf. Carpenter et al., 2004; Franke, Carpenter, Fennema, Ansell, & Behrend, 1998; Goldsmith & Schifter, 1997). The professional developers' goals thus include finding effective ways to support the teachers in revising the core assumptions of their practice and in helping them develop a need to change their classroom instruction. The analysis I present documents the extent to which these goals were accomplished in the professional development study and the means that supported learning of the professional teaching community.

Mathematical Knowledge for Teaching

To design and understand effective professional development programs for mathematics teachers, it is important to clarify what the teachers need to know and be able to do *mathematically* in order to be effective in teaching mathematics for understanding. Ball, Bass, Hill, and colleagues' work is relevant in this regard (Ball & Bass, 2003; Ball, Hill, & Bass, 2005). They analyzed a database of entire year of effective mathematics instruction in a third grade classroom, and attempted to develop a *practice-based theory of mathematical knowledge for teaching*. Based on the analysis, they developed items to measure mathematical knowledge for teaching at the elementary level and used these measures to evaluate teacher learning during extended summer workshops that were part of California's Mathematics Professional Development Institutes (Hill & Ball, 2004).

In developing a practice-based theory of mathematical knowledge for teaching, Ball and colleagues built on Shulman and colleagues' concept of pedagogical content knowledge (Grossman, 1990; Shulman, 1986, 1987; Wilson, Shulman, & Richert, 1987). As Ball and Bass (2003) clarified,

> In addition to general pedagogical knowledge and knowledge of the content, teachers need to know things like what topics children find interesting or difficult and the representations most useful for teaching a specific content idea. Pedagogical content knowledge is a unique kind of knowledge that intertwines content with aspects of teaching and learning (p. 4).

Ball and Bass attempted to identify mathematical demands of teaching mathematics or, in other words, what kind of mathematical work is routinely a part of mathematics teachers' job. They concluded that teaching mathematics "involves a steady stream of mathematical problems that teachers must solve" (p. 6) and that these problems are specific to teachers' occupation. Hill and Ball (2004) called the knowledge needed to solve these problems *specialized knowledge of content* to contrast it with *common knowledge of content*. Common knowledge of content requires that a person is able to solve a mathematical problem accurately, and many non-teachers are likely to hold such knowledge. In contrast, solving mathematical problems *of teaching* involves, for example, inspecting alternative solution methods that students present in classroom that are often unknown to the teacher. The teacher needs to examine mathematical structure and principles that underlie these methods, and judge whether or not a method is valid and can be generalized. Ability to do so effectively is not a part of what most people learn in mathematics courses, and thus it should be, Ball and Bass contend, specifically cultivated in mathematics teachers.

Among essential features of knowing mathematics for teaching, Ball and Bass (2003) identified teachers' awareness that mathematical knowledge needs conceptual *unpacking*. Given a powerful characteristic of mathematics to "*compress* information into abstract, highly usable forms" (p.11), supporting students' learning of mathematics requires helping them understand how mathematical ideas are generated, before they can be meaningfully compressed. Ball and Bass also highlighted the need for teachers to understand the "connectedness of mathematical knowledge, both across mathematical domains at given level, and across time as mathematical ideas develop and extend" (p. 11), as well as understanding that the "critical mathematical issues at play in the lesson are not merely those of the curricular topic at hand" (p. 12) but include important aspects of mathematical reasoning, use of terms, and representations. Finally, Ball and Bass suggest that teachers need to understand student learning in terms of the key mathematical practices in which students come to participate in the process of learning mathematics. This suggestion is aligned with findings from classroom design experiments that illustrated how supporting students' participation in a sequence of classroom mathematical practices resulted in significant student learning (e.g., Bakker & Gravemeijer, 2004; Bowers, Cobb, & McClain, 1999; P. Cobb, McClain, & Gravemeijer, 2003; P. Cobb, Stephan, McClain, & Gravemeijer, 2001; Confrey & Smith, 1995; Lehrer, Jacobson, Kemeny, & Strom, 1999; Lehrer & Schauble, 2002; McClain & Cobb, 2001).

Ball and colleagues went beyond analyzing the nature of mathematical knowledge that is used in teaching, and developed items to measure mathematical knowledge for teaching at the elementary level. They reported that measures were positively correlated with gains in students' mathematical achievement during the first and third grade instruction (Hill, Rowan, & Ball, 2005). In addition, Hill and Ball (2004) used these measures of mathematical content knowledge for teaching to evaluate teacher learning during extended summer workshops that were part of California's Mathematics Professional Development Institutes. The results suggested that most teachers improved their mathematical knowledge for teaching on elementary grade level during a single professional development program. Hill and Ball reported that both a strong focus on mathematical content in professional development sessions, and opportunities for teachers to work together on mathematical problems that arise during mathematics instruction were among potential explanations of the success of the Mathematics Professional Development Institutes.

These findings suggest that (a) improving mathematical knowledge for teaching should be an important goal for any professional development program, (b) content focus of such program is critical, and (c) the content should not be solely approached as what teachers need to be able to do mathematically, but should address mathematical problem solving situations that are likely to arise in classrooms. The efforts to develop *domain-specific* teacher development theories also reflect the importance of mathematical content in effective professional development programs. As a contribution to a teacher development theory in middle school statistics, attention to opportunities for teachers to improve their knowledge for teaching middle-school statistics is central to the presented analysis.

Design Research Site and Goals

The data for the proposed analysis were collected during the last three years of a five-year collaboration with a group of middle school mathematics teachers who worked in the Jackson Heights Public School District[7]. This urban school district serves a 60% minority student population and is located in a state with a high-stakes accountability program where administration responds to the accountability pressures of state testing by attempting to monitor and assess

[7] All names used in this dissertation (e.g., district, schools, and teachers) are pseudonyms.

teachers (P. Cobb, McClain, Lamberg, & Dean, 2003). The district had received an external grant to support its reform efforts prior to our collaboration with the teachers. We began working in the district to provide teacher development in statistical data analysis at the invitation of the district's mathematics coordinator who selected the initial group of nine teachers. The group participation remained stable over the first two years. At the beginning of the third year, three teachers left the group and six teachers from the same district were recruited to join. The teachers who left the group either moved out of the district, changed occupation, or left for personal reasons (for participation details, as well as reasons for teachers to leave the group see Appendix A). The group then continued to recruit new teachers every fall. Throughout years 3 – 5, the group was composed of 12 teachers, typically working in 5 different schools in the district. In addition, the district mathematics coordinator, one district mathematics specialist, and the research team members participated as members of the group. I will clarify involvement of different participants in Chapter V when I summarize the learning of the group during initial two years of the collaboration.

The district mathematics coordinator was interested in professional development that focused on the middle grades because the district's reform efforts were proving to be problematic. In particular, the district had adopted an NSF-funded *Standards*-based mathematics curriculum, but significant proportions of the middle school teachers continued to use the traditional textbook series as the primary basis for their instruction. During the five years in which we worked with teachers in the district, we conducted a two-day initial summer session, a three-day work session each summer, three one-day sessions during the first school year, and six one-day sessions during all the following school years (see Figure 1).

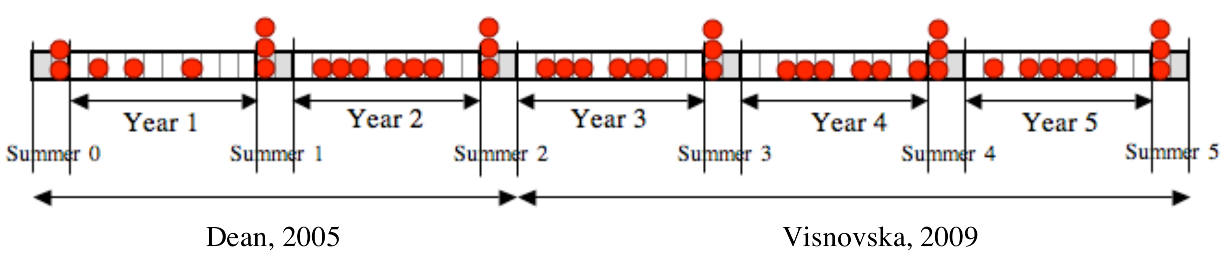

Figure 1. The Timeline of the Professional Development Work Sessions Across the Five Academic Years. Each whole-day session is depicted by a red circle. The vertical multiple circles always depict a summer session that took place in the depicted number of consecutive days of the month.

Our long-term goal was to support the teachers' development of instructional practices in which teaching is a generative, knowledge-building activity with students' reasoning at the center of instructional decision making. One of the primary conjectures underlying professional development study was that instructional sequences of the type Cobb and colleagues had developed in previous NSF-funded classroom design experiments (P. Cobb, 1999; P. Cobb, McClain, & Gravemeijer, 2003; McClain & Cobb, 2001) could serve as an important means of supporting teachers' as well as students' learning (cf., Ball & Cohen, 1996; Gearhart et al., 1999; Hiebert & Wearne, 1992). These instructional sequences were specifically designed to facilitate the kind of instructional practices envisioned by the current mathematics education reform. They required teachers to gain relatively sophisticated understandings of statistical ideas. Supporting teachers' learning of statistics was central to our research and professional development agendas.

The general approach for working with teachers included supporting teachers in: (a) deepening their own mathematical understanding, (b) attempting to make sense of individual students' mathematical interpretations and solutions, (c) locating students' mathematical activity in social context by attending to the nature of the social events in which they participate in the classroom, and (d) appreciating the pedagogical intent of instructional sequences, including the teacher's understanding of students' mathematical thinking, mathematical ideas to be learned, and how they may be learned (P. Cobb & McClain, 2001). Among the specific means that we used to support teacher learning were selected video cases[8] from the teaching experiments in which the instructional sequences were developed, videos from teachers' own classrooms, students' written solutions, and instructional materials. In designing professional development activities around these classroom-related artifacts, we conjectured that the teachers would develop reasons and motivations to change their current instructional practices in mathematics and that the artifacts would become resources for supporting such change (cf. Barnett, 1991; Barron & Goldman, 1994; Bowers, Barron, & Goldman, 1994; Franke & Kazemi, 2001a; Schifter, 1990). One of my goals is to understand how these conjectures were modified in the process of the collaboration and what role the artifacts played in supporting teachers' learning.

[8] These cases were based around specific instructional activities in statistics and associated resources (e.g., computer-based tools used as part of the statistics instructional sequences). In addition to video-recordings of classroom sessions, cases typically included transcripts of the classroom discussions, and could include additional resources such as transcripts of student interviews, copies of the students' written work, and alike.

Instructional Sequence in Statistics

The statistics instructional sequence served as the primary means of supporting the learning of the professional teaching community. The retrospective analyses conducted of the classroom design experiments in which the sequence was designed and tested (P. Cobb, 1999; P. Cobb, McClain, & Gravemeijer, 2003; McClain & Cobb, 2001) foregrounded four types of the means that were key in supporting students' learning. These included instructional tasks, the computer tools the students used to analyze data sets, organization of classroom activity, and nature of classroom discourse. I discuss these means here for two reasons. First, I intend to illustrate what the research team saw as a worthwhile statistical activity in which to engage middle-school students. Second, these means of support provided guidance in structuring professional development activities, which I introduce in analysis Chapters.

Instructional Tasks

The instructional tasks in the statistics sequences reflected the view that

> students' activity in the classroom should involve the investigative spirit of data analysis from the outset. This implied that the instructional activities should involve analyzing data sets that the students viewed as realistic for purposes that they considered legitimate (P. Cobb, Zhao, & Visnovska, 2008, p. 111).

For this reason, the designed tasks typically involved comparing two data sets in order to make a decision or judgment (e.g., analyze the T-cell counts of AIDS patients who had enrolled in two different treatment protocols to determine which treatment is more successful) and required that the students write a report with a specific audience in mind (e.g., the chief medical officer of a hospital who will use the reports to make a decision about which treatment the hospital will use).

Tools

Three computer tools were designed to support students' analyses in different phases of the instructional sequence. The first computer tool (referred to as Minitool One by the teachers) was designed to facilitate students' initial explorations of univariate data sets and provides means of ordering data values, partitioning, and otherwise organizing small sets of data in a relatively immediate way. Each individual data point is in this tool inscribed as a horizontal bar, the length of which signifies the numerical value of the data point. The color of each bar signifies to which of the two sets of data this data point belongs. In the case of the Batteries activity, which was among the first scenarios used in year three, data was generated to compare which of the two brands of batteries is better to buy if their cost is equal (see Figure 2). Each bar represented life-time of one battery in a flashlight measured in minutes.

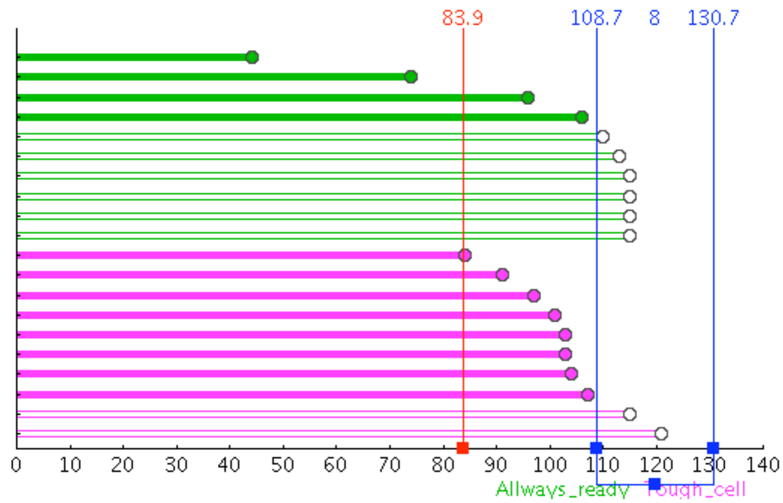

Figure 2. Batteries—Computer Tool One. Red value bar and blue range tool used.

The users could sort the data by size and by color, and could hide either data set. In addition, they could also use the red value bar to partition the data sets and to find the value of any data point by dragging the bar along the horizontal axis. Further, they could find the number of data points in any interval by using the blue range feature (P. Cobb, 1999; McClain & Cobb, 2001).

The second computer tool (Minitool Two) can be viewed as an immediate successor of the first in that the endpoints of the bars that each signified a single data point has, in effect, been collapsed down onto the axis so that a data set was now inscribed as an axis plot (see Figure 3).

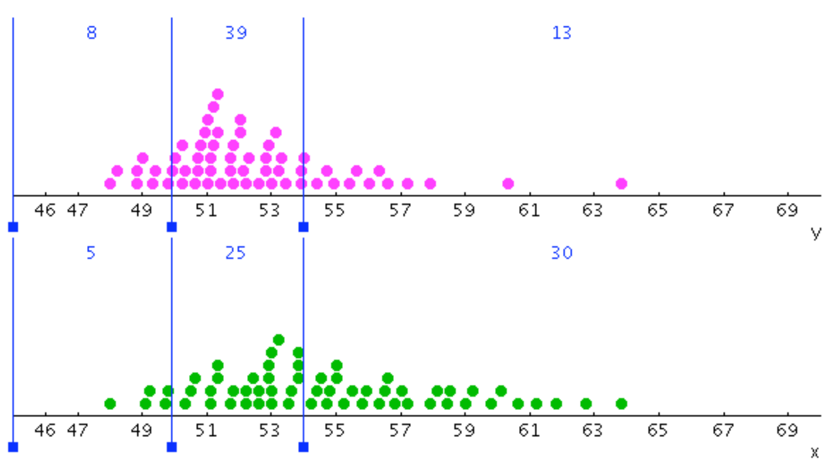

Figure 3. Speed Trap—Computer Tool Two. "Create your owns groups" feature used.

The instructional intent when designing the second tool was to support the emergence of more sophisticated ways of comparing and analyzing datasets with larger numbers of data values. The tool offered a range of ways to structure data, including (a) making your own groups, (b) partitioning data into groups of a fixed size, and (c) partitioning data into two equal groups. The first and least sophisticated of these options simply involved placing one or more vertical bars to chosen locations on the axis in order to partition the data set into groups of points. The number of points in each partition was shown on the screen and adjusted automatically as the bars were dragged along the axis. The tool also included two options that were precursors to standard ways of structuring and inscribing data and corresponded to graphs typically taught in school. These involved (d) organizing the data into four equal groups so that each group contained one-fourth of the data (precursor to the box-and-whiskers plot) and (e) organizing data into groups of a fixed interval width along the axis (precursor to the histogram) (P. Cobb, 1999; McClain & Cobb, 2001). In addition to using vertical bars to structure data, the tool included the option for hiding the data points, as shown on Figures 10 and 11 in Chapter V.

The third computer tool (Minitool Three) introduces bivariate data inscribed as a scatter plot such that each of the two measures of each case are represented on the horizontal and vertical axis (see Figure 4).

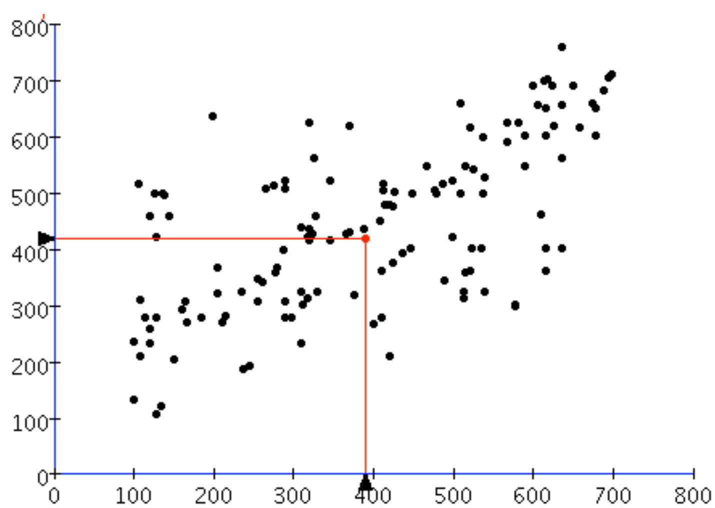

Figure 4. Speed Reading—Computer Tool Three. “Dot” feature used.

The features on the third computer tool built from those used in previous tools. A feature called Dots was parallel to a value bar used in univariate displays. It showed perpendiculars from the axes to the selected dot, highlighting values of its two measures. This feature was used to aid the teacher in ensuring that the class discussed relationships between the two measures of each of a number of cases rather than a mere configuration of dots scattered between two axes. The four ways of organizing bivariate data the tool offered were the cross, grids, two equal groups, and four equal groups. The cross option divided the data display into four cells and showed the number of data points in each cell. The center of the cross could be dragged to any location on the display, thereby changing the size of the cells. The number of data points in each cell adjusted automatically (Figure 5a). The cross can be viewed as the two-dimensional correlate of the making your own groups option included in the second computer tool. Grids option included a pull-down menu of grids that ranged in size from 3-by-3 to 10-by-10. The selected grid was shown superimposed on the data display and the number of data points in each cell was shown (Figure 5b). The grids option can be viewed as the two-dimensional correlate of the fixed interval width option included in the second computer tool.

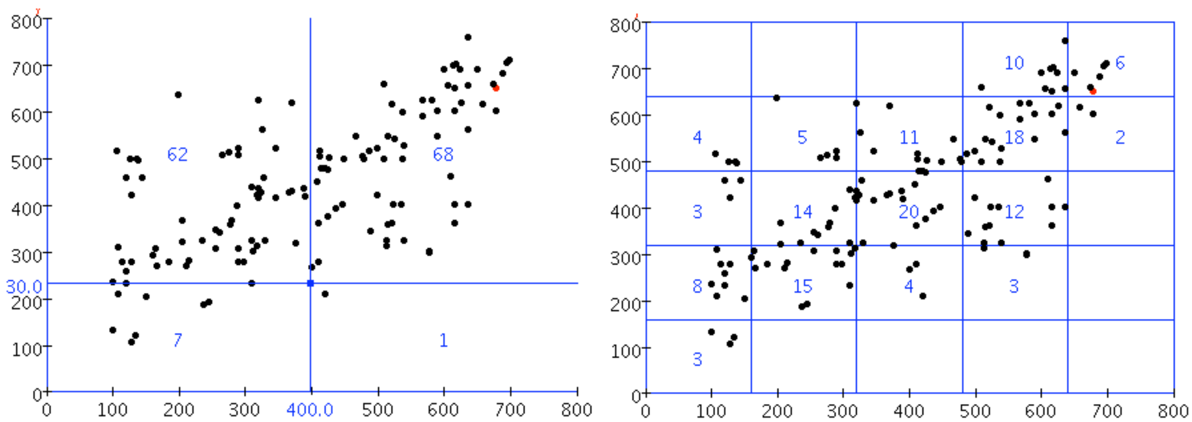

Figure 5. Speed Reading—Computer Tool Three. (a) “Cross” and (b) 5 by 5 “Grid” features used.

The two and four equal groups options partitioned the data display into columns or vertical slices, the widths of which divided the horizontal axis into equal intervals. The minimum number of slices that could be chosen was four and the maximum was ten. Within each slice, the data points were partitioned into two or four equal groups respectively (i.e., the display showed the median and the low and high values within each slice in two equal groups option, Figure 6a; and, in addition, partitions for lower and upper quartiles in four equal groups option, Figure 6b). These options can be viewed as the two-dimensional correlates of the two equal groups and four equal groups options included in the second computer tool.

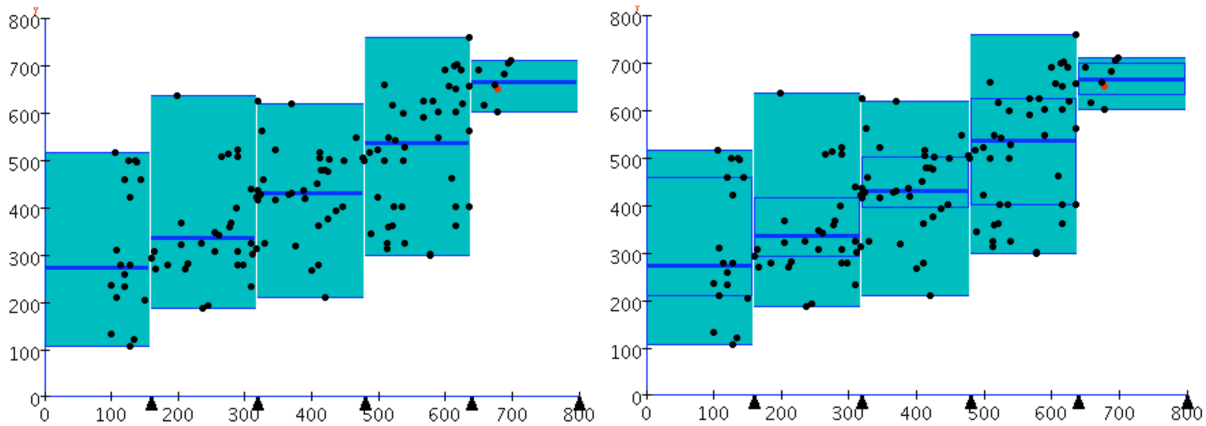

Figure 6. Speed Reading—Computer Tool Three. (a) 5 x "Two Equal Groups" and (b) 5 x "Four Equal Groups" features used.

Like in the computer tools one and two, the data points could be hidden. This option was designed to support conversations in which trends and patterns in the distribution of data are inferred from graphs (P. Cobb, McClain, & Gravemeijer, 2003).

Organization of Classroom Activities

Findings from the statistics classroom design experiments (P. Cobb & McClain, 2004) suggested that the specific ways in which classroom instruction is organized matter when attempting to productively support students' engagement in genuine data analysis in a classroom. The organization of classroom activities that proved to be effective involved (a) a whole-class discussion of the process of generating data, (b) individual or small-group activity in which the students typically used computer-based tools to analyze data, and (c) a whole-class discussion of the students' analyses.

One of the goals pursued in the designed organization of classroom activities was to ensure that the students would come to view data as measures of an aspect of a phenomenon rather than merely as numbers relatively early in the instructional sequence. This goal aligns with George Cobb and David Moore's (1997) characterization of statistics for educational purposes, in which they suggested that "data are not just numbers, they are numbers with a context" (p. 801). In their view, teaching statistics should not be reduced to a focus on statistical constructs that involves identifying patterns among numbers (e.g., the arithmetic mean as the point of balance of a set of values). They argued that students must also be able to identify the meaning and significance of those patterns in terms of the question at hand. In this characterization of statistics, “pattern and context are inseparable” (p. 803).

To support development of students’ understandings aligned with this characterization of statistical activity, the teacher in the design experiments introduced each instructional activity by talking through the data generation process with the students. During this *data generation discussion*, the teacher and students delineated the particular phenomenon under investigation (e.g., AIDS), clarified its significance (e.g., the importance of developing more effective treatments), identified relevant aspects of the phenomenon that should be measured (e.g., patients’ T-cell counts), and considered how they might be measured (e.g., taking blood samples). In addition, the teacher and students delineated a specific question that needed to be addressed (e.g., whether a new AIDS treatment is more effective than a traditional one), and proposed a specific experiment that would help them address this question (e.g., how many people to enroll in each of the two AIDS treatments, how to make sure the two groups of patients are comparable, and for how long they need to undergo the treatments before blood samples taken would reflect relative effectiveness of the treatments). The teacher then introduced the data as having been generated by this process, specified the audience for the analysis (e.g., a hospital director deciding whether to replace the traditional treatment with the new one), and the students conducted their analyses individually or in small groups, using one of the computer tools. As a consequence of participating in the data generation discussions, data sets came to have a history for the students: They were grounded in the situation from which they were generated, and reflected the interests and purposes for which they were generated (P. Cobb, 1999; P. Cobb, McClain, & Gravemeijer, 2003).

The final phase of an instructional activity consisted of a whole-class *data analysis discussion*, in which the students were required to justify their analyses by showing how they gave insight into the phenomenon under investigation. As the students attempted to formulate, understand, and resolve competing data-based arguments, they became aware of the implications of the data generation process for the conclusions that could legitimately be drawn from data. It was the teacher’s goal to organize data analysis discussions so that mathematically significant issues that advanced the instructional agenda would became explicit topics of conversation. At the same time these issues had to build from the data analyses that the students conducted.

> To this end, the teacher and a second member of the research team circulated around the classroom while the students were working at the computers to gain a sense of the various ways in which they were organizing and reasoning about the data. Towards the end of the small-group work, they then conferred briefly to develop conjectures about mathematically significant issues that might emerge as topics of conversation in the subsequent whole-class discussion. Their intent was to capitalize on the students' reasoning by identifying data analyses that, when compared and contrasted, might give rise to substantive mathematical conversations. To the extent that the teacher succeeded, students' participation in the discussions would serve as primary means of supporting their progressive reorganization of their reasoning and thus their gradual induction into the values, beliefs, and ways of knowing of the discipline (P. Cobb et al., 2008, p. 113).

Classroom Discourse

The final means of support that Cobb and colleagues identified when they analyzed students' learning in the statistics design experiments focused on the nature of classroom discourse. Building on a distinction that Thompson and Thompson (Thompson & Thompson, 1996) make between calculational and conceptual orientations in mathematics teaching, Cobb and colleagues brought attention to distinction between calculational and conceptual discourse.

> [T]he distinction concerns the norms or standards for what counts as an acceptable mathematical argument. In calculational discourse, contributions are acceptable if students describe how they produced a result and they are not obliged to explain why they used a particular method. In contrast to this exclusive focus on methods or solution strategies, the issues that emerge as topics of conversation in conceptual discourse also include the interpretations of instructional activities that underlie those ways of calculating and that constitute their rationale. (P. Cobb et al., 2008, p. 113)

A conceptual explanation of students' statistical analyses would involve describing not merely the steps of the analysis (e.g., how they structured the data using a computer tool) but also the reasons for these steps given the issue under investigation (e.g., why was it reasonable, given the question at hand, to partition the data at a specific value, or compare the two data sets within a specific interval). A retrospective analysis of the design experiment indicated that the interventions the teacher made to support conceptual explanations of this type were critical to students' learning. They contributed to the students' development of relatively sophisticated explanations in which the students compared data sets in terms of relative rather than absolute frequencies (Cobb, 1999). Cobb elaborates that

> discussions in which the teacher judiciously supports students' attempts to articulate their task interpretations can be extremely productive settings for mathematical learning. As these articulations focus on the reasoning that lies behind solution procedures, students'

> participation in such discussions increases the likelihood that they might come to understand each other's reasoning. Had the discussion in the design experiment classroom remained calculational, students could only have understood each other's explanations by creating a task interpretation that lay behind their use of the computer tool entirely on their own. In contrast, the students' participation in conceptual discourse provided them with resources that supported their understanding of other's explanations and thus their reorganization of their initial interpretations of tasks. These resources are not limited to what is said but also include inscriptions and notations that are pointed to and spoken about (cf. A. G. Thompson, Philipp, Thompson, & Boyd, 1994). In the statistics design experiment, for example, the graphs that the students developed as they used the computer tools were integral to communication as well as to their individual reasoning. (P. Cobb et al., 2008, p. 115)

To conclude this discussion of the designed means of supporting students' learning that were an integral part of the statistics instructional sequence, I would like to stress that it was not our goal for the teachers to learn how to enact statistics instruction in their classrooms by attempting to mimic instruction during the design experiment. Instead, our goal was to support the teachers in examining issues of teaching and learning statistics as they adapted, tested, and modified the sequence in their classrooms. Rather than focusing on specific teacher actions and moves that might be involved in using the statistics tasks in the classroom, our focus was on the pedagogical justifications for whatever moves or actions the teachers decided to make in specific instructional situations. We conjectured that an instructional sequence that was designed in a way that made the basis for making instructional decisions explicit would constitute an effective means of supporting the learning of the professional teaching community.

CHAPTER III

LEARNING FROM PRIOR RESEARCH

The purpose of this chapter is to clarify the significance of the goals of my study by discussing how they both build on and advance knowledge in the field. To this end, I review research that documents mathematics teachers' learning as they participated in professional development studies of an interventionist nature. The discussed studies are intended to serve as paradigmatic cases of pursuing specific type of research goals (e.g., formulating developmental theories of teacher learning, investigating teacher use of research-based knowledge, or studying ways to support generative growth of communities of teachers) while drawing on a specific set of assumptions and perspectives. The review is thus not conceived as an exhaustive synthesis. I build on the reviewed studies to outline several issues that are relevant to designing professional development for mathematics teachers, and to conducting retrospective analyses like the one presented in this dissertation.

In discussing the prior research I address two major goals. First, I elaborate my research question, taking into consideration what the field already knows about supporting mathematics teachers' development of instructional practices that place students' reasoning at the center of instructional planning and decision making. I focus on issues that are most relevant to my analysis, both in relation to how teachers' learning can be theorized and what guidance different theorizations bring to an endeavor of supporting and understanding teacher learning. In this sense, I intend this chapter to be a record of my attempts to answer my questions using the existing literature (Simon, 2004).

Second, in discussing prior research, I take a pragmatic standpoint. My goal is not to decide which theoretical perspective can best serve researchers and professional developers. Indeed, my reading of the literature was oriented by my interest in equipping myself with research tools that could help me to gain insight into the collected data. It was also oriented by a design research approach to teacher professional development (P. Cobb, Confrey et al., 2003; Simon, 2000). Within this approach, different theories and methodologies are viewed as tools that are used for specific, often complementary purposes. It is my contention that the coordinated use of a variety of research and design tools rather than a strict adherence to a specific toolbox can serve us better in answering complex questions, such as those that arise when supporting teacher learning.

I alternate sections that introduce prior research studies with my commentary of specific strengths of the research tools used with respect to the analysis I conducted. I also discuss pragmatic and methodological issues that are relevant to my analysis but were not the focus of the reviewed study. I should stress that I was able to identify these issues only after becoming familiar with the whole body of literature that I review, in which different studies asked a variety of questions from different perspectives and pursued different research goals. I thus do not see each successive study as addressing *more* of the issues that are relevant to my analysis. The studies instead address *different* issues. I explain how each of the studies helped me gain new insights that guided my analysis. For clarity of exposition, the order in which I discuss the studies is generally chronological.

Cognitive Perspectives and Supporting Teacher Learning

Research on teachers' learning has been dominated by a cognitive paradigm that focuses squarely on teachers' knowledge and beliefs.[9] Teaching is typically characterized as a problem solving activity and the metaphor of teacher as decision-maker had wide currency (cf. Ball, 2000; Borko, 2004). Much of the research conducted within this paradigm has sought to delineate the distinct knowledge bases that underlie teaching and to specify their relation to instructional practice. Shulman's (1986) analysis of pedagogical content knowledge is probably the most influential contribution of this type.

This conceptualization of teacher learning suggests that fundamental change in teaching practice – and therefore in students' learning – might be initiated by changes in teachers' knowledge (Clark & Peterson, 1986; Fennema & Franke, 1992; Putnam, Lampert, & Peterson, 1990). Consequently, a wealth of research studies that endeavored to investigate ways in which teacher learning can be *supported* drew on a knowledge-based approach (Borko, 2004; Goldsmith & Schifter, 1997; Schifter, 1998; Tirosh & Graeber, 2003; Wilson & Berne, 1999). This involved

[9] The cognitive paradigm displaced the prior process-product paradigm on teachers' learning. Studies conducted within the process-product paradigm attempted to identify causal relations between discrete aspects of teachers' observable behavior and measures of students' learning outcomes. Research conducted within this paradigm was critiqued on a number of grounds, one of the most significant being that the strategy of attempting to raise students' outcome measures by training teachers to enact the identified behaviors did not prove to be particularly effective (P. Cobb et al., 2006).

(a) identifying a relatively specific knowledge base that was conjectured beneficial to planning and conducting classroom instruction, and (b) seeking ways in which teachers could be supported in acquiring the beneficial knowledge. Justifications for what types of knowledge might be beneficial were often based on prior cognitively-oriented studies of students' learning of a specific mathematical content. Carpenter, Fennema, and colleagues' (Carpenter, Fennema, Franke, Levi, & Empson, 2000) Cognitively Guided Instruction (CGI), and Simon and colleagues' (Simon, 2000; Simon, Tzur, Heinz, Kinzel, & Smith, 2000) Mathematics Teacher Development (MTD) Project are all examples of this approach.

I discuss these particular projects because specific studies they conducted allow me to illustrate both kinds of focus questions that interventionist studies within cognitive paradigm attempt to address, and the research tools that have proven useful in addressing these questions. After giving an overview of MTD Project and research focus, I foreground two aspects of Simon and colleagues' work that I see as critical for conducting retrospective analyses. First, I discuss the importance of understanding teachers' actions *as reasonable from their perspective*. Second, I discuss the kinds of guidance that Simon and colleagues' characterization of teachers' learning provides for designing further intervention. I then turn to the work of CGI researchers to illustrate how their experiences of working in schools led them to seek a perspective that would help them account for differences in teachers' learning that they could not explain in cognitive terms. In particular, a consideration of the work of CGI researchers turns our attention to institutional support structures within teachers' schools and their relation to teacher learning.

As I explained earlier, my purpose for reviewing these studies is not to merely understand how to best work with teachers. My intention is also to gather the resources that the mathematics education research community has developed while attempting to support teacher learning that pertain to the issue of my interest – professional development designs that explicitly capitalize on teachers' current practices in order to pursue a professional development agenda.

Simon and Colleagues – Constructivist Epistemology

To situate the discussion of MTD Project, I first introduce the general orientation of this research that draws on constructivist research paradigm. Goldsmith and Schifter (1997) characterized the approach to research on teacher learning adopted by Schifter, Simon, and their colleagues as a developmental one, framing their goals as "investigating the experiences and individual processes contributing to the reconstruction of teaching practice" (pp. 20-21). In order to study these experiences and processes, the researchers engaged in specific ways of supporting and studying teacher change that they came to term the teacher development experiment (Simon, 2000). Drawing on both constructivist (Dewey, 1938a; Dewey & Bentley, 1949; Piaget, 1970; von Glasersfeld, 1991, 1995) and emergent (P. Cobb & Yackel, 1996) perspectives on learning, these researchers aimed to support changes in ways that teachers made sense of their students' mathematical thinking in order to capitalize on it in their instruction.

An aspect of teachers' knowledge and beliefs that, according to Simon and colleagues, is inherent to instructional practices advocated by reform proponents has to do with teachers' conceptions of the nature of mathematics and its teaching and learning. Because current reform recommendations are guided by constructivist perspectives on the nature of knowing, Simon (1997) argues that the mathematics education research community needs to (a) develop models of teaching that build on constructivist views of learning and (b) understand how practices represented in such models develop. In other words, with respect to the endeavor of supporting teachers' development of particular instructional practices, Simon claims that we need to clarify the conceptual endpoints to orient our work with teachers as well as possible avenues that may lead towards these endpoints.

Simon (1997) proposed a model of reform-based instruction, based both on a review of prior research and retrospective analyses of his own experience of teaching mathematics to prospective and practicing teachers. Although a detailed discussion of the model is beyond the scope of my present argument, I will highlight aspects related to documenting developmental processes of teacher learning that Simon and his colleagues set out to study. In particular, a critical aspect of the model that represents a developmental endpoint of envisioned teacher change is that the teacher holds what Simon and his colleagues term a *conception-based perspective* on learning (Simon et al., 2000). We can think of a conception-based perspective as standing for a common core of emergent and constructivist perspectives, a core that Simon and colleagues (2000) specified as sharing the following assumptions:

1. Mathematics is created through human activity. Humans have no access to mathematics that is independent of their ways of knowing.
2. What individuals see, understand, and learn is constrained and afforded by what they currently know (current conceptions).

3. Mathematical learning is a process of transformation of one's knowing and ways of acting. By using the term *transformation*, we mean to indicate that learning involves a modification of existing ideas, not just the accumulation of additional ideas (p. 584)

While a conception-based perspective stands for a way of thinking about learning that is considered powerful in the academic arena, this perspective is rare amongst mathematics teachers. Simon and colleagues (2000) point out that adopting this perspective requires a difficult epistemological shift from "we understand what we see" to "*we see what we understand*" (p. 585, see also Labinowicz, 1985), a shift that can be counterintuitive to many teachers.

In order to refine the model and promote teachers' development of the instructional practices explicated in it, Simon and colleagues engaged a group of 10 prospective and 9 practicing mathematics teachers in a 4.5-year MTD Project that combined 5 intensive semester courses (conducted as whole class teaching experiments) with case studies of individual teachers (Tzur, Simon, Heinz, & Kinzel, 2001). The courses focused on promotion of teachers' development of conceptions of key ideas in geometry and ratio as well as on their pedagogical understanding. The researchers specifically aimed to support the teachers' development of a constructivist view of learning to serve as a basis for teachers' analyses of their students' reasoning and their design of tasks that "promote certain conceptual advances in students" (Tzur et al., 2001; p. 233; cf. Simon & Schifter, 1991). To this end, Simon and colleagues attempted to support teacher change by focusing on specific cognitive aspects underlying teachers' classroom practices.

In order to trace developments throughout the professional development program, Simon and colleagues adapted the case study methodology to generate *accounts of practice* (Simon & Tzur, 1999) for each of the participating teachers. Their goal was to articulate useful ways of understanding teachers' current perspectives on teaching and learning. "Our commitment in using the accounts of teaching practice methodology is to arrive at an appropriate (given the data) articulation of the teacher's current practice in a way that portrays the reasonableness of all the teacher's observed actions" (pp. 255-256).

A key commitment in the process of generating these accounts was researchers' determination to understand teachers' actions as reasonable from these teachers' perspective. This was important for two reasons. First, attempting to understand teachers' actions as reasonable led to articulation of what the researchers considered to be *a useful way to understand teachers' current perspective*. Second, understanding reasonableness in teachers' current practices provided a foundation for the researchers to hypothesize how teachers' further development might proceed. While the latter reason was critical in designing professional development activities, the team's published work focuses on the perspective that teachers developed while attempting to align their instruction with reform recommendations. This teacher perspective on teaching and learning was distinct from both a perspective underlying more traditional approaches to teaching and from the conception-based perspective aimed for by the researchers.

In speaking of *traditional* approaches to teaching, I refer to approaches in which the teacher's overall goal is to support students' development of a range of mathematical proficiencies, attempting to achieve this goal by (a) demonstrating procedures, (b) giving students' opportunities to exercise the procedures by (usually) assigning textbook problems, and (c) assessing students' learning on the basis of correctness of their responses to homework problems, quizzes, and tests. The perspective on mathematics and mathematical activity that underlies such an approach involves what Skemp (1976) referred to as *instrumental understanding*. Skemp pointed out that for many teachers, students' familiarity with a mathematical rule and their ability to use it to arrive at a correct solution represented students' understanding.

In contrast to this traditional approach, the teachers in MTD Project used open-ended problems and hands-on tasks, encouraged collaboration, asked about students' understanding, and – importantly – themselves considered connections amongst mathematical ideas as a key part of mathematics. If the teachers' goal was to support their students' ability to use rules to arrive at correct solutions, their organization of classroom instruction would seem to be unusually complicated and inefficient. On the other hand, the teachers' approach did not align with a conception-based perspective. They held specific expectations for students' responses to open-ended questions and became increasingly directive when students responded in unexpected ways. They ascertained *whether* students understood before moving on with instruction. However, the nature of *how* students understood did not play a role in these considerations. Simon and colleagues postulated this as a *perception-based* perspective that presented a distinct phase in teachers' development:

> A perception-based perspective is grounded in a view of mathematics as a connected, logical, and universally accessible part of an ontological reality. From this perspective, learning mathematics with understanding requires learner's direct (firsthand) perception of relevant mathematical relationships. …teaching involves creating opportunities for students to apprehend (perceive) the mathematical relationships that exist around them (Simon et al., 2000, pp. 579, 594).

This perspective orients a teacher to introduce instructional activities that, in his or her opinion and experience, make mathematical relationships that students should understand *observable*. It was not unusual that teachers who gained new powerful mathematical insights through collaboration and solving open-ended mathematical problems as part of their professional development tried to reproduce these insights for their students by using some of the same problems and encouraging student collaboration (Heinz, Kinzel, Simon, & Tzur, 2000; Silverman, 2005; Tzur et al., 2001). From a conception-based perspective, this approach is problematic in that the teachers did not consider what they already knew and were able to do *that made it possible for them to* gain the valued insights. For example, Silverman (2005) documents how pre-service teachers planned their classroom instruction by choosing to discuss the last problem situation from an elaborated instructional sequence they encountered in a teacher education course. While the entire instructional sequence was designed to support the teachers' development of increasingly sophisticated forms of mathematical reasoning, the teachers associated the insights that they developed with the final few problems in which their experience of a coherent understanding of the mathematical relationships was most tangible. Perceiving mathematical relationships as inherent, observable aspects of specific problem situations rather than a result of a protracted, constructive learning process lies at the core of the distinction between perception-based and conception-based perspectives. Tzur, Simon, and colleagues (2001) clarify that,

> even those who we would characterize as having a conception-based perspective operate, at times, as if there were a universally accessible reality. … What is different about the conception-based perspective is that individuals who have developed that perspective have the possibility, at any time, to step back from this assumption of a universally accessible reality to question the differences in learners' experiential realities (Tzur et al., 2001, p. 249)

The implications of a perception-based perspective for supporting students' learning are problematic in that the teacher's attention to his or her students' current understanding does not appear critical or particularly useful. For the teacher who views mathematics as *universally accessible* part of an ontological reality, students' understanding of mathematical relationships is afforded and constrained by the opportunities to perceive that reality. It is therefore the teacher's job to provide students with such opportunities. However, if students' mathematical learning depends critically on what students already know, then what counts as an opportunity to perceive mathematical relationships is by no means universal. Instruction that from the teacher's point of view seems like providing opportunities for students' learning of specific mathematical ideas with understanding might, from students' point of view, relate to a distinctly different set of ideas (e.g. Bauersfeld, 1980).

Simon and colleagues contend that the perception-based perspective was not an idiosyncratic perspective developed by the participants in the MTD Project, but is instead common among mathematics teachers who attempt to change their instruction in response to reform proposals. Their explication of this perspective therefore contributes to our understanding of more general developmental patterns of teacher change. The distinctions outlined by Simon and colleagues in teachers' underlying perspectives on teaching and learning provide a useful analytical tool when documenting changes in teachers' practices over extended periods of time.

I find it important to highlight two issues this work brings to the fore with respect to supporting teacher learning. The first issue is the importance of researchers' understanding teachers' perspectives and the implications this has for conceptualizing differences in teachers' and researchers' beliefs from a design perspective. Second, although an outline of different developmental phases provides us with a big picture of teacher learning, it is not by itself sufficient to guide efforts to support teacher change.

1) Understanding Teachers' Practices as Reasonable from their Perspective

The assumption that teachers' classroom instruction is reasonable from the teachers' perspectives allowed Simon and colleagues to generate accounts of practice for the participating teachers that were useful both theoretically and pragmatically. While recommendations to view teachers' instruction as reasonable are a repeated theme in teacher education literature (e.g., Leatham, 2006; McIntyre & Hagger, 1992; Skott, 2001; Thompson, 1992), developing such a view can often seem counterintuitive. This is true especially in cases when teachers' instructional practices differ significantly from those advocated by the reform proponents. However, if we do not commit to view teachers' current instruction as reasonable from their perspective, we run into significant problems from point of view of design as well as equity.

First, we lose an important resource to orient professional development design. In the absence of a perspective within which teachers' practices are assumed to be coherent and reasonable, these practices can appear as a random collection of instructional decisions and values. There is then no basis on which to anticipate whether the teachers will find designed professional development activities relevant to their instruction, and whether they will be willing to engage in them. Moreover, teachers' rationales to engage in some but not in other activities would seem equally random, making it impossible to generalize design research findings beyond the specific contingences of the research site.

Second, by rejecting teachers' current instructional practices as inadequate or objectionable, we are likely to overlook opportunities to build on those practices. The task of professional development would then comprise filling the gaps between teachers' current – "deficient" – instructional practices and the envisioned ones. The problematic nature of this approach is well documented by the frustrations of both teachers who participated in professional development programs that were hard to justify within their current understanding of teaching and learning (Putnam & Borko, 2000), and professional developers who struggled to earn participating teachers' cooperation and enthusiasm (Franke, Kazemi, Carpenter, Battey, & Deneroff, 2002). The resulting mismatch in professional developers' and participating teachers' views of ways to improve classroom mathematics instruction has been discussed in the literature in terms of incongruence in beliefs (e.g., Tillema, 1995), and getting teachers to adopt the researchers' beliefs has repeatedly been reported a challenging task (Thompson, 1992).

Simon and colleagues illustrated that understanding teachers' current instructional practices as a coherent system, rather than as a random conglomerate of teaching moves, makes it possible to take these practices "as a valuable starting point, not as something to be replaced, but a useful platform on which to build" (McIntyre & Hagger, 1992, p.271). Other professional development studies (e.g., Confrey, Makar, & Kazak, 2004; Makar & Confrey, 2002) substantiate the MTD Project's finding that the approach helped to significantly reduce problematic mismatches between researchers' expectations and teachers' actual participation in professional development activities. It is important to stress that these researchers did not downscale their goals and expectations for teachers' learning. Instead, they tried to understand issues of concern and importance that were grounded in the teachers' current practices, along with ways in which to support progressive reorganization of these practices so that they would eventually align with the professional development goals for teachers' learning. In this way, these studies addressed the phenomenon of teachers' "constraining" beliefs as a problem of professional development design.

2) Understanding Teachers' Practices in order to Support Teacher Learning

Analyses of teachers' current practices have served a range of different purposes in prior professional development studies. In addition to differing in terms of their research agendas, these studies often differ in the extent to which they explicitly establish and pursue a professional development agenda for teacher learning. My analysis is oriented by the goal of formulating and refining domain-specific teacher development theories while working with groups of teachers. I am therefore especially interested in research tools that can directly feed into the efforts of supporting teacher learning. To discuss research tools from this perspective, I draw on Tzur and colleagues' (Tzur et al., 2001) suggestion that research constructs, theories, and heuristics provide guidance for supporting teacher learning at different levels of design.

At a broad level, an understanding of the different kinds of perspectives that teachers hold on teaching and learning mathematics can help to highlight some of the key characteristics of instructional practices that professional development might aim to support. In this sense, the distinction that Simon and colleagues explicated between perception-based and conception-based perspectives specifies a general direction for professional development of mathematics teachers. For example, it allows us to see that with respect to a perspective on teaching and learning that underlies "traditional" teaching practices, the development of a perception-based perspective is an important accomplishment. However, with respect to the forms of instructional practice that are the goal of current reform efforts, further support is needed if teachers are to develop conception-based perspectives. Understanding teachers' practices at this broad level also gives insight into the general nature of some of the challenges that professional development facilitators might encounter when pursuing their goals. For example, the realization that teachers have developed a perception-based perspective can orient us towards helping them to develop a more encompassing interpretation of their own mathematical learning and how that learning was supported. Lastly, understanding teachers' practices at this level provides a useful analytical tool to document changes in teachers' practices over long periods of time.

Fine-grained analyses of teachers' practices as reasonable from their perspective, such as those afforded by the accounts of practice developed in the MTD Project, is especially useful in anticipating teachers' interpretations of professional development activities. To illustrate why this is the case, the MTD Project researchers' initial view of teachers' classroom instructional practices was that the teachers were not inquiring into the nature of their students' understanding in their daily instruction. The development of detailed accounts of practices of all 19 participating teachers helped the researchers understand that the teachers, indeed, saw themselves as taking students' reasoning in consideration. However, they were only doing so as long as students' reasoning corresponded – in teachers' view – to observable mathematical reality. The teachers did not build their instruction on student contributions that were inconsistent with that reality. Simon and colleagues proposed that the sense that the teachers were making of opportunities to explore students' reasoning both in their classrooms and in professional development sessions was constrained by their current perspectives on teaching and learning. Promoting MTD Project teachers' inquiry into their students' reasoning would be likely interpreted by the teachers as something they were already doing in their classrooms and would therefore not lead to the envisioned changes in these teachers' instructional practices.

This fine-grained level of teachers' practices is especially useful when conducting retrospective analyses of teachers' learning and when developing a record of shifts in teachers' practices during their participation in professional development. However, because of the time demands of data collection and analysis, detailed accounts of each individual teacher's practice do not provide the most practical means of informing ongoing, session-to-session design decisions about supporting the learning of the entire group of participating teachers. There is a need for another level of analysis of teachers' practices, a level that would yield insights that can directly inform the design of subsequent professional development activities. I refer to this level as the *meso-level* of professional development design.

The meso-level of design concerns the session-to-session design decisions related to planning specific interventions in response to both the teachers' actual participation in prior professional development sessions and the professional development agenda. Insights into teachers' current practices are informative at this level of design if they are specific enough to help developers discern aspects of the teachers' practices that might provide a springboard for further intervention. At the same time, the insights are beneficial if they clarify how patterns in the practices of the group, rather than those of individual teachers, are shaped. In the reminder of this section, I provide justification for each of these two requirements and propose an initial orientation for addressing both requirements effectively at the meso-level of professional development design.

It is significant that the expectation that the teachers' current instructional practices can and should serve as a basis on which to build in supporting their further learning spans different research traditions in mathematics education. Researchers working within constructivist, emergent, and situated traditions all aim to design professional development activities that both engage teachers' current professional expertise and supports its transformation (Ball & Cohen, 1999; Kazemi & Franke, 2004; McIntyre & Hagger, 1992; Simon et al., 2000; Wilson & Berne, 1999). The MTD Project experiences illustrate that this is not a trivial task. For the teachers whose underlying perspective on teaching and learning could be characterized as perception-based, further learning would involve a shift in paradigm with respect to how mathematical knowledge develops. In what ways could teachers' current practices, oriented by a paradigm we want them to overcome, serve as a resource in supporting the envisioned shift? As designers of teacher professional development with an ultimate goal of improving students' mathematical learning, we need to understand teachers' current practices in ways that would allow us to answer this question. A systematic understanding of teachers' practices that would enable us to formulate revisable conjectures about ways of supporting teachers' learning on an ongoing basis would be of both theoretical and pragmatic value.

The argument for the usefulness of understanding teachers' current practices in ways that allow for consideration of how practices of the entire teacher group can be supported parallels arguments made within research on designing classroom instructional resources in mathematics. Cobb et al. (2008) argued that exclusive focus on designing resources to support individual students' reasoning is instructionally problematic. This is because an approach to instruction that is based on the development of a single student's reasoning necessarily ignores the diversity in students' reasoning that is present in any classroom at any point in time. In addition, basing instruction on the development of multiple individual students' mathematical reasoning is not a manageable possibility for mathematics teachers. If we expect effective professional development programs to support the learning of multiple teachers at the same time, similar concerns apply. For this reason, research tools that could guide the design of specific means for supporting the collective learning of mathematics teacher groups are valuable.

Thus far, I have explained why analyzing teachers' practices solely in terms of individual accounts of practice has limitations at meso-level of PD design. To propose an initial orientation that addresses these concerns, I highlight aspects of instruction that remained in the background in Simon and colleagues' study, but that were documented by others to significantly influence teaching from teachers' point of view, often by shaping the setting in which teachers work. These aspects include the instructional resources and assessment tools that are available for use in classrooms (Ball & Cohen, 1996; Bowen & McClain, 2005; P. Cobb, McClain, Lamberg et al., 2003; Confrey et al., 2004; Makar & Confrey, 2002; Remillard, 1999, 2000, 2005), teachers' views of student motivation and classroom misbehavior (Dean, 2006; Visnovska, 2005; Zhao, Visnovska, Cobb, & McClain, 2006), and the overall institutional contexts in which teachers work (P. Cobb, McClain, Lamberg et al., 2003; Coburn, 2003; Elmore, 2000; Gamoran et al., 2003). Each of these aspects constitutes a source of insight to the reasonableness of teachers' current instructional practices (Zhao, 2005). More importantly, each of these insights can serve as a resource in designing professional development activities that the teachers would recognize as highly relevant to their current instruction. A more encompassing range of instructional issues on which to draw in professional development design also broadens the ways in which the professional development designers can envision a trajectory for teachers' learning that not only starts within the issues of relevance to the teachers but that is also justifiable with respect to the potential end points for their learning.

It is important to note that many of the aspects that shape teachers' current practices are common across the teachers who typically participate in professional development activities together. It is, for example, reasonable to expect that most teachers from the same school or school district would develop a relatively similar sense of the expectations that others in their school and the district have for their students' learning and for aspects of instruction that are viewed as most critical. From the perspective of a designer, these similarities would allow for planning professional development activities where concerns that are currently perceived as relevant by the most of the teachers could become a topic of discussion. The teachers' individual responses to these common concerns might then provide the professional development facilitator with a diversity of ideas on which to build in supporting further learning of the teacher group.

I should clarify that this elaboration of the meso-level of professional development design is not motivated by a quest for an ultimate theoretical account. The studies that I review in the following section suggest that we cannot expect that all teachers characterized as having developed a certain perspective on teaching and learning could be further supported in the same way, independently of the institutional context of their work, the instructional resources available in their schools, and major impediments to instruction as seen from teachers' perspectives. As I illustrate by focusing on the CGI project, this more encompassing perspective is implicitly present in professional development designs developed within different theoretical paradigms that could be claimed effective in supporting teacher learning.

3) Understanding Teachers' Practices as Profoundly Shaped by Institutional Context of their Work

I first introduce a CGI study (Fennema et al., 1996) conducted under a cognitive research paradigm. I chose this study based on the detailed picture that the researchers provided of the concerns that played a role in their design and research efforts. Concerns that related to the institutional context of teachers' school were treated as background issues and were not accounted for within the cognitive framework adopted for the study. Nevertheless, it would be hard to overlook the design efforts explicitly devoted to shaping the institutional context in which the teachers worked.

I then follow the development of the CGI research by discussing one of their more recent studies in which the researchers explicitly drew on situated theories of learning to account for developments in their collaboration with a group of mathematics teachers. In doing so, I further elaborate what I mean by understanding teachers' practices at the meso-level of professional development design. Shifts in the CGI research approach also illustrate the manner in which research paradigms are used as tools to be evoked and developed in order to address specific questions encountered in a research work.

CGI: Research-based Knowledge for Teaching

The CGI researchers first developed their program of research in the mid 1980's to investigate how mathematics teachers might capitalize upon research-based knowledge of student reasoning in their classroom instruction. In terms of content, most of the CGI research work was grounded in a substantial body of research that provided a consistent and coherent picture of students' development of basic number concepts (Carpenter, 1985; Carpenter, Fennema, Franke, Levi, & Empson, 1999; Fuson, 1992). Over the years, CGI researchers conducted a number of research and professional development projects in which they collaborated with a variety of mathematics teacher groups. In the particular study that provides a background for this discussion, the CGI researchers engaged a group of 21 elementary mathematics teachers in a longitudinal 4-year teacher development program that focused on "helping the teachers understand the development of children's mathematical thinking by interacting with a specific research-based model" (Fennema et al., 1996, p. 403).

The teachers' active part in the professional development was in deciding how to make use of the knowledge in the context of their own classroom instruction. The researchers conjectured that by providing teachers with an operationalized model of how children's thinking develops, the teachers would become competent in identifying different forms of students' reasoning in their own classrooms, as well as in planning appropriate follow up instruction that would capitalize on the identified forms of student reasoning. As part of the professional development activities, the teachers were regularly asked to assign specific word problems in their classrooms and bring their students' work to professional development sessions for analysis. In this way, student work was conceptualized as a resource for grounding work session discussions and for supporting teachers to "understand the mathematical thought processes of their students" (Fennema et al., 1996, p. 432). For CGI researchers, classrooms became more than spaces for teachers' to exercise and apply new research-based knowledge; they became spaces where teachers actively made this knowledge real in the context of their practice while interacting with their students.

The success of the professional development efforts was framed in terms of changes in the teachers' beliefs and instruction. Findings from case studies of individual teachers led the researchers to conclude that "developing an understanding of children's mathematical thinking *can be* a productive basis for helping teachers to make the fundamental changes called for in current reform recommendations" (p. 403, stress added). Such studies served as an existence proof of what could be achieved with teachers through focusing on a research-based framework of student thinking. Teachers' knowledge of students' developmental processes and their ability to understand their own students' reasoning were both framed as instrumental to the changes documented in teachers' instructional practices.

In terms of the means used to support the teachers' learning, the early CGI reports focused on two issues: (a) the research-based model of student thinking, and (b) teachers' use of that model in their classrooms. It is important to clarify that *supporting collaborating teachers' learning* also included following:

> A CGI staff member and a mentor teacher were assigned to each school. Their responsibilities included participating in the workshops, visiting classrooms, engaging the teachers in discussions, and generally providing support as the teachers learned to base instruction on their students' thinking. Both staff members and the mentor teachers were trained to focus most of their interactions with teachers directly on children's thinking and its use. Insofar as possible, these interactions concerned specific children (Fennema et al., 1996, p. 409).

In their plan of action, the CGI program did not focus solely on teachers' knowledge and beliefs. It also involved significant interventions with both school principals and mathematics support staff based in the teachers' schools. In order to generate evidence of the usefulness of research-based knowledge to teachers' instruction, the researchers took seriously the *institutional context* within which the teachers worked. In a very real sense, the CGI work involved designing for a particular type of institutional context that the researchers conjectured would be supportive of teachers' learning. Yet, at this point, these considerations were conceptualized as a background to the primary focus of the project, rather than as key support for teachers' developing practices. The distinction is critical with respect to generalizability of the research findings or, in other words, with respect to the orientation the findings provide for design and facilitation of other professional development programs. As the following CGI study vividly illustrates, the adaptation of the CGI program to new contexts was indeed problematic and not immediately viable. The CGI researchers resolved the unexpected problems by drawing on both situated theories of learning and their understanding of the institutional context of teachers' work. In discussing this study, I further clarify the usefulness of understanding teachers' practices on the meso-level of design.

CGI: The Contrasting Case of Algebraic Reasoning

After years of conducting and studying professional development programs that focused on early number concepts, Franke and colleagues (Franke, Carpenter, & Battey, 2008; Franke et al., 2002) engaged in professional development and research efforts that focused on early algebraic thinking. Using their intimate understanding of CGI principles and findings, they aimed to support elementary teachers in enhancing students' ability to generate, use, represent, and justify generalizations about fundamental properties of arithmetic. As was the case in their previous work, the researchers intended to support teachers in understanding a research-based model of students' algebraic reasoning and in developing practices that place their students' reasoning in the center of classroom instruction. However, in the spirit of building upon – rather than abandoning – what they learned previously, the researchers came to view teachers' cognitions as being inherently social and their development as being inseparable from the institutional aspects of teachers' work.

The case for my discussion comes from Franke and colleagues' collaboration with a group of teachers in one of the lowest achieving elementary schools in the state of California, Lincoln Elementary (Franke et al., 2002). The researchers intended to use the discussions of student work as leverage in supporting teachers' appreciation of understanding students' algebraic reasoning in instruction. To the researchers' surprise and frustration, even after many work-sessions, student reasoning did not become something teachers wanted to learn about and use in their instruction: "All the teachers at Lincoln see is the answer and while this occurred initially in our earlier work the teachers quickly began to see on the paper and in their questioning what students did to solve the problem" (2002, p. 28). The teachers continued to check for the correctness of students' responses and did not find it useful to conduct classroom discussions in which students explained how they had solved tasks. Instead, they requested that the researchers provide them with more "worksheets" for students to practice until they ceased making mistakes.

In order to support these teachers' learning effectively, the researchers needed to understand *why*, despite their best efforts, it continued to be reasonable from the teachers' perspective to support their students' learning by providing them with abundant opportunities to practice, and by correcting their mistakes. Simon and colleagues' focus on teachers' conceptions locates the source of the reasonableness of teachers' actions within individual teachers' cognition. According to an analysis conducted from this viewpoint, the Lincoln teachers could be characterized as making instructional decisions within a traditional perspective, based on a view of algebra as a collection of rules and facts that can be best learned by repetition. Although such a characterization might capture teachers' actions quite accurately, it does not clarify why sustained efforts at supporting these teachers' change failed. This point is critical because, according to Franke and colleagues (2002), teachers had initially focused on correctness and practice in the earlier CGI collaborations as well. However, supported by the CGI team, they soon came to appreciate student reasoning as an instructional resource. It appears that although Simon and colleagues' characterization of teachers' perspectives provides a useful and specific orientation in terms of goals for teacher learning, it is not specific enough to guide the ongoing process of designing for teacher learning. The exclusively cognitive focus of this characterization seems insufficient to explain why the means of support that have proven effective earlier were not effective with Lincoln teachers.

Franke and colleagues' (2008) analysis instead located the difficulties they encountered in both the content-specific demands of the teachers' learning, and the institutional setting of the teachers' work. They documented how professional development in early algebra differed in the demands for teacher learning from the prior CGI professional development programs in numerical reasoning. This analysis allowed the researchers to propose specific adaptations to the professional development design that took into account the unique characteristics of the professional development context. Specifically, the first of the differences in demands on teachers concerned the extent to which the sequencing of problems was critical to effectively supporting student learning. In developing students' understanding of whole number operations, teachers succeeded in enhancing their students' understanding by posing a variety of word problems and asking students to share a range of strategies. Teachers were able to support students' learning by asking questions that focused on the details of the strategy they had used and thus scaffolding students' development of a repertoire of flexible strategies. In contrast, the sequencing of problems proved to be a critical consideration in early algebra when supporting students' generalizations about fundamental properties of arithmetic. It was no longer sufficient for teachers to elicit students' explanations of their solutions to a single problem. Supporting students' development in this domain required that the teachers make explicit the relationships across students' solutions and problem sequences. I the cases of both early number and early algebra, teachers had to develop ways of capitalizing on students' reasoning in their instruction. However, the demands differed in the extent to which they needed to understand the rationales underlying the two instructional sequences in order to adapt them successfully in their classrooms.

The second content-specific dimension along which the demands on teachers' learning differed related to the institutional setting of teachers' work. It concerned the extent to which the content area addressed in professional development was central (or peripheral) in the curriculum used in the teachers' schools. Franke and colleagues documented that the emphasis that the curriculum put on a specific content area had consequences for development of teacher's expertise in that area. Specifically, differences in curricular emphasis affected the extent to which (a) the teachers' current practices in the content area could be a resource for professional development work, and (b) their classrooms afforded opportunities for the teachers' further learning in that content area. To elaborate the first difference, number development directly related to the early-grades curricula that were in place in the collaborating schools. However, the ideas of relational thinking and formulating conjectures that were central to the CGI model of development of students' early algebraic reasoning were not central aspects of the typical mathematics curricula. The teachers had therefore few opportunities to hear their students work with the ideas and to deepen their own algebraic understanding. Consequently, the teachers often doubted that they could learn to address the content issues that might arise in their classrooms effectively, and productively engage students in algebraic thinking.

To address the second difference, the central position of early number development content in the curriculum provided teachers with plenty of opportunities to pose CGI word problems and consider student solutions. In contrast, to make seemingly "extracurricular" algebraic reasoning an instructional focus in their classrooms, the teachers had the additional challenge of coordinating the mathematical content addressed explicitly in the required curriculum with activities in which students could make generalizations, notice relations, and justify conjectures.

The researchers' understanding of these critical content-related demands on teachers' developing instructional practices and how these demands related to the institutional context in which the teachers worked oriented the researchers' conjectures about viable means of supporting teachers' further learning. For example, the researchers reported that to help the teachers learn to identify opportunities for algebraic thinking, they brought examples of interactions they observed in the teachers' classrooms to the group for discussion. In addition, they started to create structured opportunities for the teachers to reflect on "where their own students are in their understanding of the various ideas of algebraic thinking" (Franke et al., 2008), as students' progress in this content area did not feature on the district quarterly benchmark assessments. These adaptations, while open for further testing and modifications, serve as examples of the flexibility that understanding teachers' practices as situated in the institutional contexts of teachers' work affords to professional development designers. This perspective enabled the CGI researchers' capacity to maneuver on the *meso-level* of design, where pragmatic decisions of how to proceed are informed by systematic ongoing analyses.

Summary: Cognitive Perspectives and Supporting Teacher Learning

In reviewing high quality professional development studies conducted within cognitive paradigm, I discussed the usefulness of understanding teachers' practices in ways that yield resources that directly feed back to designing further means of support. Although developmental approaches can help us delineate worthwhile end points for teacher learning, it appears that studies conducted under a situated paradigm are especially useful when developing means for supporting teacher learning on the meso-level of professional development design. The usefulness of these studies stems in part from the manner in which they frame teachers' practices as situated within the institutional context of their work. I now turn to introduce the major tenets of the situated paradigm and examine the ways in which they orient design and research in mathematics teacher education.

Situated Perspectives and Supporting Teacher Learning

In recent years, efforts to support teachers' development of sophisticated instructional practices have brought to the fore the social contexts of teachers' work and, in particular, the opportunities for learning that these contexts afford. Ways of theorizing teachers' learning that draw on situated theories of activity have become prominent in research on teacher professional development. The theoretical underpinnings of this perspective on teacher learning are derived primarily from the work of Rogoff, Lave, and Wenger (Lave, 1991; Lave & Wenger, 1991; Rogoff, 1995, 1997; Wenger, 1998). This orientation conceptualizes learning as a process that is inherently related to the social and cultural contexts in which it occurs.

> Situated cognition theorists challenge the assumption that social process can be clearly partitioned off from cognitive processes and treated as external condition for them. These theorists instead view cognition as extending out into the world and as being social through and through. They therefore attempt to break down a distinction … between the individual reasoner and the world reasoned about (P. Cobb, 2001, p. 14126).

The assumptions of situated cognition have face validity to many researchers and professional developers working with groups of teachers with a goal of supporting development of new instructional practices. Consistent with empirical findings, situated theories portray the process of teachers' learning as profoundly influenced by the context in which it occurs. From this perspective,

> the physical and social contexts in which an activity takes place are an integral part of the activity, and… the activity is an integral part of the learning that takes place within it. How a person learns a particular set of knowledge and skills, and the situation in which a person learns, become a fundamental part of what is learned (Putnam & Borko, 2000, p. 4).

On the one hand, this perspective has enabled researchers to account for cases in which the same designs resulted in differential learning opportunities for teachers with similar initial knowledge and beliefs of effective mathematics instruction by bringing differences in the institutional settings in which these teachers worked into the picture (e.g., Franke et al., 2008). On the other hand, this view foregrounds both theoretical and pragmatic questions about supporting teachers' learning across different settings. The need to understand the ways in which teachers' learning in professional development settings might transfer to their classroom instruction has long been a concern (e.g., Kagan, 1992). However, framing teachers' learning explicitly as situated in settings in which it occurs required that researchers explicate how teachers' participation in professional development sessions might result in changes in their classroom practices.

Centering Teacher Learning in Instructional Practices

Aiming to support improvements in teachers' classroom instruction, rather than merely their professional development session performance, Ball and Cohen (1999) explored the idea of *centering teachers' learning in their instructional practices* as an overarching principle for professional development design and analysis. They clarified that

> Centering professional education in [teachers' instructional] practice is not a statement about either a physical locale or some stereotypical professional work. Rather, it is a statement about a terrain of action and analysis that is defined first by identifying the central activities of teaching practice and, second, by selecting or creating materials that usefully depict that work and could be selected, represented or otherwise modified to create opportunities for novice and experienced practitioners to learn (p. 13).

The idea of centering teachers' professional development learning in instructional practice broadly addresses the need to coordinate teachers' learning across the two settings, and departs from a focus on knowledge and beliefs that teachers were expected to develop in work-sessions and apply in their classrooms. Ball and Cohen (1999) argue that to learn anything relevant to professional performance, teachers "need experience with the tasks and ways of thinking that are fundamental to the [instructional] practice" (p. 12). This view builds on characterizations of teaching as a reflective practice, changes in which can be supported through the process of focused inquiry (Dewey, 1910, 1938b; Schön, 1983, 1987). As Ball and Cohen point out, engaging teachers in inquiry into instructional practice requires the development of appropriate means of support, such as suitable professional development activities, tools of professional analysis, professional discourse, and teachers' engagement in communities of practice.

Some of the professional development programs that succeeded to engage teachers in inquiry into instructional practice used activities that focused on explicating the mathematical potential of instructional tasks while considering learners' perspectives (McClain, 2003), and on understanding students' mathematical reasoning from their written work (Kazemi & Franke, 2004). In these activities, instructional materials and student work played the role of what Ball and Cohen call "strategic documentation of practice" (p.13), and were established as tools for professional analysis. The inquiries designed around these activities of teaching required teachers to develop and compare their conjectures about how students might reason in the situations under investigation. The purpose of these inquiries was to support teachers' development of specific forms of instructional practice, in which drawing on their students' reasoning would become central. Importantly, the collaboration among teachers that would allow them to productively compare and discuss their conjectures could not be adequately cultivated without the development of more substantial professional discourse and teachers' engagement in communities of practice (Ball & Cohen, 1999; Gamoran et al., 2003; McLaughlin & Talbert, 1993; Putnam & Borko, 2000; Zhao et al., 2006). As a result, studies that investigate the process of supporting the emergence and further learning of *professional teaching communities* (Achinstein, 2002; Dean, 2005; Franke & Kazemi, 2001b; Grossman, Wineburg, & Woolworth, 2001; Kazemi & Franke, 2004; Little, 2002; Nickerson & Moriarty, 2005; Stein & Brown, 1997; Warren & Rosebery, 1995; Westheimer, 1998) have become increasingly common in this line of research.

In the following sections, I will first discuss in more detail research related to supporting the development of communities of practice – or professional teaching communities – of mathematics teachers. Specifically, I will briefly discuss how a professional teaching community differs from a group of mathematics teachers who, for example, meet regularly for lunch, and why the distinction matters. I will then go on to discuss rationales that led to developing a framework that situates learning of professional teaching communities in the institutional context of teachers' schools and districts. Against this framework, I will then discuss the methodology that Dean and colleagues (Dean, 2004, 2005, 2006; Zhao et al., 2006) used to understand some of the critical issues that arose when supporting the initial establishment and further development of a professional teaching community. I conclude this chapter by discussing strengths and limitations in how the means of supporting teacher learning have been conceptualized in professional development studies. As previously, my goal in reviewing the research is to both equip myself with potentially useful analytic tools for gaining insights into productive ways of supporting teacher learning, and to clarify the research questions that remain significant in the field.

Professional Teaching Communities

As I indicated previously, the instructional practices that are the goal of ambitious professional development efforts are complex, demanding, uncertain, and not reducible to predictable routines (Ball & Cohen, 1999; Clark, 1988; Lampert, 2001; McClain, 2002; Schifter, 1995; Smith, 1996). The findings of a number of investigations indicate that teachers' participation in a professional teaching community and, more generally, strong social networks can be a crucial resource as they attempt to develop instructional practices of this type (P. Cobb & McClain, 2001; Franke & Kazemi, 2001b; Gamoran, Secada, & Marrett, 2000; Kazemi & Franke, 2004; Lachance & Confrey, 2003; Lehrer & Schauble, 1998; Little, 2002; Stein, Silver, & Smith, 1998).

Building on empirical research, Carpenter and colleagues (2004) assert that professional teaching communities can "provide a climate for engaging in inquiry, sharing knowledge of student thinking, sharing norms for what counts as effective instruction and student achievement, and building social supports for managing uncertainty" (p. 8; cf. P. Cobb, 1999; Gamoran et al., 2003; McLaughlin & Talbert, 1993; Quiroz, 2001). Recent CGI studies (e.g., Franke & Kazemi, 2001a) can serve as an illustration. These studies indicate that it was through participation in professional teaching community discussions that the teachers developed understandings of students' strategies, and how these related to each other and to the mathematical ideas being developed. In addition, teachers began attending to pedagogical practices that supported specific forms of students' reasoning both in their own and in classrooms that they observed. A key function of the social support provided by participating in communal activities with colleagues was that the "teachers began to see themselves as able to contribute to the group as teachers of mathematics with knowledge and skills to share" (p. 107). This was a significant development in light of Spillane's (Spillane, 2005) observation that teachers are typically much less likely to initiate and contribute to discussions directed at instructional improvement in mathematics than in literacy, and often attribute improvements in mathematics to the mathematics curriculum and other external programs. It is therefore important that from the teachers' perspective, trying new ideas in their classrooms became a valued part of their instruction that provided them with opportunities to meaningfully contribute to the development of the group, as well as to their students' learning. As a consequence of promising empirical findings, designs for supporting teachers' learning that involve guiding the initial emergence and subsequent development of professional teaching communities have become increasingly common, yielding a range of approaches. The challenge of developing an interpretive framework that would enable researchers to document the learning of both a professional teaching community and the participating teachers is therefore pragmatically as well theoretically significant.

Additional observations strengthen the justification for studying the learning of professional teaching communities. The systematic design effort of researchers has often been critical for the initial emergence of professional teaching communities (Putnam & Borko, 2000). However, several of these communities have persisted and have continued to learn many years after the researchers withdrew from the site. For example, Franke and Kazemi (2001a, p. 107) reported that the teacher work groups with which they collaborated for a period of four years continued to collaborate two years after the researchers discontinued their participation. Similarly, the professional collaboration with a group of 25 second-grade teachers supported by Cobb and colleagues (P. Cobb, Yackel, & Wood, 1991) over a three-year period resulted in a productive teacher collaboration that was reported to continue for ten years (P. Cobb & McClain, 2001). The teachers in both cases continued to meet to address instructional issues, treating teaching as a learning profession (Darling-Hammond & Sykes, 1999).

While these cases can serve as an existence proof of generative growth resulting from teachers' participation in professional teaching communities, they are by no means typical at present time. Professional teaching communities frequently have to cope with institutional constraints – such as withdrawal of resources, or lack of alignment with school and district policies – that become more salient after the researchers withdraw from the collaboration (Gamoran et al., 2003). The goal of establishing professional communities that continue to grow has led researchers to better understand the key aspects that influence the sustainability of professional teaching communities, such as those related to the institutional setting of teachers' work. To more effectively support teachers in changing their instructional practices, the researchers needed to develop tools to design professional development programs that would take institutional setting of teachers' work in consideration.

An Interpretive Framework that Situates Teachers' Learning in the Institutional Setting of their Schools and the District

Institutional context of teachers' work has come to the fore in discussions of both development and the sustainability of professional teaching communities. Professional development and research experiences created examples of professional development designs that, although highly effective in one setting, did not prove viable in supporting learning of a professional teaching community in another setting (e.g., Franke et al., 2008; Franke et al., 2002). As discussed previously, cognitively-oriented professional development studies were conducted at carefully selected research sites that allowed for introduction of significant changes to the structure of teachers' working environment (e.g., Fennema et al., 1996). Current reform implementation and sustainability research (e.g., Carpenter et al., 2004; Coburn, 2003; Elmore, 2004) suggests that designing specific institutional supports can provide critical resources for teachers' learning. Understanding teachers' learning as situated within the affordances and constraints of the schools and districts in which they work became important within efforts at supporting effective and sustainable changes in teachers' instructional practices.

I will discuss a framework developed by Cobb and colleagues (2003) in response to the practical need to account for teachers' learning in the social context of the professional teaching community as it was enabled and constrained by the broader context of the institution. When characterizing the environments in which teachers develop and refine their instructional practices, Cobb and colleagues focused on the *functions of teaching* and how they are accomplished in schools and school districts. This focus highlights how a number of persons in various designated positions within the school and the district are involved in accomplishing these functions, thus shaping students' mathematical learning. Specifically, these functions include

- *Organizing for mathematics teaching and learning* by, for example, delineating instructional goals and by selecting and adapting instructional activities and other resources, and
- *Making mathematics learning and teaching visible* by, for example, interpreting test scores or posing tasks designed to generate a record of students' mathematical reasoning (p. 14)

Mathematics teaching is thus seen as an activity that is distributed across different people and their actions. Importantly, a resulting view of the institutional setting in which mathematics instruction takes place is that of groups of people attempting to achieve at times complementary, and at other times competing agendas as they organize for mathematics teaching and learning. Cobb and colleagues used the framework to analyze the institutional setting in which they collaborated with a group of middle school mathematics teachers. This analysis focused on the initial year of the same professional development project for which I analyzed the data collected in years 3-5 in this dissertation study. Their findings will therefore help me to situate the dissertation study in the history of the professional teaching community and institutional setting in which it evolved.

Cobb and colleagues identified distinct communities of practice (Wenger, 1998) whose enterprises were concerned with teaching and learning of mathematics in the district. These were a district-wide mathematics leadership community, school leadership communities, and a professional teaching community comprised of mathematics teachers. The *mathematics leadership community* included a mathematics coordinator and three mathematics specialists as core members, and a number of teachers as more peripheral members. This group aimed to improve mathematics performance of all students, and of minority students in particular, by helping teachers use the reform textbook series as the basis for their mathematics instruction. The *leadership communities in teachers' schools* consisted of a principal and two or more assistant principals. One or more mathematics teachers served as peripheral members of each school leadership community. The enterprise of these school leadership communities was to raise students' scores on the state-mandated achievement test. Lastly, a group of 9 mathematics teachers from 5 different middle schools in the district collaborated with the research team during the first two years of the project to develop a common agenda of improving instruction so that all students learn mathematics with understanding in their classrooms. After 19 months of the collaboration, this group has evolved into a *professional teaching community.* All groups attempted to influence both mathematics instruction and students' learning in the district. To understand the co-existence and potential alignment of the agendas, the researchers analyzed three types of interconnections among the groups: (a) *boundary encounters* in which members of different groups engaged in activities together, (b) the role of *brokers* who were at least peripheral members of two or more groups, and (c) the role of *boundary objects* that have been incorporated into the practices of two or more groups (cf. Star & Griesemer, 1989; Wenger, 1998).

Boundary encounters between the mathematics leadership community and the teacher group occurred when mathematics leaders conducted work sessions and study-group meetings with teachers. These sessions and meetings focused on the use of the reform textbook series that the district had adopted to be used along with an established traditional textbook series. The collaborating teachers valued these sessions and indicated that, as a result, they were better prepared to discern the mathematical intent of the nonstandard problems in the new textbooks. In contrast, a typical boundary encounter in which teachers and the school leaders engaged were frequent drop-in classroom visits that the school leaders conducted to monitor and assess teachers' instructional practices. These boundary encounters led teachers to develop a view of classroom observations as situations for assessment rather than assistance, along with a preference for keeping their instruction private. Institutionalization of teaching as a private activity in which the teachers relied almost exclusively on their own resources was documented to reduce the extent to which the teachers' classrooms could be sites for their learning (Gamoran et al., 2003).

With respect to brokers who can bridge the activities of different groups by facilitating the alignment of perspectives, it was significant that there were none to be found among the three identified groups when the research collaboration started. This absence of brokers could partially account for the misaligned agendas of the mathematics leadership group and the school leadership groups, which led to tensions for teachers attempting to fulfill both of the competing agendas.

To provide an illustration of a boundary object, I will focus on the *pacing guide* that the mathematics leaders in the district had produced as they organized for mathematics teaching and learning. The pacing guide mapped the two textbook series used in the district (i.e., a traditional textbook series and a reform textbook series) onto the State Mathematics Standards produced by the State Department of Education. The State Mathematics Standards specify the mathematical objectives that teachers should address at each grade level. The mathematics leaders' rationale for creating the pacing guide was to facilitate teachers' use of the reform textbook series, which, though valued by the mathematics leadership group, did not address all objectives mandated by the State Mathematics Standards. By creating the pacing guide, the district leaders hoped to facilitate teachers' transition towards more "reformed" teaching while ensuring that all state-mandated objectives would be addressed in the course of instruction. When the researchers first began working in the district, the teachers used the pacing guide to develop lessons that tended to focus on performing and applying mathematical procedures. For the teachers, the usefulness of the pacing guide resided in the way in which it helped them to locate procedures the students were required to master.

This example highlights two aspects that make analyses of boundary objects productive. First, it cautions us against assuming that an artifact automatically carries the intentions of the group that created it to another group that uses it as part of everyday practice. A careful analysis of boundary objects can help us understand practices of different groups with respect to different purposes that boundary objects serve. At the same time, boundary objects, even when used differently by different groups, contribute to coordination of the groups' activities. In the illustration above, although the teachers' use of the pacing guide did not facilitate teachers' development of types of instructional practices envisioned by the mathematics leaders, it enabled many of them to use the new textbook series as one of the resources for planning instruction. In this sense, the pacing guide facilitated connections between practices of the teacher and mathematics leader groups, though the nature of those connections could not be understood by studying practices of only one of the groups.

An Interpretive Framework for Documenting Learning of Professional Teaching Communities

Building from this analysis of the institutional setting, Dean's work (2005; 2006) focused on documenting the learning of the same group of middle school mathematics teachers during the initial two years of professional development collaboration and the means by which that learning was supported. In addition to developing a framework for documenting the learning of a group of teachers in a professional development context, Dean's analysis provides critical insights into the process of the evolution of a professional teaching community. The term "professional teaching community," as used by Dean and colleagues, is not synonymous to a "group of mathematics teachers who collaborate with each other in some way." Specifying the distinction between a group and a community is important given that not every group composed of mathematics teachers would provide them with the climate, the need, and the resources for a deep, systematic engagement in issues relevant to their profession. Some groups, nevertheless, have been documented to develop such resources (Carpenter et al., 2004). Based on review of the literature on professional communities and professional teaching communities (Bellah, Madsen, Sullivan, Swidler, & Tipton, 1985; Gamoran et al., 2003; Grossman et al., 2001; Lave & Wenger, 1991; Lehrer & Schauble, 1998; Newmann & Associates, 1996; Rogoff, 1995; Secada & Adajian, 1997; Stein et al., 1998; Warren & Rosebery, 1995; Wenger, 1998), Dean and colleagues (P. Cobb, McClain, Lamberg et al., 2003; Dean, 2004, 2005) articulated the salient characteristics of a professional teaching community of mathematics teachers:

- *A shared purpose or enterprise* such as ensuring that students come to understand central mathematical ideas while simultaneously performing more than adequately on high stakes assessments of mathematics achievement
- *A shared repertoire of ways of reasoning with tools and artifacts* that is specific to the community and the shared purpose including normative ways of reasoning with instructional materials and other resources when planning for instruction or using tasks and other resources to make students' mathematical reasoning visible
- *Norms of mutual engagement* encompassing both general norms of participation as well as norms that are specific to mathematics teaching such as the standards to which the members of the community hold each other accountable when they justify pedagogical decisions and judgments (Dean, 2004, pp. 3-4)

This characterization of a community gives designers of professional development a more specific orientation to the substance of the professional collaborations, the emergence of which they attempt to support. The characterization can also help us understand some of the reasons why teachers' membership in groups that, for instance, meet regularly to socialize during lunch time might not adequately support the teachers in improving their students' learning of mathematics with understanding.

It is important to note that supporting the emergence of a professional teaching community characterized by a shared purpose, repertoire of ways of reasoning with tools, and substantial norms of mutual engagement is not a trivial matter. According to Dean's analysis, it was not until after approximately 19 months of the collaboration (during which the group met in 12 full-day sessions) that the group emerged as a professional teaching community. In order to trace the emergence and how it was supported, Dean analyzed the development of four interrelated types of norms of mutual engagement that became established in the group: norms for (a) general participation, (b) pedagogical reasoning, (c) mathematical reasoning, and

(d) institutional[10] reasoning. These norms were documented empirically by discerning patterns or regularities in the ongoing interactions of the members of the group. A norm is therefore not an individualistic notion but is instead a joint or collective accomplishment of the group members (Voigt, 1995).

The analysis of *norms for general participation* documented the evolving participation structure of the community (Lampert, 1990; Shulman, 1986). As an illustration, this analysis documented whether it became an established norm in the group for the teachers studied by Dean to question and critique each others' reasoning or whether norms involved what Grossman et al. (2001) term pseudo-agreement in which the teachers refrained from confronting issues that relate to their instructional practices.

The analysis of *norms for pedagogical reasoning* documented the norms that became established as the teachers both reflected on their instruction and planned for instruction. In focusing on the key norm of what counts as an acceptable pedagogical argument, for example, Dean documented the extent to which the teachers became obliged to justify their pedagogical judgments in terms of analyses of students' mathematical reasoning.

The analysis of *norms for mathematical reasoning* documented both the norms for mathematical argumentation and the norms for reasoning that became established as the teachers explored particular mathematical domains. When the teachers engaged in activities that involved analyzing data, for example, Dean documented whether the norms that became established for statistical reasoning involved additive or multiplicative reasoning.

The analysis of *norms for institutional reasoning* documented the evolution of the teachers' understanding of the institutional setting and its influence on their instructional practices. As part of her analysis, Dean focused both on the extent to which the teachers viewed their practices as shaped by the institutional settings in which they worked and on the aspects of these settings that they believed they could change.

The first three types of norms are well regarded as relevant to development of professional teaching communities or mathematics teachers' learning in the research community (e.g., Bransford, Brown, & Cocking, 2000; Grossman et al., 2001; Ma, 1999; Shulman, 1986). Among

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[10] In her earlier work, Dean called these norms the norms for *strategic* reasoning. In this proposal, I use her revision of the term – norms for *institutional* reasoning.

the major contributions of Dean's analysis is an explicit attention to development of norms of institutional reasoning along with the focus on supporting deprivatization of teachers' practices.

> The importance of focusing specifically on norms of [institutional] reasoning emerged in the course of collaboration with the teachers and the concept proved to have considerable explanatory power and practical significance. The teachers' changing views of the institutional context and how it supported or constrained their instructional practices was an important aspect of their learning (Dean, 2006, p. 25).

To illustrate the significance of both focusing on norms for institutional reasoning and their cultivation in the group, I discuss two important developments that occurred during the initial two years in which the research team collaborated with the teachers. Because this discussion also helps to situate my analysis, I provide considerable details. Dean documented that at the beginning of the collaboration, all the teachers perceived that it was beyond their control to influence the institutional context, and pointed to pressures associated with the state-mandated end-of-year standardized testing and lack of instructional support. Accustomed to being monitored rather than supported, teachers actively worked to keep their practices private. They avoided both discussing examples from their instructional experiences in the work-sessions, as well as the presence of researchers' video cameras in their classrooms.

The first development occurred when the teachers discussed their students' statistics work during the third work-session of the year one. As I noted earlier, the professional development collaboration was grounded in the statistics instructional sequences developed by the research group in prior classroom design experiments (P. Cobb, 1999; McClain & Cobb, 2001). The teachers used some of the designed statistics tasks in their classrooms and brought their students' solutions for analysis in the professional development sessions. When the teachers discussed their students' work, they shared their frustration from principals' disapproving reaction when they saw that the teachers devoted time to a task that the principals did not immediately perceive as relevant to the approaching test. However, as Dean (2005) stressed,

> there were instances where even as teachers were expressing frustrations about pressures they felt, they were starting to ask others for advice. This was the first instance of the deprivatization of their classroom instructional practices. … For the first time, the teachers openly discussed events from their classrooms as a way of describing problems and sharing solutions related to their perception of the institutional setting and how it affected their teaching (pp. 98-99).

Explicit discussions of the institutional context opened up an avenue to teachers' classroom instructional practices that teachers cautiously guarded when the discussions related to their pedagogical reasoning.

The critical role of the cultivation of norms for institutional reasoning in supporting the emergence of a professional teaching community became even more pronounced during the summer session at the end of the year one. At that time, the group analyzed a video case from the seventh-grade statistics classroom design experiment, during which one of the statistics instructional sequences was developed. This was intended to initiate discussion of the different ways the students in the video reasoned about the statistics. In contrast, the teachers focused solely on, and made jokes about, student behavior. In the ensuing discussion, the teachers explained that

> they focused on student behavior and classroom management issues because this was the primary criterion on which they were evaluated by school administrators. In other words, teacher effectiveness was equated with teachers' ability to manage their classrooms. Jeremy [a teacher] added that for the principals, "learning happens when everyone is paying attention." Other teachers confirmed Jeremy's statement explaining that they were evaluated based on number of students on task and whether the … [State Mathematics Standards] objective was written on the board. Lisa [a teacher] added that their focus on student behavior could also be related to the fact that they are isolated and have never had an opportunity to observe other teachers, even on video. Other teachers confirmed Lisa's statement and voiced frustrations of having no chance to observe others' teaching practices (pp. 117-118).

While the teachers had previously discussed their limited professional contacts and the pressures they felt due to state-mandated end-of-year tests, they now referred to the institutional context to explain how they engaged in activities during the session. As Dean documented, making institutional context of teachers' work a point of discussion and examination significantly enhanced learning of the group. The teachers started to develop ways to contribute to the negotiation of what they considered should be good mathematics instruction within the institutional setting of their schools and the district.

Interpretive Frameworks and Viewing Teachers' Actions as Reasonable

Understanding how functions of mathematics teaching got accomplished within the institutional setting of the teachers' work proved to be a critical resource for understanding teachers' actions in their classrooms, as well as in the professional development sessions *as reasonable* *from their perspective*. For example, when the group initially worked on the statistics tasks, the teachers focused on procedures for creating graphical representations and calculating measures of center. The teachers' contributions at this point differed from what the research team envisioned as normative ways of mathematical reasoning that could be supportive of productive teacher learning. The envisioned ways of reasoning included that teachers provide justifications for their arguments, reason about the data in terms of distribution, and above all, analyze data in order to gain insights into the situation under investigation. However, the norms for mathematical reasoning that emerged within the group were understandable when we consider how the mathematics that teachers taught and had access to was shaped by the institutional context of their work. When the teachers initially participated in our professional development sessions, they demonstrated mathematics that they were accountable for teaching in their classrooms. This mathematics was specified by state objectives that, in domain of statistics, focused on students' skills in construction and reading of graphs, and on calculations of measures of center. Teachers' adherence to these objectives was monitored by school leaders' weekly drop-in visits to classrooms, and the teachers were held accountable for their students' scores on the annual state test.

Second, the ways that the two textbook series were used in the district did not guide the teachers to develop more sophisticated forms of statistical reasoning. The teachers used both the traditional and the reform textbook series in ways that fitted with their understanding of statistics they needed to teach, that of helping students know what to calculate and how to construct graphs when given a set of data. Teachers' interpretation of the pacing guide as a tool for locating textbook units that address specific objectives also reinforced the teachers' understanding of what students needed to learn.

Third, teachers' opportunities to develop deeper mathematical knowledge for teaching in the district were limited to professional development organized by the mathematics leaders that focused on teachers' use of the reform textbook series and, at the time, was in place for less than a year. There were no formal or informal networks or sources of assistance in the district on which the teachers could draw to improve their mathematical reasoning.

From perspective of professional development design, interpreting teachers' contributions with respect to the institutional context was important for two reasons. First, it enhanced researchers' understanding of teachers' participation. For example, the teachers' initial engagement in statistical discussions could not provide the research team with detailed insights into their cognitive skills in the area of statistics. Partially, this was because the teachers engaged in a different "game" than the researchers when they proposed solutions to the statistics tasks at the beginning of the collaboration. As I illustrated, what statistics meant for the teachers at that time was profoundly shaped by the institutional expectations for statistical learning of students in their schools.

Second, the analysis of the institutional context in which the teachers work became an important means of supporting the learning of the teacher group. As Dean (2005) reported, teachers' increased awareness of how the institutional setting shaped their instructional practices was indicated by both the teachers' desire to make changes in their practices and by their perception of the institutional setting as something they could influence. As the teachers started to actively examine what they previously perceived as insurmountable constraints, they started to draw on each other's ideas and gradually transformed ways and purposes of group interactions, establishing a professional teaching community.

Conceptualizing Teachers' Learning Across Two Settings

Before I summarize the interpretive framework that I adopted for my analysis, I need to discuss one more issue of importance in supporting learning of professional teaching communities, an issue brought to the fore by Zhao (2007). This issue concerns how we theorize the relationship between the teachers' learning in professional development settings and their instructional practices in their classrooms. For example, it is not obvious whether, how, and under what circumstances tools for professional analysis developed in professional development settings might become relevant to teachers' everyday instruction. A reciprocal problem concerns the question of making teachers' instructional practices readily available for their reflections and analyses in the professional development settings, that is, outside of their classrooms.

These two pragmatic concerns arise from the assumptions that (a) teachers' current practices should serve as valuable resources in their learning and (b) relations between teachers' activities across the settings of classroom and professional development should be *interrelated*. Zhao (2007) contrasted perspectives on professional development built on these underlying assumptions with both *uni-directional* conceptualizations that are typical of professional development programs that treated teachers' current practices as inadequate and in need of repair, and with *dichotomized* conceptualizations "in which what teachers learn in professional development and what they do in the classroom are divided along the lines of theory and practice" (p. 12, cf. Lampert & Ball, 1998). Zhao discussed how professional development efforts guided by uni-directional conceptualizations consider professional development sessions the site of teachers' learning, while they view classrooms as places of relatively straightforward application of teachers' new knowledge. In the case of dichotomized conceptualizations, "[t]he theoretical knowledge addressed in the professional development is presumed to support the emergence of more productive beliefs about teaching and learning, which are then expected to lead to fundamental changes in classroom practices" (p. 13).

In contrast, a number of more recent studies of teacher professional development—including the studies I discussed in prior sections of this chapter—rejected the linear, causal conceptualizations of changing teachers' instructional practices, in which teachers' learning would be considered to travel from professional development setting to the classroom. They instead argued that professional development should be situated in the context of teachers' classroom practices. To suggest how that could be accomplished in the professional development sessions that take place outside of teachers' classrooms, Ball and Cohen (1999) advocated for the use of "strategic documentation of practice," such as

> copies of students' work, videotapes of classroom lessons, curriculum materials, and teachers' notes … [These] could locate the curriculum of teacher education "in practice," for they could focus professional learning in material taken from real classrooms that present salient problems of practice (p. 14).

In this formulation, Ball and Cohen clarified that it is not necessarily a classroom practice in its entirety that the professional developers need to make a part of the professional development sessions. Rather, the teachers need to have access to "salient problems of practice" that could become a focus of communal discussions and analyses. Professional development activities that focus on salient problems of practice were conjectured to support teachers in making explicit the bases for instructional decisions involved in solving these problems, and in examining these bases critically (Barnett-Clarke, 2001).

Drawing on non-interventionist studies of teacher communities that she conducted, Little (2007) highlighted the role that teachers' accounts of classroom practice play in a community's ability to attend to problems of practice and to learn from them. She argued that in some instances, these accounts of practice "serve to render classroom practice with greater transparency and to identify important problems of practice for collective attention" (p. 221; cf. Little, 2003).

In her view, the importance of teachers' accounts of practice as a resource in their learning is increased by the fact that teaching is one of the occupations "in which members work largely out of sight and hearing of one another and must rely on narrative accounts and related material artifacts to construct a shared understanding of the technical and social character of the work" (Little, 2007, p. 220). Little also pointed out that given the relatively isolated nature of teachers' work in classrooms, "one might wonder about what specific aspects of work come to be visible through teachers' participation with one another and with what fullness and specificity" (Little, 2003, p. 918).

This last point is especially relevant for both design and analysis of professional development interventions that strive to make salient problems of practice available for teachers' examination: We need to remember the ambiguities involved in teachers' interpretations of accounts and records of classroom practice. When teachers participate in professional development activities that center on records of practice, they necessarily rely on what Goldsmith and Schifter (1997) call "useful images from their personal experiences" (p. 25). This yet again points to the importance of professional developers' understanding of teachers' current practices in order to anticipate the spectrum of teachers' possible interpretations and the subsequent opportunities for communal learning that could arise from them. It also suggests that in some cases, it might be useful to design specific professional development activities in order to provide the teachers with personal experiences of the kind that could fuel the subsequent professional development discussions if the teachers decided to draw on them. These suggestions stem from the belief that the question is not whether we should use records of practice as tools in professional development programs. Rather, it is what might be involved in using them effectively in supporting the learning of professional teaching communities.

Little's observations also suggest that when we conduct retrospective analyses of teachers' participation with records of practice in professional development activities, we need to understand both the teachers' *use* of these records as they attempt to accomplish certain goals, and the *meanings* that these artifacts come to have as a result of the teachers' activity (Kaput, 1994; Meira, 1995, 1998; van Oers, 1996). In this way, the records of practice are not seen as standing apart from the activity of teachers. Instead, they are a part of teachers' activity in professional development sessions. To the extent that they enable the teachers to pursue specific goals relevant to improving classroom instruction and students' learning, they become established as tools for professional analysis within the community.

A number of empirical studies have attempted to bridge between the professional development and classroom contexts by using records of instructional practice, such as student work (Chamberlin, 2004, 2005; Kazemi & Franke, 2004; Saxe, Gearhart, & Nasir, 2001), written cases (Barnett-Clarke, 2001; Doyle, 1990), and video and multimedia cases (Bencze, Hewitt, & Pedretti, 2001; Brophy, 2004; Richardson & Kile, 1999; Sherin, 2004; van den Berg, 2001). Frequently, these records of practice were conjectured to provide a means of supporting teachers in both re-constructing and examining instructional practices in professional development settings (Barnett-Clarke, 2001). Kazemi's (2004) discussion of student work and written cases illustrates how these artifacts were conceptualized in studies conducted from a situated perspective to serve as a means of supporting teacher learning.

> Both cases and student work allow teachers to focus on particular instances of interaction or reasoning in the classroom. At the same time, examining those particular moments presses teachers to elaborate the disciplinary knowledge required to make full sense of what happened or what students did. Moreover, discussions of these artifacts of practice can potentially initiate questions about the instructional context in which the work was produced or pedagogies which can advance students' thinking. Such professional inquiry can allow teachers to form conjectures about what their students are learning and to test out those conjectures when they return to their classrooms. This cycle of analysis and experimentation in the classroom can be generative, establishing norms for inquiry through which teachers can continually learn from their students and improve their practice (p.1).

As Kazemi explains, these studies used student work and cases to provide teachers with a common ground in activities where, in order to submit a specific problems of instructional practice to reflection and scrutiny, the teachers first needed to re-construct these problem situations outside of a classroom setting.

It is important to stress that Kazemi describes teacher-researcher interactions, in which participating teachers found it reasonable to see student work and cases as depicting instances of student reasoning in the classroom. This is an important clarification with respect to Little's (2003) and Goldsmith and Schifter's (1997) notes about the role that participating teachers' interpretations play in what kinds of instructional problems a group might re-construct as its members use records of practice in specific professional development contexts. In Kazemi's case, the researchers built on the spectrum of the teachers' interpretations of records of practice and succeeded in guiding the group to focus on students' reasoning. To accomplish this, both the spectrum of the teachers' interpretations and the facilitators' role in building on it towards the professional development agenda were of the utmost importance.

The episode reported by Dean (2005) in which the group of teachers focused on behavior of students in analyzed video and made jokes about it, further illustrates this point. As Dean documented, this teacher group did not have a means to interpret and analyze classroom instruction in a way that would make students' reasoning visible and relevant to what counted as a good instruction for the teachers at the time. Discrepancies of this kind between the professional developers' expectations and the actual participation of teachers might be relatively frequent when the problems of instruction that professional developers aim to target involve students' reasoning. This is because many US teachers' practices do not currently involve student reasoning as a focus of instruction (Carpenter et al., 2004). Cases like Dean's suggest that we should not expect that sophisticated ways of using records of practice (e.g., those discussed by Kazemi) would be viable at the outset of every professional development collaboration. It is therefore important that we seek to understand how records of practice can *become* an effective means for professional analysis as the teachers engage with them in the course of professional development.

Beyond Records of Practice

As illustrated above, the idea of situating professional development designs in the context of teachers' classroom practice has inspired promising approaches to supporting teacher learning. Zhao (2007) identified two broad types of approaches

> in terms of the specific manner through which the alignment is brought about between teachers' learning in the setting of professional development and their instructional practices in the classroom. The two approaches include (a) bringing the classroom into professional development and (b) carrying professional development into the classroom (p. 17).

As Zhao illustrated in the case of CGI studies, *bringing classrooms into professional development* approaches center on scrutinizing records of practice in professional development setting. In contrast, *carrying professional development into the classroom* approaches involve providing teachers with external resources to examine classroom instruction in the professional development setting, such as researcher-selected video recordings intended to exemplify students' thinking. Insights into classroom instruction that teachers developed through their engagement in professional development activities were conjectured to eventually lead to improvements in their classroom practices.

It is important to point out with Zhao (2007) that both these types of professional development design address primarily pragmatic concerns of professional developers related to professional development strategies that can best support teachers in connecting their learning to their classroom teaching. As Zhao commented,

> [t]o this end, researchers [who work within the interrelated conceptualization] explore a number of important questions that include: What are the central aspects of teaching that need to be addressed in professional development? What are the critical records of practice that have the potential to lead to meaningful learning opportunities? How should the discussion in professional development be structured in order to effectively address teaching? Answers to these questions, although undoubtedly significant, do not lend themselves to developing a fully elaborated theoretical framework through which a fuller, deeper explanation of teacher development can be generated (p. 22).

Zhao argued that the contributions of these studies remained mostly limited to innovating design strategies on the level of concrete ideas of how teacher professional development should be carried out. She called this level the *level of action* of professional development design and contrasted it with the more tacit and often unexamined *level of justification.* This latter level

> is typically composed of suppositions and assumptions held by researchers—for instance, how teachers learn, how teachers' learning in professional development relates to their classroom practices, what supports or hinders such learning, and what are the critical aspects in motivating changes in teachers' practices. This bottom level involves justifications that undergird the action plan, and ideally, gives rationality and unity to the seemingly discrete activities on the level of action. (pp. 5-6)

Zhao then went on to propose a foundation for conceptualizing the relations between teachers' learning in the professional development and their practices in the classroom on the level of justification that I now introduce. My motivation for doing so is to outline a theoretically grounded framework for conceptualizing teachers' learning within which I then situate my analysis.

Bi-directional Interplay

In proposing a framework for supporting teachers' learning across the settings of professional development and teachers' classrooms, Zhao and colleagues (2005; 2007; Zhao & Cobb, 2006) built on Beach's (1999) notion of *consequential transitions*. Beach defined transition as a "developmental change in the relation between an individual and one or more social activities" (Beach, 1999, p. 114), and clarified that

> [a] change in relation can occur through a change in the individual, the activity, or both. Transition, then, is the concept we use to understand how knowledge is generalized, or propagated, across social space and time. A transition is *consequential* when it is consciously reflected on, struggled with, and shifts the individual's sense of self or social position. Thus consequential transitions link identity with knowledge propagation. A college student becoming a teacher, a worker trying to adapt to a management-reorganized job, a middle school student doing well in math for the first time in his life, and high school students taking part time work in fast food restaurant are all potential examples of transitions that are consequential both for the individual and for the particular social organization (Beach, 2004, p. 2, italics in the original).

In case of teachers' learning, transitions occur as teachers shift from engaging in classroom teaching to participating in professional development activities, and vice versa. Zhao specified that these transitions can be consequential for teachers and their schools if and only if (a) teachers' participation in professional development sessions is oriented towards reworking their classroom practices, and (b) if their classroom teaching constitutes the context for them to make sense of, reflect on, and apply what they learn in professional development. Zhao adapted two methodological tools for purposes of both design and retrospective analysis of consequential transitions in the learning of professional teaching communities: *leading activity* and *developmental coupling*. Here I discuss relevance of these tools to conducting retrospective analyses of teachers' learning. I later illustrate the extent to which they inform my analysis.

Leading Activity. In illustrating these methodological tools, I draw on a study that Beach (1999) conducted to "understand how adolescents' and adults' arithmetic reasoning changed during transitions between school and work in a Nepali village" (p. 120). He contrasted two groups of individuals (a) shopkeepers who entered adult education classes to learn arithmetic that would be of benefit in their shops, and (b) graduating high school students who became apprentices to local shopkeepers and who learned arithmetic as part of their high-school instruction.

Beach illustrated that the high school students saw activities of schooling and shopkeeping as disconnected as they made a transition from school to work. When they developed and later used arithmetic strategies in the adult education class, they followed the conventions of written algorithms that they had been taught in school. However, as the shopkeepers engaged in transition between work and school, their participation in the adult education class was oriented by a leading activity of shop keeping. As a result, they developed a repertoire of arithmetic strategies that were directly relevant to running their business and disregarded superficial features of written algorithms that were not vital for their practical use. In other words, the students engaged in learning arithmetic as an isolated skill relevant to school whereas the shopkeepers attempted to develop arithmetical strategies that would enable them to be more effective in doing their job.

Teachers may engage in professional development activities for a variety of reasons and the prior illustration suggests that these reasons may be consequential to what these teachers learn. Common reasons for teachers' participation in professional development include a need to satisfy district requirements in order to be able to keep or progress in their jobs. Teachers might be driven to address this need by engaging in professional development activities where the leading activity is that of complying with district mandates in order to satisfy job requirements. In such cases, it is reasonable that teachers would treat professional development activities as unrelated to their classroom instruction. Another common reason for teachers' participation is their desire to improve their mathematical competences. In such cases, the leading activity might be that of learning mathematics. Leading activity of this kind might be equally problematic if the teachers remain focused on their own understandings and do not consider whether and how it might become a resource in supporting their students' learning (Ball & Bass, 2003; Borko & Whitcomb, 2008).

From the perspective of professional developers, it seems obvious that teachers need to approach professional development with the goal of improving their classroom practices if their learning is to be consequential for their classroom instruction. However, for many teachers this goal is not viable at the outset of their participation, since their views of goals for professional development are shaped by the institutional settings in which they work and by the specific demands they face. Therefore it seems important that in attempting to understand the learning of professional teaching communities and how this learning was supported, we attend to the *evolution* of members' leading activity in the course of their participation. Zhao (2007) reported that the process by which the leading activity is negotiated in professional teaching communities remains a black box in the teacher education literature. For this reason, documenting this process as part of the analysis is both pragmatically and theoretically important.

Developmental Coupling. Beach proposed the concept of coupling by drawing on Varela, Thompson, and Rosch's (1991) description of the changing relationships between individuals and the social activities in which they participate.

> The coupling itself is the primary unit of study and concern rather than the individual or the activity per se. The coupling assumes that individuals move across space, time, and changing social activities rather than being hermetically situated within an unchanging context. If a context does appear unchanging, it is because much collective effort is being put into maintaining it in place (Beach, 1999, p. 120).

Beach illustrated that neither the activity of shopkeeping nor that of formal schooling alone could satisfactorily explain the differences in forms of arithmetic reasoning that the Nepalese students and shopkeepers developed. Both groups engaged in formal schooling and both groups chose shopkeeping as their job. However, the different *transitions* among activities involved in these groups' learning usefully accounted for the differences in the arithmetic practices that these groups developed (see Figure 7). First, the shopkeepers moved between activities of schooling and shopkeeping when they participated in adult education classes with the goal of improving their shopkeeping skills. As a result, their participation as shopkeepers informed their interests as learners of arithmetic and enabled them to modify written algorithms to serve their shopkeeping needs. In contrast, the students first participated in activities of schooling and then used the arithmetic skills they had learned in this context when they participated as shopkeepers. They were reluctant to modify notational conventions that belonged to the world of schooling.

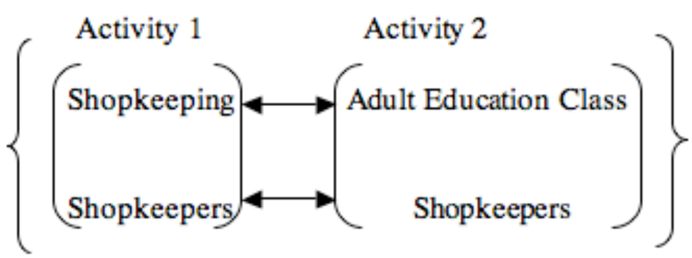

Figure 7a. Developmental Coupling as a Unit of Analysis: the Case of Shopkeepers

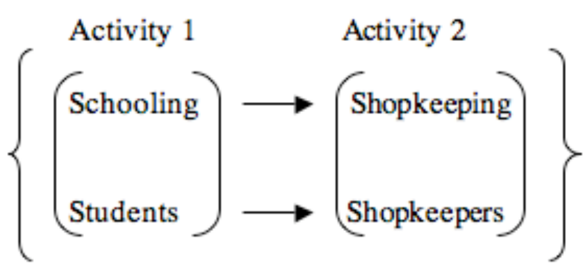

Figure 7b. Developmental Coupling as a Unit of Analysis: the Case of Students[11]

[11] Figure XX reprinted with permission from (Zhao, 2007).

Beach argued that these contrasting forms of arithmetical reasoning emerged in transitions between the activities and that for this reason, both activities involved in a transition should constitute the unit of analysis (Zhao, 2007).

Mathematics teachers develop new forms of reasoning that are relevant to their profession as they participate in multiple activities. A unit of analysis that brings to foreground not only multiple activities but also their transitions between them thus appears highly appropriate. By making transitions visible, the construct of developmental coupling facilitates a perspective within which teachers' learning in different settings is construed as a bi-directional interplay.

When documenting developmental coupling, Beach paid special attention to artifacts (e.g., notation systems in case of Nepali learners) that reify practices and transcend different social activities in which people participate. Zhao highlights the role of artifacts in documenting teacher learning and elaborates that using the construct of developmental coupling as a methodological tool

> implies that teachers' learning in professional development needs to be interpreted against the background of their classroom practices; likewise, changes in teachers' classroom practices cannot be sufficiently accounted for without an explicit reference to their learning in the setting of professional development. As an example, when using developmental coupling to analyze how teachers reason with records of classroom practice (e.g., student work, classroom tasks and assignments, video recorded episodes of student problem solving) in professional development, it becomes imperative to look at how similar artifacts or activities are constituted in context of classroom teaching. (Zhao, 2007, pp. 27-28)

When a bi-directional interplay framework is adopted, an analysis of teachers' use of classroom artifacts in the professional development setting is no longer sufficient (Goldsmith & Schifter, 1997; Little, 2003). Instead, it becomes necessary to consider teachers' histories of participation with these artifacts across different activities and different settings that contributed to teachers' construction of meaning.

In adopting a bi-directional framework of teacher learning and using developmental coupling as a unit of analysis, Zhao highlighted conceptual (rather than solely pragmatic) rationales for drawing on analyses of teachers' institutional context and classroom instruction to inform analyses of teachers' learning in professional development settings. In doing so, she addressed the questions of teachers' learning across two settings on the level of justification. A framework of this kind is important if we are to understand the consequences of teachers' participation in professional development activities for their classroom instruction and thus the mathematical learning of their students.

Bi-directional Interplay and Studies of Professional Development Programs

I have discussed a number of issues relevant to design and analysis of professional development programs on the level of action (e.g., use of records of practice). I will now illustrate how the theoretical framework of bi-directional interplay proposed by Zhao (2007) enables us to bring the pragmatic concerns to the fore on the level of justification.

The bi-directional interplay framework builds on situated perspectives on learning and suggests that the focus on transitions between activities in professional development and in classroom in which teachers participate concurrently is the most appropriate unit of analysis of teacher learning. This is because, as Zhao convincingly argues, if we are to learn about the shifts in teachers' professional development practices that are *consequential* for their classroom instruction, we need to understand these shifts in relation to teachers' transitions between the two settings. In the remainder of this section I first outline the complex system of analyses implied in bi-directional interplay conceptualization that would fully account for teachers' learning across two settings (Figure 8). I then propose an adaptation of this conceptualization intended to provide guidance to studies that primarily draw on data from teacher professional development, as is the case in my analysis (Figure 9).

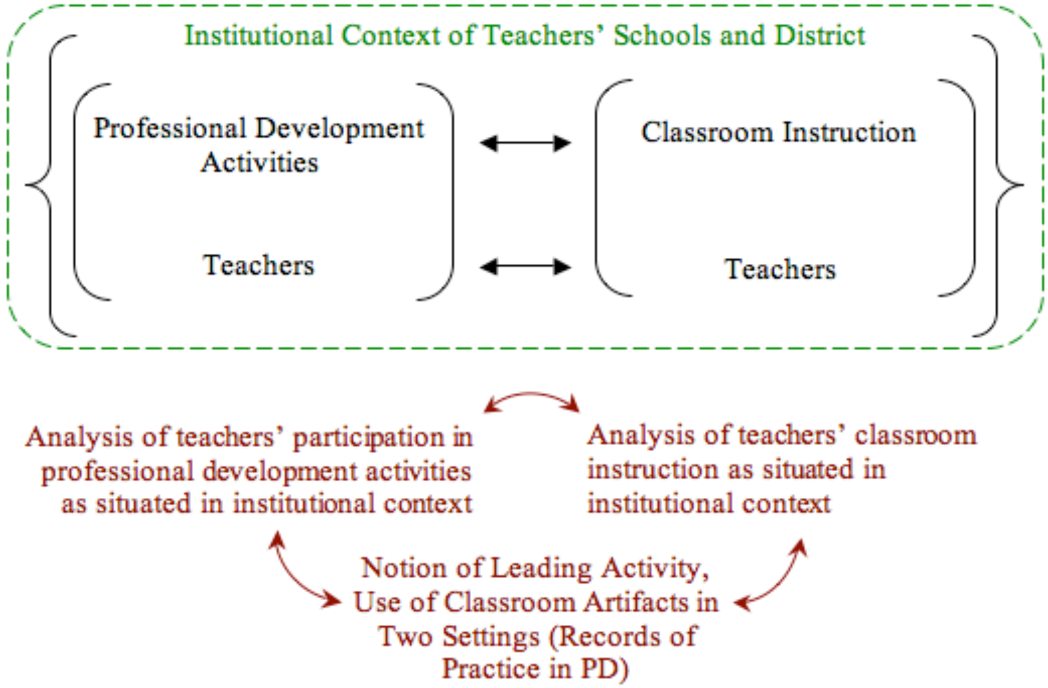

Figure 8. Developmental Coupling as a Unit of Analysis when Teacher Learning is Conceptualized as a Bi-directional Interplay.

Given the ample evidence of the impact of institutional context in which teachers work on their participation in both professional development sessions and their classrooms (Franke et al., 2008; Franke et al., 2002; Goos, Dole, & Makar, 2007; Zhao, McClain, & Visnovska, 2007), I find it imperative to view teachers' participation in both professional development and classroom settings as situated in the institutional context of their schools and the district. The bi-directional interplay conceptualization indicates that three types of analyses are useful in accounting for teachers' learning: (a) analysis of teachers' participation in professional development activities as situated in institutional context, (b) analysis of teachers' classroom practices as situated in institutional context, and (c) analyses intended to highlight the nature of teachers' transitions between the settings such as analysis of leading activity and of teachers' use of classroom artifacts (see Figure 8). My goal in specifying these different types of analyses is not to suggest that meaningful contributions about teachers' learning can only be made when researchers account for all aspects of bi-directional interplay framework. Instead, I outlined these analyses to explicate the sources of explanation that this conceptualization makes visible, and to suggest that these need to be anticipated also in partial analyses of teachers' learning (e.g., in analysis of teachers' participation in professional development). I further elaborate this assertion by outlining an adaptation of bi-directional interplay framework geared to analyses that primarily draw on data from professional development sessions.

It is important to clarify that in the case of many analyses attempts to address the broad landscape proposed by the bi-directional interplay framework would be neither manageable nor the most useful. In case of my analysis, this is because the overarching goal is to contribute to development of a domain-specific teacher development theory. It is my priority to understand shifts in normative practices of the professional teaching community and how these shifts were supported by the research team in professional development sessions. However, because I am interested in learning about those shifts in teachers' professional development practices that are *consequential* for their classroom instruction, I need to understand these shifts within the transitions inherent to bi-directional interplay conceptualization. One possible solution that supports a focus on professional development sessions without departing from the bi-directional interplay view involves capitalizing on complementary formal and informal analyses conducted in the same study setting. Figure 9 illustrates how complementary analyses conducted as part of the study at Jackson Heights District allow for adaptation of bi-directional interplay framework:

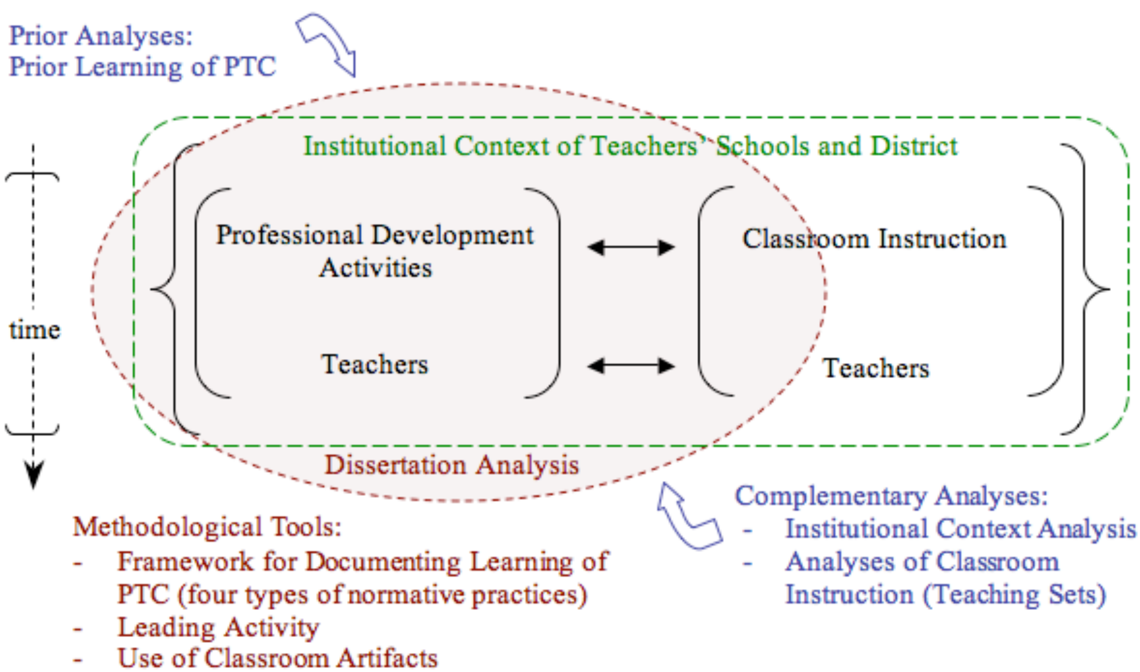

Figure 9. Delineation of the Dissertation Analysis within the Broader Analytical Framework of Bi-directional Interplay

The complementary analyses—the analyses of institutional context (P. Cobb, McClain, Lamberg et al., 2003; Lamberg, 2004) and the yearly formal and informal ongoing analyses of teachers' instructional practices[12] (e.g., Zhao et al., 2006, design log)—can inform my analysis of teachers' participation in professional development sessions in a manner that most closely, given the data, approximates the analysis of teachers' transitions. In addition, we designed the learning trajectory for the professional teaching community so that reworking of classroom instructional practices would become established as a leading activity in the group. As a consequence, a wealth of evidence about teachers' classroom instruction became available in professional development sessions in years 3-5. For example, the teachers' classroom video recordings that the community analyzed, as well as teachers' contributions in these analyses provided a window into teachers' instructional practices.

I would like to clarify that in drawing on partial and informal analyses, the goal is not to treat these as a comprehensive replacements for formal accounts of teachers' classroom instruction. Instead, the goal is to make a researcher's understandings of teachers' instructional practices explicit along with the evidentiary basis on which those understandings rest, and to use them as a resource in determining the nature of teachers' transitions across the settings. Even though my data comes almost exclusively from the professional development sessions, complementary analyses and records of classroom practice provide the insights necessary for adopting the bi-directional interplay conceptualization of teachers' learning.

Summary: Analysis Framework

In this section, I summarize the lessons learned from the prior research and in doing so, elaborate the analytical framework that I will use to conduct the retrospective analysis.

Dean's (2005) analysis of the initial two years of teachers' participation in professional development provides the starting point for my analysis. Additionally, the envisioned end points for the normative practices of reasoning about mathematics and pedagogy in the community were compatible with Simon and colleagues' (2000) proposition that teachers should develop a

[12] The analyses of teachers' instructional practices built on Simon and colleague's (1999) notion of accounts of practice and treated teachers' instruction as reasonable from their perspectives. In addition, they treated instructional practices as situated in institutional context of teachers' schools and the district. Detailed treatment of these analyses is beyond the scope of this dissertation.

conception-based perspective on learning in which students' current forms of reasoning form a basis for what they can see, learn, and understand. The distinction between perception-based and conception-based perspectives that teachers may hold about teaching and learning mathematics therefore provides a guidance in interpreting the broad shifts in teachers' participation over time.

To trace shifts in normative practices of the professional teaching community on what I called meso level of analysis, the interpretive framework developed by Dean (2005) proved to be most suitable. This specifically involves documenting changes in normative practices and joint enterprise of the professional teaching community, while viewing teachers' participation in the professional development sessions as reasonable from their perspective and situated in institutional context of their schools and the district. It also involves documenting the means by which the learning of the community was supported, such as records of practice used in professional development activities and the statistics instructional sequence.

Two methodological tools appear suitable for keeping the bi-directional nature of teachers' learning in the picture even when the analyzed data are constrained to professional development sessions. First, the bi-directional interplay framework necessitates that attention is paid to leading activity in teachers' participation in professional development sessions. With respect to my analysis it is of interest to understand how leading activity is negotiated in the professional teaching community across time. Second, when examining the teachers' use of records of practice such as student work and instructional tasks in professional development activities, it is imperative to draw on complementary analyses in order to understand what kinds of instructional practices might these artifacts represent for the teachers across the classroom and professional development settings.

Given that this approach to analysis is well aligned with Dean's analysis of the early developments in the group, the two analyses combined contribute to a domain-specific teacher development theory of supporting teacher learning.

CHAPTER IV

METHODOLOGY

In previous chapters, I introduced the overarching questions and conjectures that oriented the design-research study that produced the data I analyze. I now describe the data and explain how I conducted the analysis. While discussing the data that I have analyzed, I argue that it allows me to both document the actual learning of the teacher group and the means by which this learning was supported. I then discuss how the adopted analytical framework guides me in accomplishing each of the two analysis subgoals. Lastly, I outline the specific ways in which I approach the analysis and discuss the trustworthiness of findings generated in this way.

Data Collection

Teachers' instructional practices were documented twice a year by generating a modified teaching set (Simon & Tzur, 1999) for each participating teacher. A modified teaching set entailed videotaping each teacher's lesson and then conducting follow-up audio-recorded teacher interviews that focused on issues that emerged in the course of instruction. The observed classroom session served as a context within which the teacher could be oriented to address issues that were of relevance to the researchers' overarching goals. Our purpose in collecting teaching sets (as opposed to classroom observations or teacher interviews alone) was to understand teachers' practices, including their rationales for making decisions, and their accounts for successes and difficulties in their instruction. The interview questions were therefore purposefully grounded in concrete episodes from the teachers' classroom instruction, instructional planning, and real-time adjustments. Specifically, we asked the teachers

- to characterize the class they were teaching,
- to describe instructional concerns they had when they planned the lesson,
- what they anticipated would be easy or difficult for their students in this lesson and how they planned to support their students in overcoming the difficulties,
- to what extent their expectations were realized in the lesson,
- whether they needed to change or adjust some parts of the lesson and why,

- how they made decisions about the pacing of the lesson, and
- how the lesson fit in what happened in the previous lesson and how would they proceed in the coming lesson.

We also asked the teachers how they decided on ways to organize and structure the observed lesson (e.g., as individual student work, small groups, whole class discussion), and what students needed to do in order to be successful in their mathematics class. The interview data collected in this way allowed us to interpret teachers' responses within the context of their classroom instruction, and helped us understand the key instructional goals and struggles the teachers faced. Issues that related to the institutional context of teachers' work often surfaced in these discussions. In some interviews we also talked to the teachers about their concerns and valuations of their participation in our professional development sessions.

The primary purpose of the teaching sets was to develop an understanding of teachers' instructional practices that could inform planning for the further learning of the professional teaching community. Retrospective analyses were not systematically conducted from these data. Nevertheless, it is important that I discuss the details of this data corpus here. As I explained in Chapter III, making explicit the basis on which the research team members developed understandings of the teachers' instructional practices is intended to provide the reader with a means to assess the plausibility of these understandings.

Several members of the research team worked together to collect the teaching sets. I took part in this data collection in years three through five. During the interviews, we were cautious about the social context that was co-constructed between the interviewee and the interviewer and strove to ensure that the teachers did not perceive themselves as being evaluated or feel their professional status threatened. The interviewers met after each day of interviews to formulate tentative interpretations of the collected data, evaluate and revise the previous conjectures, and possibly formulate new ones. When needed, the semi-structured interview protocol was adjusted to include issues emerging as potentially significant.

The regular professional development sessions during the school year were held either at the district professional development center or at one of the middle schools in the district where the participating teachers worked. The summer session during the third year was held at the research team's university. The last two summer sessions were held at a convention center in the city where the teachers worked. We video- and audio-recorded all work sessions using two video cameras and three audio-recorders. Audio-recordings were particularly useful when the teachers worked in several groups. A set of field notes generated during the sessions by one member of the research team (usually a post-doc) accompanied the video and audio data. After each session, the person who originally created the field notes for the session produced a summary of the field notes. Data also included copies of all material artifacts used or produced by the teachers during work sessions such as copies of students' work the teachers analyzed, their written analyses of statistics tasks, group-work posters, individual reflection sheets, and chart paper records of ideas and issues raised by the teachers during whole group discussions.

Because my goal was not limited to documenting the learning of the professional teaching community but also focused on the means of supporting that learning, another data source was significant in my analysis: the log of the research team's ongoing conjectures. The log was compiled throughout the professional development collaboration to document the learning of the research team. It consists of a record of ongoing conjectures about the teachers' learning and audio-recordings of all research-team debriefing meetings that were conducted after each professional development session. During these meetings, members of the research team shared and debated their interpretations of professional development session events in order to plan for the subsequent sessions. The resulting log specifies episodes that provided an evidential basis for revising ongoing conjectures about the teachers' learning and means of supporting it. It also contains design rationales for all planned activities.

As a member of the research team, I was involved in all aspects of professional development design and research. This included contributing to the development of the semi-structured interview protocols, conducting interviews and classroom observations, participating in ongoing analyses and design of professional development activities, and leading selected activities during the professional development sessions. As a member of the team, I was also responsible for composing field notes, making the video and audio recordings, and collecting and organizing teacher's work.

Data Analysis

The first goal of my analysis was to specify the process of the collective learning of the community and the means by which it was supported in order to understand how the documented changes could be replicated with other groups of teachers. The second goal was to understand the role of tools in supporting learning of the professional teaching community. I explain what pursuing both of these goals entailed and then discuss the specific methods I used to analyze the data.

To develop an account of the actual learning trajectory for the professional teaching community, I capitalized on the framework developed by Dean and colleagues (P. Cobb, McClain, Lamberg et al., 2003; Dean, 2004, 2005), which situates the documented learning of a teacher community in the institutional context of teachers' schools and the district. I took Dean's account of the learning of the professional teaching community at the end of year 2 to establish the starting points for the group's further learning. The purpose for summarizing the starting points was to understand the resources on which we could have capitalized at the beginning of year three, as we worked towards the envisioned endpoints. I include a summary of these starting points in Chapter V. Against the starting points, I then identified the successive norms that became established in the community for (a) general participation, (b) pedagogical reasoning, (c) mathematical reasoning, and (d) institutional reasoning. I discuss the method for documenting these norms shortly. The resulting actual learning trajectory of the professional teaching community accounts for the difference between the starting points and the actual end points of the community learning.

Along with the successive norms for mutual engagement in the community, I analyzed the artifacts that served as additional means of supporting the learning of the professional teaching community. To understand the role that classroom-related artifacts (such as classroom video or instructional materials) came to play in supporting the learning of the professional teaching community, I documented the ways of using and reasoning with artifacts that became normative in the group at different points in time. Consistent with Zhao's framework for understanding teachers' learning as a bi-directional interplay (Zhao, 2005, 2007; Zhao & Cobb, 2006), my primary interest is in the contributions of classroom artifacts to the process of negotiation of meaning (Wenger, 1998) around various aspects of classroom instruction. I therefore analyzed developments in the practices of the community across the three-year period to understand (a) the meanings that the classroom artifacts represented in the community when they were first used in a professional development activity, (b) how the meanings of the artifacts evolved as community members continued to use them to advance the shared purpose, and (c) whether and how the artifacts supported reorganization of teachers' mathematical and pedagogical reasoning. In this analysis, I focused on the teachers' activity with the artifacts rather than on the characteristics of the artifacts alone.

The two goals combined enabled me to contribute to a teacher development theory of supporting the learning of a group of middle school mathematics teachers in the domain of statistics. The analysis of the use of artifacts furthers our understanding of how the learning of the group of teachers was supported. It also substantiates the theoretical conceptualization of teachers' learning as a bi-directional interplay.

Method of Analysis

My analysis of the data collected to document the learning of the professional teaching community involved a method described by Cobb and Whitenack (1996) for analyzing longitudinal data sets of the type generated during design experiments. This method is a variant of Glaser and Strauss's (1967) constant comparative method and is specifically tailored to the *systematic* analysis of longitudinal data sets in mathematics education. The distinguishing feature of their method is that as new episodes are analyzed, they are compared with currently conjectured themes or categories. This process of constantly comparing episodes leads to the ongoing refinement of theoretical categories, which remain grounded in the data. As Glaser and Strauss note, cases that appear to contradict a current category are of particular interest and are used to further refine the emerging categories.

Analysis of Norms of Mutual Engagement

It is important to clarify that norms are identified by discerning patterns or regularities in the ongoing members' interactions. A norm is therefore not an individualistic notion but is instead a joint or collective accomplishment of the members of a community (Voigt, 1995).
A primary consideration when conducting an analysis of changes in norms of a professional teaching community is to be explicit about the types of evidence used to infer that a particular norm has been established so that other researchers can monitor the analysis. A first, relatively robust type of evidence occurs when a particular way of reasoning or acting which initially has to be justified is itself later used to justify other ways of reasoning or acting (Stephan & Rasmussen, 2002). In such cases, the shift in the role of the way of reasoning or acting within an

argument structure from a claim that requires a warrant, to a warrant for a subsequent claim provides direct evidence that it has become normative and beyond justification.

A second, robust type of evidence is indicated by Sfard's (2000) observation that normative ways of acting are not mere arbitrary conventions for members of a community, which can be modified at will. Instead, these ways of acting are value-laden in that they are constituted within the community as legitimate or acceptable ways of acting. This observation indicates the importance of searching for instances where a teacher appears to violate a proposed communal norm in order to check whether his or her activity is constituted as legitimate or illegitimate. In the former case, it would be necessary to revise the conjecture whereas, in the latter case, the observation that the teachers' activity was constituted as a breach of a norm provides evidence in support of the conjecture (cf. P. Cobb et al., 2001).

Finally, a third and even more direct type of evidence occurs when the members of a professional teaching community talk explicitly about their respective obligations and expectations. Such exchanges typically occur when one or more of the members perceive that a norm has been violated.

Analysis and Professional Teaching Community Membership

The research team members[13] were, along with the teachers, central members of the professional teaching community. We participated in the community activities while actively pursuing an agenda for teachers' learning[14]. We shaped the activities and discussions, in the process influencing the goals the group pursued, tools used to pursue them, and normative ways of interacting. There were times at which we intentionally held back from the group discussions and other times when we proactively shaped what was talked about. Because of the interventionist nature of our work with the teachers, it is worth clarifying that the goals and practices we actively promoted as part of our agenda for teachers' learning only became *established in the*

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[13] The typical constellation of research team members who regularly participated in professional development sessions with the teachers included Paul Cobb, one post-doc, and three doctoral students. In years 3-5, the post-docs were Teruni Lamberg (years 3 & 4) and Melissa Gresalfi (year 5), and graduate students were Qing Zhao, Lori Tyler, and myself. Additional team members joined the group during summer institutes.

[14] It is worth clarifying that besides participation in the professional teaching community, we – the researchers – were members of another community of practice that is relevant to my analysis: the community that we constituted as members of the design research team. While we participated in the activities of the professional teaching community, we also pursued a research agenda. Within the design research community of practice, we pursued goals related to investigating how to best support further learning of the professional teaching community.

community at the point when they became normative in the teachers' interactions. It is for this reason that in the analysis of norms of mutual engagement, I foreground the teachers' participation while the researchers' participation often remains in the background, or is discussed as part of the means of support.

As I explain in Chapter V in more detail, district mathematics leaders Ruth and Esther participated as members in the professional teaching community during years three through five (Lamberg, 2004). Ruth attended all the professional development sessions and Esther visited more than half of the one-day sessions for approximately two hours each, and participated during the summer sessions. For purposes of the analysis, I included Ruth's and Esther's participation in professional development sessions with that of the teachers. Specifically, whenever Ruth or Esther participated in a professional development activity together with the teachers (i.e., contributed at least one comment in whole group discussion, or were part of small group activities), they are included in the "teacher" counts. Whenever differences in the teachers' and district mathematics leaders' participation became evident, I took note. In the report of my analysis, I include the differences that proved consequential to the learning of the community.

Three Phases of Analysis

The first phase of documenting the actual learning trajectory of the group entailed (a) reading the log of ongoing conjectures to identify specific goals and plans for each professional development session, and (b) watching videos and reading through the field notes of the work sessions in chronological order to create a record of the work sessions organized by theme of conversation or activity. In other words, I documented the major issues that emerged episode-by-episode, where the determining characteristic of an episode is that a particular mathematical or pedagogical theme was the focus of the teachers' and researchers' public activity and discourse. In this phase, my purpose was to index what happened in the sessions without differentiating between episodes in terms of how well they might reveal normative practices of the group. My intent was to develop a broad overview of what transpired in the work sessions and to create an organizational structure and brief summaries to facilitate subsequent analysis of the data. In documenting the episodes, I kept track of specific classroom artifacts that the researchers conjectured would serve as tools for supporting the teachers' learning.

In the second phase of the analysis, I worked through the entire data corpus generated during the last three years of the collaboration in chronological order. I first made summaries of the work-session goals that we pursued for each school year of the collaboration in order to understand relative importance and challenges involved in pursuing different goals for the teacher professional development. Against this understanding of the pursued goals, I identified episodes that contained direct evidence of the four types of norms as they were established within the group at the time. In addition, I identified episodes that provided evidence for the ways of using classroom artifacts that became normative in the group, and the goals the group pursued in using them. As a result, I supplemented the overview created in the first phase by providing detailed descriptions of the identified episodes and transcripts of the illustrative examples for kinds of evidence identified in the video and audio data. I then formulated, tested, and refined conjectures about (a) the evolution of the four types of norms as they were successively established within the group, (b) how this evolution was supported by the designed tools and activities, and, (c) how teachers' use of tools and classroom artifacts changed as the norms evolved. For each claim, I substantiated it with further evidence, refuted, or modified it while I analyzed subsequent episodes. Two members of the research team read the summaries of this stage of the analysis, checked for inconsistencies with their notes and recollections of the developments in the group, provided additional comments, and asked for elaborations. This phase resulted in a chain of conjectures, refutations, and revisions, which are grounded in the details of the specific episodes.

In the third phase of the analysis, I analyzed and reorganized the resulting chain of conjectures and refutations from the second phase. Two types of patterns became apparent in the evolution of normative practices of the group: (a) *periodic* re-constitution of general participation norms and norms of institutional reasoning in the community at the beginning of each year when new members joined the group, and (b) *progressive* evolution of norms of mathematical and pedagogical reasoning that spanned across the years of collaboration. These patterns gave rise to two strands of claims. The first strand of claims, which I report in Chapter VI, documents the continuation of the community across years and changes in membership. The second strand of claims, which I report in Chapter VII, foregrounds the pedagogical and mathematical learning of the professional teaching community and how this learning was supported. In this phase of analysis, I further searched for evidence to disprove the identified patterns in evolution of normative practices. In the following paragraphs, I discuss methods specific to each strand of analysis.

Analysis of Continuation of the Community

In phase two of the analysis I corroborated my claims about the evolution of general participation norms and norms of institutional reasoning at the beginning of each year by re-analyzing data to compare participation patterns of the groups of the newcomers[15] and the old-timers in the community. In this analysis, I built on work of Stein, Silver, and Smith (1998) in the QUASAR[16] project. They proposed using the construct of *communities of practice* (Lave & Wenger, 1991) "as a theoretical framework for describing how teacher learning occurs in collaborative, school-based communities" (p. 48). Stein and colleagues elaborated how the construct of *legitimate peripheral participation* helped them understand ways in which newcomers learned as they participated in increasingly central ways in activities of the community, and how formation of identity as a reform teacher was an inherent part of their leaning.

To understand whether the group of teachers continued to function as the community of practice after inclusion of the newcomers, I first looked for evidence that would suggest (a) whether norms of general participation were explicitly negotiated with the newcomers, (b) whether old-timers participated in normative ways, thus modeling participation for the newcomers, (c) whether the newcomers participated in normative ways or breached the norms of the community, and (d) how what I identified as a breach of a norm was constituted in the group (i.e., whether it was constituted as a breach). It was my conjecture that if the group continued to function as a professional teaching community, the general norms of participation would be re-generated in the newcomers' interactions gradually, but in relatively short period of time. I looked for evidence of the newcomers voicing disagreement (rather than pseudo-agreement), actively making sense of discussions, and building on others' arguments in their contributions.

With respect to the norms of institutional reasoning, I examined issues related to deprivatization of newcomers' practices. Supporting deprivatization of teachers' practices was a major challenge in establishing the professional teaching community with the original group of nine teachers in the first two years. I was therefore particularly interested in whether the newcomers

[15] In this dissertation, I use "newcomers" and "old-timers" to refer to the teacher members of the professional teaching community. This is because to infer group norms, I focused on teacher interactions that were becoming normative.

[16] QUASAR stands for Quantitative Understanding: Amplifying Student Achievement and Reasoning, a national educational reform project which aimed at fostering and studying the development and implementation of enhanced mathematics instructional programs for students attending middle schools in economically disadvantaged areas.

made their practices available for group purposes, and, if so, how the old-timers and research team supported deprivatization of the newcomers' practices.

Dean (2005) reported that the evolution of normative practices during the first two years differed at times with respect to the type of activity in which the teachers engaged. For example, the teachers still interacted as a pseudo-community (e.g., never interrupted each other) when they were engaged in activities dealing with pedagogy (e.g., discussion of their students' work) during year one, session three. However, when they were engaged in statistical data analysis, they built on others' contributions and directed their comments to each other, not the researcher. I therefore conjectured that while the newcomers might have participated similarly to the old-timers in some professional development activities, their participation might have differed significantly in others. To identify activities in which the newcomers' participation was different from that of the old-timers, I first looked at the relative *frequency* of the newcomers' contributions to the group discussions. I generated exact participation counts for the group of the newcomers and the old-timers in those cases in which the newcomers' lack of participation was noticeable. In generating the counts, I counted every uninterrupted participant's comment or question as a contribution to the discussion. Given that we designed the professional development activities with the intention of providing the newcomers with ways to actively participate and contribute (e.g., ask questions) from the very beginning, I conjectured that the relative frequency of newcomers' contributions should soon become proportionally similar to that of the old-timers[17].

It is important to clarify that while some newcomers and old-timers were talkative, others contributed less frequently but often indicated their intellectual presence throughout the debate. For this reason, comparing counts of individuals' contributions would be a poor indicator of the extent of their participation. However, sum of the utterance counts across the group of newcomers relative to that of the group of old-timers[18] indicated whether the newcomers' participation in the activity was adequately supported. I compared the utterance counts of the two groups to indicate the extent to which the newcomers as a group (a) had access to the task at hand and the

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[17] I weighted the counts against the numbers of the newcomers and the old-timers who were *actually present* during an analyzed activity. I report the counts for an activity as a ratio that signifies (number of old-timers' contributions per a participating old-timer : number of newcomers' contributions per a participating newcomer), for example (7.5 : 8.2).

[18] In years three and four, both groups (the newcomers and old-timers) were sufficiently large to warrant this comparison. Six newcomers and six old-timers participated in year three, and four newcomers and eight old-timers in year four.

means to address it, and (b) were positioned as a resource during the whole group discussions. Opportunities for the newcomers to make contributions and pose questions are critical to their participation becoming more central. This is because such opportunities facilitate newcomers' development of both community-specific competencies, and identities as valued members of the community (Stein et al., 1998). I therefore used the relative participation counts as an indicator of the types of professional development activities in which the newcomers might not have had access to legitimate participation. I then further analyzed these activities to understand the reasons for disparities in the newcomers' and old-timers' contributions and how the newcomers' participation could have been better supported.

To further understand differences in the newcomers' and the old-timers' participation, I identified episodes in which the newcomers' *ways of reasoning* about mathematical and pedagogical situations differed from those that were normative among the old-timers. In particular, I looked for situations in which the group members noticed differences in interpretations and engaged in negotiations of meaning. Such situations constitute a variation of the first type of evidence that a norm is being established, in which meanings that have previously been constituted as normative are challenged and must be re-negotiated in the whole group discussion.

I used this methodology to analyze continuity of the professional teaching community at the beginning of years three and four, when the ratios of newcomers to old-timers present were relatively high (6 : 6 and 4 : 8 respectively, when all teachers were present in the session). I conjectured that the lower proportion of newcomers in the group in year four might contribute to a more seamless re-constitution of the norms of the professional teaching community. To better understand the induction process, I looked for patterns that spanned these two years.

In the last year of collaboration, only two new teachers were invited to join the community, which, at the time, included 10 old-timers. Moreover, one of the newcomers participated in only three work sessions before she left teaching. The small proportion of newcomers in the group made it reasonable to expect that there would be no significant discontinuities in the normative practices of the professional teaching community. In the data, I checked for indications whether the new teacher who remained in the group was included in the professional development activities and provided access to the goals, purposes, and justifications for community decisions. I found that she did become more central member of the community over time. I have not included this year in the analysis of continuity of the professional teaching community.

As I will document in Chapter VI, the norms for general participation were re-established in the community during the first three sessions after the new teachers joined the group. Overall, these norms (such as obligation to ask for clarification and respectfully disagree) appeared to remain relatively stable across years three through five, as did the norms of institutional reasoning. In contrast, norms for mathematical and pedagogical reasoning continued to evolve as the teachers participated in the activities of the community. These were the focus of the second analysis presented in this dissertation.

Analysis of Professional Learning of the Community

In phase two of the analysis, I corroborated my claims about the evolution of norms of institutional, mathematical, and pedagogical reasoning in relation to specific professional development activities and in relation to classroom artifacts used in those activities. In phase three, I documented *progressive* evolution of norms of pedagogical and mathematical reasoning that spanned last three years of collaboration. Norms of institutional reasoning remained relatively stable in years 4 and 5, and thus I focused on their evolution only in year three. In conducting the analysis, I drew on the three types of evidence described in detail above.

Pedagogical reasoning. I documented the norms that became established as the teachers both reflected on their own and others' classroom practices, and planned for instruction. I primarily attempted to understand the shifts in the role of *students' reasoning* in instructional planning and decision making as it played out in the teachers' activity. On the broad level of analysis, my goal was to understand whether, to what extent, and in what situations the contributions made within the group indicated the teachers' development of a conception-based perspective on their students' learning.

Mathematical reasoning. Shifts that occurred in the normative ways of mathematical reasoning established by the group during first two years pertain to how the teachers *themselves* reasoned statistically (Dean, 2005). The forms of statistical reasoning developed by end of year two provided an adequate resource in subsequent pedagogical discussions and in teachers' classroom instruction, at least in the context of univariate data analysis. For this reason, supporting teachers' statistical reasoning per se was not an explicit part of the professional development agenda in years 3 through 5. Nonetheless, important shifts did occur in normative ways of mathematical reasoning that pertain to what Ball & Bass (2003) refer to as mathematics for teaching. I focused on documenting the normative meanings constituted by the teacher group for what is a *statistical activity*, and how these meanings influenced how they organized lessons.

I accounted for the means that supported the emergence of normative practices by documenting instances where some of the teachers started to reason in ways that were novel within the teacher group. I interpreted instances of this kind as an indication that the tasks and other means supported the emergence of a new diversity in teachers' contributions that made possible the subsequent emergence of a new normative practice. When I illustrated new forms of individual reasoning within the community, I provided counts to illustrate the extent to which such reasoning was typical in the professional development discussions. I either indicated how many teachers' contributions reflected the new form of reasoning or how many provided an evidence of not having access to this form of reasoning. I then documented the process by which a particular way of reasoning became normative after some of the teachers began to reason in this way.

Two aspects of the final phase of the analysis are noteworthy. First, four types of norms did not evolve independently. Dean (2005) documented the inter-related nature of the four types of norms and described how shifts in one type of norm can create affordances for shifts in another type of norm. Although I first analyzed different norms separately, it proved critical to understand them as an interdependent system. For example, the views of statistical activity that were normative in the group at certain point (i.e., mathematical reasoning) played out in ways that the teachers interpreted and talked about both classroom videos and purposes for organizing a statistical activity in specific ways in the classroom (i.e., pedagogical reasoning).

Second, I drew on the prior analysis of the institutional setting (P. Cobb, McClain, Lamberg et al., 2003) and on informal analyses of teachers' classroom practices (e.g., Zhao et al., 2006) to situate the chain of conjectures and refutations from the second phase. This helped me further elaborate explanations of the teachers' participation in professional development sessions and eliminate competing conjectures. The final product of the third phase of analysis is thus an empirically grounded account of the learning of the professional teaching community that consists of a network of mutually reinforcing assertions that span the entire data set.

Generalizability and Significance

One of the products of the cyclic process of design, analysis, and revision is a sequence of tasks and artifacts that constitute a means of establishing and sustaining professional teaching communities as sites for teachers' continuous professional learning. Although this sequence of tasks is inherently specific to the setting in which it was developed, it is accompanied by an explanatory framework (Steffe & Thompson, 2000). The explanatory framework serves two important functions: (a) It justifies the sequence in terms of a substantiated trajectory for teachers' learning, and the documented means of supporting learning along that trajectory (P. Cobb, McClain, & Gravemeijer, 2003), and (b) it enables researchers and teacher educators to adjust the sequence to the specific circumstances of sites in which they are working. It is in this sense that the explanatory framework, capable of informing the interpretation of events and thus the researchers' planning and decision-making in different settings, allows for generalization of design research findings (Steffe & Thompson, 2000).

While a new site with similar characteristics will require relatively small adaptations to the sequence, it is reasonable to expect that a site with significantly different characteristics will require that further design research will be conducted. The results of my analysis most adequately address the question of supporting groups of middle-school mathematics teachers in urban school districts, in which administration responds to the accountability pressures of state standardized tests by attempting to monitor and regulate teachers' instructional practices. These results also inform development of *initial* designs for different sites where conjectures inherent in the design will need to be tested in the course of ongoing work with teachers.

The analysis is pragmatically useful in that it provides relatively detailed insights into the learning of a professional teaching community and the means by which that learning was supported. The major theoretical contribution of the explanatory framework is to the development of a teacher development theory in the domain of middle-school statistics.

Trustworthiness

The analysis of the teachers' learning is trustworthy because (a) the method of analysis was systematic and involved refuting conjectures, (b) the criteria for making claims are explicit, thus enabling other researchers to monitor the analysis, (c) final claims and assertions can be justified by backtracking through the various phases of the analysis, if necessary to video-recordings and other data sources, and (d) the analysis was critiqued by other researchers, some but not all of whom were familiar with the setting from which the data was generated.

CHAPTER V

PREVIOUS LEARNING OF THE GROUP

The purpose of this chapter is to summarize the findings on (a) the institutional setting of the Jackson Heights schools and the district documented by Cobb and colleagues (2003), and (b) the norms of mutual engagement established in the professional teaching community by the end of the second year as documented by Dean (2005; 2006). This summary provides a baseline for my analysis of the years three through five of the collaboration.

Institutional Setting

As I discussed in Chapter III, Cobb and colleagues (2003) identified distinct communities of practice whose enterprises concerned the teaching and learning of mathematics in the district. The district-wide mathematics leadership community included a mathematics coordinator and three mathematics specialists as core members and a number of teachers as more peripheral members. The mathematics leaders viewed teaching mathematics as a complex endeavor aimed at students learning mathematics with understanding. Their view of high quality mathematics instruction included engaging students in demanding mathematical tasks in ways that were meaningful to students. This community viewed their role as supporting teachers in improving their instructional practices and attempted to do so by helping the teachers use the reform textbook series as the basis for their mathematics instruction. District mathematics specialists mentored new teachers in their classrooms, and organized district wide grade-level study groups that aimed at unpacking big mathematical ideas of instructional units in the reform textbooks. These activities were supported by an external grant that the district had received to support its reform efforts prior to our collaboration with the teachers.

The leadership communities in the schools consisted of a principal and two or more assistant principals. One or more mathematics teachers served as peripheral members of each school leadership community. The school leaders viewed mathematics teaching as a routine activity of assisting students to master specific skills outlined in the state objectives by effectively covering textbook units that addressed those objectives clearly. In their view, teaching mathematics did not involve complex decisions and did not require specialized knowledge. Instead, high quality mathematics instruction comprised both keeping students quiet and on task in order to cover lessons effectively, and addressing all objectives in timely manner. The school leaders did not focus on the methods of instruction that the teachers used. The enterprise of these school leadership communities was to raise students' scores on the state-mandated achievement test. To achieve their goal, the school leaders monitored teachers by conducting frequent, sometimes daily drop-in classroom visits during which they checked for both student behavior and whether the state objective that was being covered was written on the board.

The lack of alignment in enterprises of the mathematics leadership and the school leadership communities generated a less than optimal setting for teacher learning. In particular, the differences in views of high quality mathematics instruction were counterproductive to instructional improvement. For example, the mathematics leaders often faced resistance to using the reform instructional materials from middle school mathematics teachers who were held accountable by their principals for covering state objectives, irrespective of instructional materials they used. As the district had adopted one traditional and one reform textbook series, the teachers could choose to continue using the traditional textbooks despite mathematics leaders' efforts. The differences in views of high quality mathematics instruction and how to achieve it between the mathematics leadership and the school leadership communities were sustained by nonexistence of brokers and scarcity of boundary encounters between members of these communities. As a consequence, the school leaders did not have opportunities to develop an alternative view of high quality mathematics instruction and what teaching mathematics might entail. They therefore did not come to view time for teacher collaboration and use of the reform textbook series as potentially key resources for instructional improvement in mathematics.

Teachers' Initial Practices

A group of 9 mathematics teachers from 5 different middle schools in the district collaborated with the research team during the first two years of the project. The research team began working in the district to provide teacher development in statistical data analysis at the invitation of the district's mathematics coordinator who selected the initial group of teachers.

The institutional setting in which the teachers worked lacked informal teacher networks, provided limited means of support for instructional improvement, and the teachers' work was monitored by school administrators who attempted to regulate their instruction practices in terms of content coverage. The teachers worked in isolation and treated their own and other teachers' practices as private. This had consequences for the teachers' participation in the initial professional development sessions, where they avoided discussing their own classroom instruction and agreed that different instructional methods might work for different teachers. Despite the teachers' isolation, their instructional practices were largely homogenous, and reflected the school leaders' requirements for high quality mathematics instruction that were enforced during frequent drop-in visits to classrooms. In particular, the teachers attempted to cover the objectives outlined in the State Mathematics Standards[19] and achieve good behavior of their students, which was interpreted in terms of students being on task.

In organizing instruction, the teachers drew on textbooks, State Mathematics Standards, and a district-created pacing guide that matched specific objectives in the standards to units from two textbook series used in the district. To assess whether they were achieving their instructional agendas, the teachers focused almost exclusively on the correctness of student solutions and responses in class, homework, and quizzes. At the same time, school administrators inferred student learning from students' scores on the annual state-mandated test. Students' current mathematical reasoning remained largely invisible for the teachers as it was not integral to the endeavor of teaching mathematics in their classrooms and in their schools.

Emergence and Initial Learning of the Professional Teaching Community

Dean (2005) documented learning of the teacher group during the initial two years of the professional development collaboration. She concluded that nineteen months into the collaboration[20], the group had evolved into a professional teaching community that would satisfy Wenger's (1998) criteria for community of practice: a joint enterprise, mutual engagement, and a shared repertoire. I first outline developments during the first nineteen months that led to the emergence of the professional teaching community, and then summarize the normative practices that were in place at the end of year two of the professional development collaboration.

Among the initial challenges in the professional development collaboration were that (a) the teachers had limited experience in conducting statistical analyses of data, especially in dealing with variability and distribution, (b) interacted as a pseudocommunity (i.e., avoided to

[19] "State Mathematics Standards" stands for the standards document issued by the State Department of Education.
[20] This corresponds to the 13th collaborative whole day work-session (i.e., year two, session five) if each day of the two summer sessions counts as a separate work-session day (see the timeline on Figure 1).

challenge each other's contributions, pretended agreement; Grossman et al., 2001), (c) held their practices private, and (d) held the unarticulated assumption that they had no influence over aspects of the district that were a source of their frustration (e.g., perceived pressures of the state-mandated standardized test). While I organized the upcoming summary of the group developments according to the four types of norms that Dean documented, it is important to note that these norms evolved simultaneously and were interdependent.

Norms of Mathematical Reasoning

Dean documented how the teachers' engagement in activities from a statistics instructional sequence as learners provided an initial entry for the collaboration, and supported the teachers in developing data based arguments and providing justifications for their solutions. The teachers also became competent in using computer tools designed as part of the statistics instructional sequences to organize and analyze data, reasoned about distributions multiplicatively (i.e., in terms of percentages and part-whole relationships), developed a variety of strategies to compare two sets of data based on their distributions, and could infer shapes of these distributions from several different representations of data. Specifically, the teachers could infer the shape of univariate distributions from representations in which the data were partitioned into intervals of equal width (i.e., precursor of a histogram, Figure 10) by envisioning relative frequency of data as height of the distribution in each interval.

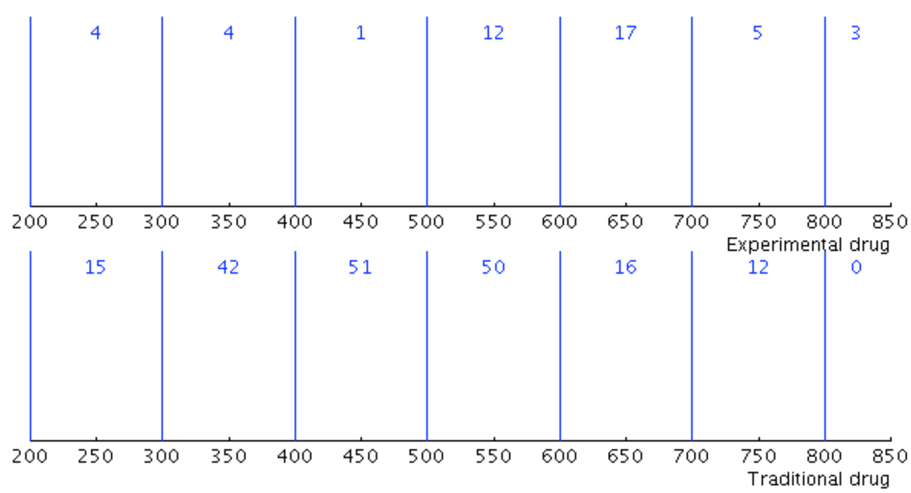

Figure 10a. Computer Applet Tool Two—Equal Interval Width Representation—Data Hidden. Each of the two univariate datasets is partitioned into intervals of width 100. The blue numbers within intervals each depict a number of data points in the interval.

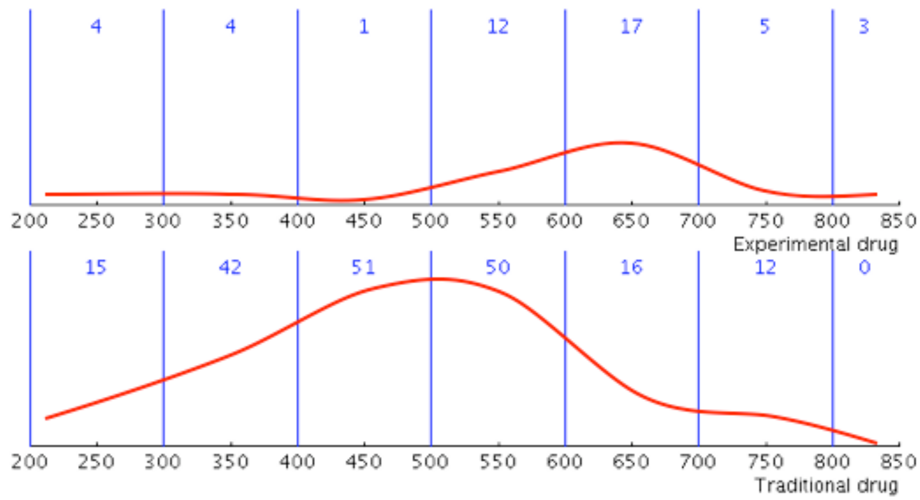

Figure 10b. Equal Interval Width Representation with Shapes of the Distributions Inferred.

The teachers could also infer the shape of univariate distributions from representations that partitioned data into four groups with equal numbers of data points (i.e., precursor of a box plot, Figure 11) by taking in consideration relative density of the data within each of the four created intervals. These ways to reason about univariate distributions in terms of shape became normative in the group by year two, session two.

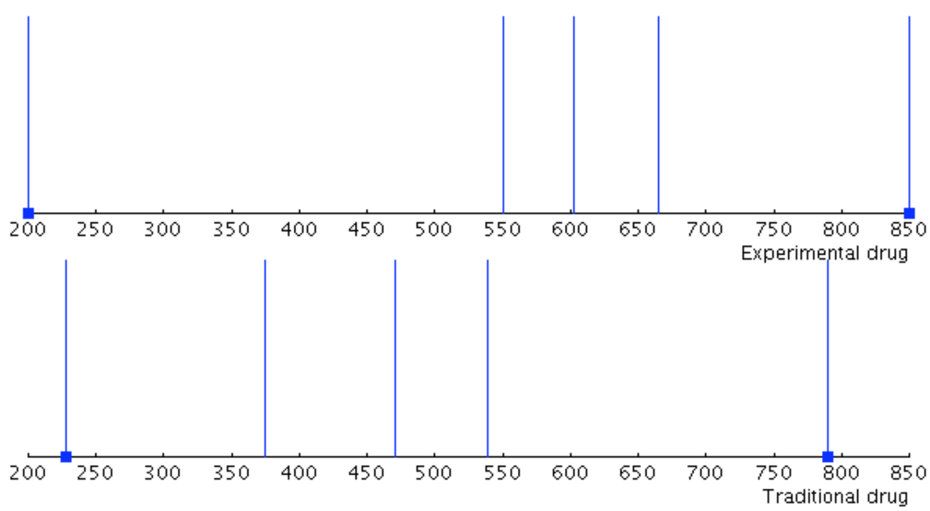

Figure 11a. Computer Applet Tool Two—Four Equal Groups Representation—Data Hidden. Each of the two univariate datasets is partitioned into intervals that each contain one fourth of its data.

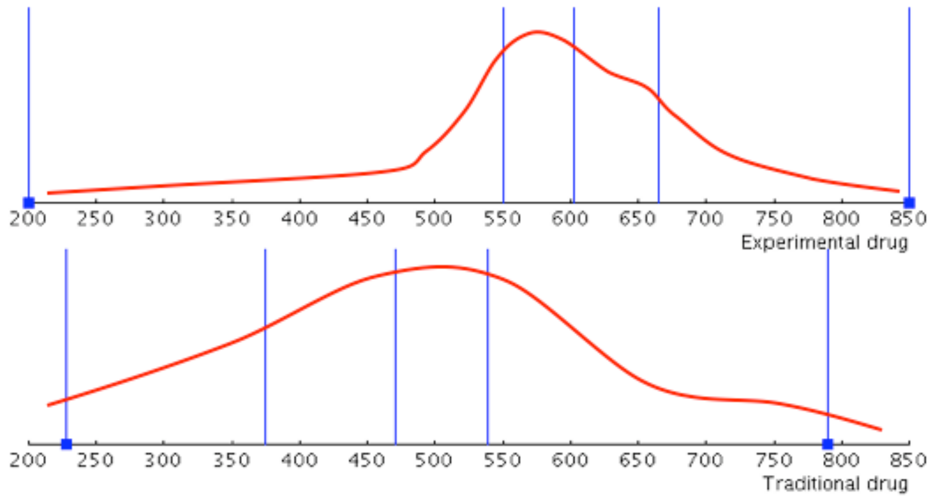

Figure 11b. Four Equal Groups Representation with Shapes of the Distributions Inferred.

The tools the group used to organize and interpret bivariate data inscribed in scatter plots built from the Equal Interval Width and the Four Equal Groups representations that the teachers used when reasoning about univariate data (Figures 12, 13). The goal was to support the teachers to "come to view bivariate datasets as distributed within a two-dimensional space of values" (P. Cobb, McClain, & Gravemeijer, 2003, cf. Wilensky, 1997). We conjectured that the teachers would come to envision the shape of bivariate data distributions as "hills" in the missing third dimension (i.e., the dimension perpendicular to the sheet of paper in Figures 12, 13). There was no conclusive

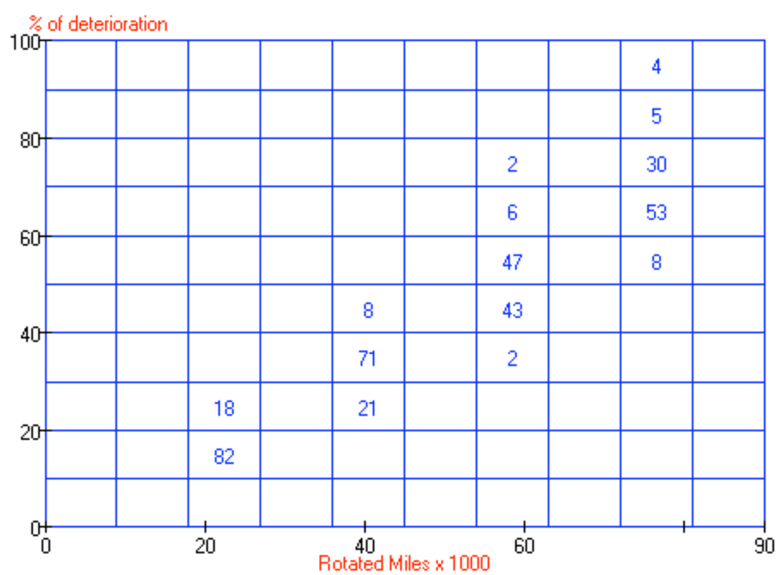

Figure 12. Computer Applet Tool Three—Grids Option—Data Hidden.

evidence that reasoning about bivariate distributions in terms of shape became normative in the group by the end of year two, even though at least four teachers could infer shape of distributions from graphical representations of bivariate data when prompted to do so by the researchers.

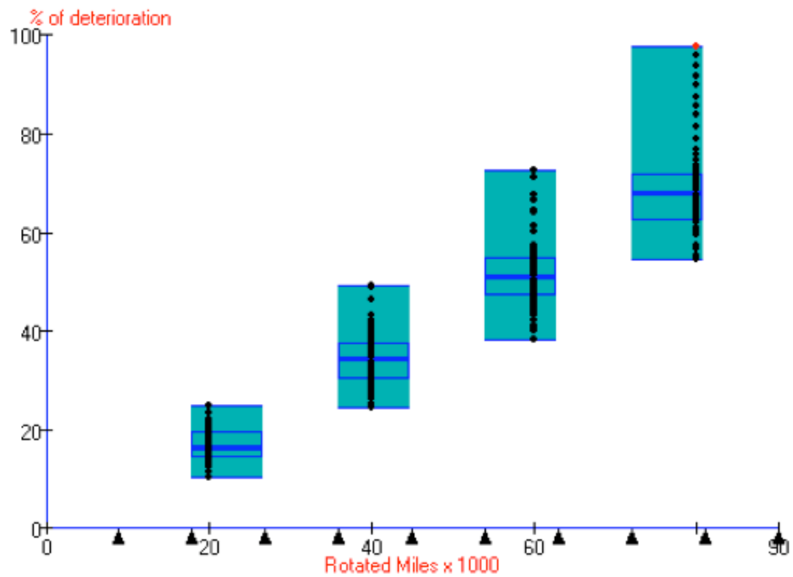

Figure 13. Computer Applet Tool Three—Four Equal Groups Option.

Norms of General Participation

Through the engagement in professional development activities, in particular those where the teachers discussed their analyses of statistical data, the group gradually renegotiated the general norms of participation in the professional development sessions. The teachers started to address clarifying questions to each other rather than directing them to the researchers. By year one, session three, challenges of others' contributions and interruptions ceased to be constituted as a breach of norms in the discussions of statistical tasks. However, in the discussions that involved pedagogical reasoning, such as analysis of students' written work, the teachers did not interrupt or challenge each other. Dean explained this contrast in norms of participation by taking into account the private nature of the teachers' instructional practices.

> When working on the statistics activities, the teachers perceived themselves as learners, students of mathematics. However, discussing issues of pedagogy, particularly when examining their students' work, was a high risk activity that they perceived as involving the evaluation of their classroom practices. As a consequence of this privatization of instructional practices, the group continued to be a pseudocommunity when engaging in activities that were related to pedagogy. (Dean, 2005, p. 93)

At the beginning of the collaboration, the private nature of teachers' instructional practices was also evidenced by their resistance to allowing their instruction to be video recorded as part of the data collection. In addition, when asked to bring their students' statistical work to the professional development sessions, most of the teachers initially pre-taught methods they hoped their students would use prior to assigning the task. The teachers' conduct was reasonable given the institutional context in which they worked, where similar situations were constituted as occasions to monitor and assess teachers' work. However, it influenced the teachers' initial participation in the professional development sessions, in particular evolution of norms for general participation and pedagogical reasoning. Forms of participation where the teachers addressed questions directly, challenged and finished each other's ideas became normative in the group across different professional development activities by the end of year one summer session. Developments related to norms of pedagogical and institutional reasoning, that I summarize next, contributed to further deprivatization of the teachers' practices. These developments occurred in year two.

Norms of Pedagogical Reasoning

In pedagogical discussions, the teachers often inquired about general prescriptions for the implementation of statistics activities in classrooms. These inquiries suggested that in their view, it was possible to prescribe instruction, irrespective of students' actual participation. The teachers initially justified instructional practices based on students' outcomes, that is, whether or not students "got" what was intended. In contrast, mathematics education researchers who studied instructional practices that proved effective in classroom situations (e.g. Ball, 1993; Hiebert et al., 1997; Lampert, 2001) argue that instructional decisions should be justified in terms of both students' current forms of reasoning and the big mathematical ideas that are the goal of instruction.

The teachers' focus on what students were supposed to "get" was characteristic of the teachers' epistemology, and it permeated their interpretations of professional development activities during the initial two years. For example, when the group co-constructed benchmarks of what the researchers saw as instructional intent of the statistics sequences (for the benchmarks, see Appendix B), the teachers interpreted them as a list of things that they should ensure students "got" as they went though the sequence. The teachers' analysis of students' work also involved sorting solutions according to what different students “got”. When analyzing students’ work, the teachers focused on students’ solution methods, rather than on their underlying reasoning. This approach aligned with their instructional focus on helping students to learn how to solve particular types of tasks in particular ways.

Supporting the teachers to focus on students’ reasoning continued to be a challenge. In year two, the teachers repeatedly discussed *student thinking* in terms of the importance of “getting students to think,” because many students in their classrooms expected to be told what to do. An important shift occurred in year two, session four, when the teachers interviewed several students using fraction tasks[21] (see Appendix C). The students’ surprising solutions to these tasks motivated the teachers to look not only for the methods the students used but also for why they might have reasoned in these ways. In the remaining sessions of year two, while the teachers continued to focus on the solution methods the students used in statistics tasks, they started to make conjectures about why certain students used certain methods, how they understood the task, and how they came up with a solution. While student work initially functioned in the sessions as a record of what had happened in the teachers’ classrooms, it became a tool for reasoning about students’ solution methods by year two, session five.

Ways that the teachers came to reason about classroom organization of the statistics instructional activities (i.e., whole class data generation discussion, small group analysis of data, and whole class data analysis discussion; see Chapter VII for elaboration) constituted an important part in the evolution of norms of pedagogical reasoning. During the first two years, the group activities focused primarily on the initial phase: the data generation discussion. From the researchers’ perspective, the major goal during the data generation discussion was to ensure that the students’ mathematical activity remained grounded in situation-specific imagery (McClain & Cobb, 1998). In other words, it was key that the students would come to view data as measures of an aspect of a phenomenon rather than merely as numbers (cf. G. W. Cobb & Moore, 1997). In contrast, a view that became normative among the teachers regarding usefulness of data generation discussions was that they provided “real world” context for the problems and helped to solicit student engagement. It was initially acceptable for the teachers if the students addressed the problem solely by manipulating numbers and attempting to produce pre-taught graphs. From

[21] Dean clarifies that use of fractions rather than statistics as content for student interviews was chosen because while the teachers reasoned very similarly to their students on statistics tasks, the researchers conjectured that they would view fraction tasks as self-evident and would become interested in unanticipated students’ solutions.

year two, session five, it became normative that the numbers that the students generated must be a measure of something, and the calculations produced as a part of data analysis must be produced for a reason. Student production of calculations alone was no longer seen as sufficient to demonstrate understanding.

Lastly, the community developed a professional discourse within which terms such as "shared planning" and "re-teaching" came to have specific meanings as a result of the teachers' histories of participation in professional development activities. For example, the meaning for "shared planning" as it was established within the community at the end of year two, was greatly influenced by the teachers' learning about Japanese lesson planning. The community-specific professional discourse became more visible at the beginning of year three, when the group recruited new teacher members. I therefore discuss some of the specifics of the discourse that the group developed in the initial two years when I contrast the newcomers' and the old-timers' interpretations of pedagogical situations in Chapter VI.

Norms of Institutional Reasoning

One of the important contributions of Dean's analysis was in documenting how the focus on the institutional context of the teachers' schools and district in the professional development activities in year two became a resource in supporting both the emergence of a professional teaching community and the further learning of the group. From the beginning of the collaboration, the teachers often expressed frustration about limited professional contacts and pressures they felt due to state-mandated standardized tests. However, as I discussed in Chapter III, during year one summer session, they started to draw on the institutional context to explain their participation in professional development activities. Specifically, the teachers related their focus on students' behavior in classroom video used in the professional development activity to what they were accountable for in their instruction, as well as to their lack of experience with observing others teach. The teachers started to realize that the institutional context of their work influenced what they attended to in mathematics instruction. Dean reported that

> the nature of the research team's relationship with the teachers changed in two ways: 1) the researchers took the teachers' explanations seriously and began to attend to the institutional context explicitly during sessions and 2) although the researchers were still viewed as experts in statistics, the teachers began viewing themselves as authorities on what teaching entailed in their specific district. (Dean 2005, p. 118)

As the teachers increasingly viewed themselves as valued contributors to the group endeavors, they adopted more active roles within the professional development sessions. They started to perceive the conflict between content coverage and focusing on student thinking, and started to view their instruction as problematic. As they were coming to understand that teaching that focused on student thinking was difficult, they started to think about the importance of collaboration, which in turn supported further deprivatization of their practices. Professional development activities such as (a) collaborative planning and teaching of a statistics lesson, (b) analysis of a video-recording from the Third International Mathematics and Science Study (TIMSS) that showed typical eighth-grade geometry lessons in three countries, and (c) student fraction task interviews all spurred discussions about why re-teaching the same content many times in different grades does not help students to learn it. These activities further reinforced the teachers' valuation of collaboration.

Dean highlighted that the evolving norms of pedagogical reasoning in the group influenced the changes in norms of institutional reasoning, specifically in what the teachers were coming to consider useful resources for their learning as they became interested in changing their instructional practices. They invited the researchers' proposal to collaboratively generate evidence for school leaders that would make the consequences of typical instruction that focused on content coverage evident. By year two, session five, it became normative in the group to perceive the institutional context as something the teachers could affect.

Professional Teaching Community

It was at this point (i.e., year two, session five) that the group satisfied the criteria Dean used to distinguish a group from a community: a shared purpose, a shared repertoire, and norms of mutual engagement. According to Dean (2005),

> the shared purpose that emerged had a two-fold purpose: 1) ensuring that students come to understand central mathematical ideas while simultaneously performing more than adequately on high stakes assessments of mathematics achievement and 2) identifying, acquiring, and controlling resources to make that possible.
>
> The shared repertoire, which was specific to this professional teaching community and the shared purpose, included normative ways of reasoning with computer applet activities, student work, and student interviews when planning for instruction or making students' mathematical reasoning visible.

> The norms of mutual engagement included the general norms of participation such as building on others contributions, asking clarifying questions, and challenging others assertions. In addition, the norms of mutual engagement encompassed the norms that were specific to mathematics teaching such as the standards to which the members of the community held each other accountable when they justify pedagogical decisions and judgments. For example, it was unacceptable to justified pedagogical decisions based on the need to cover the content for the standardized test. It is important to note that these criteria take as a given the deprivatization of teachers' instructional practice as necessary for the emergence of a professional teaching community and also acknowledge the situatedness of the professional teaching community within the institutional setting of the school and district. (pp. 166-7)

It is worth reiterating that the professional development group was composed of the teachers whose initial instructional practices were rather typical for the US teachers, and for 8 of the 9 teachers could be characterized as "traditional." The teachers developed and sustained these practices while they worked in a relatively typical institutional setting where their practices were monitored and instructional assistance was limited (P. Cobb, McClain, Lamberg et al., 2003). By documenting the development of a shared purpose, a shared repertoire, and norms of mutual engagement, Dean provided insights into the process in which this group of teachers (a) started to question the adequacy of their instructional practices, (b) developed a need to modify instruction, and (c) began to seek resources to make the modifications possible. Finally, towards the end of year two, the teachers started to identify each other and the community as key resources.

The final shifts in the norms for institutional reasoning in the initial two years occurred during year two summer session. The teachers engaged in professional development activities that helped them identify and examine what they perceived as the major affordances and constraints to teaching mathematics in their institutional context. The teachers determined that lack of time to collaborate on instructional issues and instructional time (e.g., short mathematics periods) were the greatest hindrances to teaching in the district. They then jointly invested time in the summer session to plan how to gain access to these resources. First, the researchers helped the teachers understand that the school leaders' definition of high quality mathematics instruction, which was based on content coverage, involved the assumption that mathematics teaching was a routine activity rather than a highly complex and demanding activity that required specialized knowledge. The teachers then conjectured that if the school leaders understood the complexity involved in teaching mathematics, they would be more inclined to provide the resources for instructional improvement. They therefore produced a conjectured trajectory for supporting the school leaders' development of deeper understandings of mathematics teaching (for an outline of the trajectory, see Appendix D).

It was important that as a consequence of the active roles the teachers were adopting in relation to their institutional context, their views of the affordances and constraints placed on their classroom instruction changed, even as the institutional context of their work itself remained largely unaltered. The teachers now perceived themselves as active players who could and ought to influence how high quality mathematics teaching would be defined in their schools. These developments were encouraged and supported by the district mathematics coordinator and the district mathematics specialist who participated as peripheral members of the professional teaching community. The teachers' perception of possibilities for instructional improvement expanded.

The ways that the teachers planned to shape the school leaders' understandings of mathematics instruction indicated that rather than preparing themselves to interact with their school leaders from a position of individual mathematics teachers, the teachers began to reason both about the school leaders and about themselves in terms of respective communities of practice and their collective enterprises. This evidenced that the group members were beginning to reason about responsibility for instructional improvement in collective and systemic, rather than in individualistic terms. The old-timers' participation at the beginning of year three, documented in Chapter VI, provides further evidence that shift of this type took place in the professional teaching community towards the end of year two.

District Leaders' Membership in the Professional Teaching Community

It is important to clarify the involvement of the district mathematics coordinator and the district mathematics specialist in the professional teaching community. In this summary, I draw on work of Lamberg (2004) who documented evolution of the district mathematics leaders' involvement during the initial three years of the collaboration.

Pat, a district mathematics specialist, initially helped with the logistics of the sessions and did not herself participate. However, starting from year two, session five, Ruth attended each professional development session. She actively participated in some of the professional development activities with the teachers, while in others she participated peripherally, as an observer.

Her motivations for joining the sessions included (a) using the sessions as an additional forum to communicate with the teachers, and (b) learning about building capacity for instructional improvement in the district by developing teacher leaders. Ruth continued to actively participate in the community throughout the years that are the focus of my analysis.

Esther, the district mathematics coordinator, started to participate in the professional development sessions at the end of year two. She joined the community for two days of the three-day summer session in order to keep up with the developments in the teacher community. During years three through five, she visited more than half of the one-day sessions and participated in all summer institutes. During the one-day sessions, she usually joined the community for lunch and stayed for an additional hour or two. For most of her attendance, Esther participated in activities together with the teachers.

According to Lamberg, along with their increasingly substantial participation, the mathematics leaders started to offer resources to support the enterprise of the professional teaching community, and began taking actions to bring about changes within the institutional setting. My analysis in Chapters VI and VII corroborates these claims.

Institutional Setting at the End of Year Two

The practices and enterprise of the school leadership communities remained relatively stable during the first two years that we worked in the district. No significant events occurred that would have supported changes in the school leaders' views of high quality mathematics instruction. They continued to monitor teachers via frequent drop-in classroom visits during which they primarily checked for content coverage and student on task behavior, attempting thus to ensure that students would perform adequately on the state standardized tests.

A shift occurred in the district mathematics leadership community, in particular in the district mathematics leaders' views of the professional teaching community. Initially, the district mathematics leaders' goal was to enhance middle school teachers' statistical knowledge and classroom instruction. For this reason, they selected the initial group of teachers from among those who would, in the district leaders' view, most benefit from opportunities to learn statistics. While participating as peripheral members, the district leaders started to realize that as the group evolved into a professional teaching community, it could become a resource in promoting changes in mathematics instruction in the district by building mathematical, pedagogical, and teacher leadership capacity. The group of teachers who were selected to join the community in year three reflected this shift in the mathematics leaders' perception of the potential of the community. It included two ambitious elementary-certified teachers who currently taught middle school (and who were conjectured to benefit from gaining more mathematical competence), a former psychological statistician who recently changed occupation to become a middle-school mathematics teacher (and who was conjectured to benefit from pedagogical focus of the community), and three accomplished middle school mathematics teachers who were active participants in study groups organized by the mathematics leaders. In addition, the mathematics leaders started to draw on some of the community members to help them organize the study groups in their schools.

CHAPTER VI

CONTINUATION OF THE PROFESSIONAL TEACHING COMMUNITY

The purpose of this chapter is to provide justification for subsequent analysis of developments of the professional teaching community as it continued to evolve over a three year period despite changes in membership. Need for such justification was primarily motivated by significant changes in the community membership that occurred at the beginning of the year three, when six (out of nine) of the original teachers remained in the group and six new teachers from the same district were recruited to join. The group then continued to recruit new members at the beginning of years four and five (see Appendix A for details). In addition to documenting the continuation of the professional teaching community across changes in membership, the analysis reported in this chapter also brings insights into the processes by which the old-timers facilitated the induction of new members, and how these processes were supported.

Given the relatively high proportion of newcomers to the group, it was important to question the continuation of the professional teaching community. This is because the developments that were supported in the initial two years of the collaboration with the teacher group in Jackson Heights were significant and involved a major departure from the view of high quality mathematics instruction communicated to teachers by the school leaders in both word and deed. The scope of the shift was evidenced by a relatively long period of time in the group history before it started to function as a professional teaching community. It was thus reasonable to expect that the normative ways of participation in the community would be, at least initially, alien for the six newly recruited teachers. In addition, the professional teaching community had no prior experiences with induction of new members. It was therefore an open question whether and to what extent would the normative practices of the community be re-generated at the beginning of year three.

Grounding of the Analysis

As I indicated in Chapter IV, I build on work of Stein, Silver, and Smith (1998) in the QUASAR project to situate this analysis. Stein and colleagues proposed to use the construct of *communities of practice* (Lave & Wenger, 1991) "as a theoretical framework for describing how teacher learning occurs in collaborative, school-based communities" (p. 48). Using a case of one middle school, they illustrated how changes that occurred over a period of five years during which a growing group of mathematics teachers started to use a reform-based curriculum, can be understood productively in terms of these teachers' participation in a community of practice. Stein and colleagues especially elaborated how the construct of *legitimate peripheral participation* helped them understand the ways in which newcomers learned as they participated in increasingly central ways in activities of the community, and how formation of identity as a reform teacher was an inherent part of their leaning.

We adopted a community of practice lens, elaborated by Dean (2005), to teacher professional development in the Jackson Heights District. While the normative practices and the enterprise of a professional teaching community are always generated and re-generated in the participation of its members, the term "professional teaching community" does not primarily refer to group membership. Rather, it refers to the collection of practices that are established as normative through teachers' participation in communal activities as the community pursues its purpose. Documenting the norms of mutual engagement allowed me to determine whether the professional teaching community continued to be re-generated. The findings of presented analysis make it reasonable to talk of the evolution of a single community where the changes in the group membership are conceptualized in terms of induction of new members, rather than in terms of an emergence of a new community at the beginning of every year. More importantly, this analysis justifies that subsequent shifts in the normative practices that occurred in different years of the collaboration should be interpreted as learning of the single professional teaching community.

Institutional Support Structures and Professional Teaching Community

Stein and colleagues documented that the accomplishments of the school-based teacher community they analyzed were made possible by a wide array of conditions and factors that included, but were not limited to professional development sessions provided for the teachers as part of the QUASAR project. These factors included supportive school administration that collaborated with the researchers to provide resources for teacher learning, and well-qualified teachers interested in instructional improvement and in adoption of a reform-oriented curriculum.

The analyzed community of practice was described when it was already established and appeared to grow seamlessly. At that time, it was sustained primarily by the numerous interactions, formal and informal, of the teachers who regularly worked together in their school setting, and who initiated and drove these interactions.

To better understand the professional teaching community that I analyze, it is important to highlight the differences in the support structures available to the teachers Stein et al. studied and the teachers at Jackson Heights. For example, the teachers at Jackson Heights had minimal opportunities to work together outside professional development sessions because usually only two professional development participants worked in the same school, and there were no professional networks in place in their schools. Lack of alignment between school leadership and district mathematics leadership communities resulted in less than effective support, and time to collaborate was a scarce resource for the teachers. The image of the professional teaching community that emerged at Jackson Heights would be therefore inaccurate if I tried to describe it as being driven and sustained primarily by the teachers. While the teachers critically contributed to the emergence of the community, it is important to acknowledge that the research team members were, along with the teachers, its central members. As we participated in the community activities, we were actively pursuing an agenda for teachers' learning. We shaped the activities and discussions in which we engaged with the teachers, the goals that the group pursued, tools used to pursue them, and normative ways of interacting. Our role in designing and participating in professional development activities was especially important when the community attempted to induct new members.

Recruitment of Newcomers

While four of the six new members, Helen, Brian, Erin, and Kate, were recruited from schools where at least one old-timer taught, the remaining two teachers, Jane and Hazel, came from a school that was newly added to participate in the professional development program. The recruitment decisions were made jointly by the continuing teachers, Naomi, Marci, Muriel, Wesley, Amy, and Lisa, and by Ruth, the district mathematics specialist who participated as a member in the professional teaching community at the end of the year two (Lamberg, 2004). Where necessary, I use (O) after the teacher name to indicate an old-timer, and (N) to indicate a newcomer.

Year Three Analysis

Introduction: Old-timers' Goals and Valuations

In order to facilitate newcomers' inception, the group met for a 2-hour informational meeting in the district mathematics specialist's backyard a month before the first professional development session of year three. The purpose of this informal meeting was to welcome the newcomers, introduce them to the goals and workings of the group and answer their questions. While three members of the research team were present at this meeting, the old-timers introduced the topics they wanted to share with the newcomers. During substantive 90-minute discussion, the 8 old-timers contributed 87 times, the 3 researchers 41 times, and the 5 newcomers 29 times (10.9 : 13.7 : 5.8). The majority of the newcomers' contributions were questions related to the workings of the group and requests for clarification. During this meeting, as well as during the initial professional development sessions, the old-timers talked about both the history and the goals of the group as they understood them at the time. The following excerpts are illustrative of both the nature and the content of the old-timers' contributions. In terms of content, the old-timers especially described and elaborated four themes: (a) how the group attempted to understand students' thinking:

Wesley: What I've been so impressed with is how … what we can find out about what students learn and the assessment part of that. It's not just we stand in front of them and they learn. How do we get them to learn and how do we know what they've learned.

Amy: I'm watching these kids think, I can actually see them think, it is novel to me [she laughs], it is great.

(Year Three, Informal Meeting, Aug 2002)

(b) how the group attempted to use both curriculum and statistics instructional sequences as resources to plan instruction, and "redo" the reform textbook unit on statistics:

Marci: We're going through the [reform textbooks] and looking at where we want to go with it and how the books or the questions or the activities that need to be changed-

Wesley: [jumps in] to use the Minitools[22].

Esther: It's not redo all the [reform text]. It's the statistics sequence. Because from learning so much, from using Minitools and doing so much statistics together the first year it shows the weaknesses in the materials we are using and want to get rid of them, pick some, add few, whatever.

Amy: I think basically with the statistics in [reform textbook] is doing a fairly good job of showing different ways of data collection and presentation but they do not do nearly enough data analysis, and what is it used for.

(Year Three, Informal Meeting, Aug 2002)

(c) how the group learned about Japanese lesson study and attempted to understand collaborative improvement of lessons over time:

Amy: Something else that blew me away, because we also spent some time looking at videos of the lessons that were taught in Japan. It was flabbergasting, the amount of time that the teachers worked together, working up a lesson before it ever got near kids. And how creative they were and how many things [they considered] in wording, and it was all very subtle.

Marci: But not only that. The time they've spent fine tuning it over the years to make it work for kids.

Amy: It wasn't a done deal. They worked it up, they did it once, and then they went back and fixed it.

(Year Three, Informal Meeting, Aug 2002)

and (d) how the group worked on issues related to institutional context, and especially on supporting principals' understanding of what high quality mathematics instruction involved:

Muriel: … But some of the environments that we are working in at some of the schools, it's not a place that that [experimenting, improving instruction] is easily done. So that's one of the things we've been working on. Trying to figure out ways to show the principals or whoever else that this is valuable, and what we've learned, what we are learning, we try to lead the way into making it more accessible to change.

…

Amy: We've always been aware of [state tests], the question is, are we gonna let the [tests] drive the curriculum or are we going to teach what kids need to know

[22] The teachers referred to the statistics instructional sequences developed by Cobb and colleagues as "Minitools," according to the name of the computer-based applet tools that were used to analyze statistical tasks.

and have access to? And if we do that doesn't it automatically take care of [test scores]?

(Year Three, Informal Meeting, Aug 2002)

The teachers continued to describe the goals and history of the group around these four themes also during the initial professional development session in year three. For example, Wesley elaborated that his ability to honor student thinking has really risen as a result of both learning to understand mathematics more deeply and examining students' solutions in the sessions.

When Esther, the district mathematics coordinator, summarized her view of the professional development session goals, she highlighted the collective nature of teacher learning, and importance of keeping student learning in the forefront.

Esther: I have not been very involved, I am a balcony person here, but I think the point is improving our practice together and learning more about how kids learn mathematics together and choosing statistics as a focus is just [an example]. It's a way to connect things but it's not the main point. The main point is the student learning.

(Year Three, Informal Meeting, Aug 2002)

The alignment between the researchers' and the district mathematics leaders' views of how to improve mathematics instruction provided legitimacy to the goals and foci of the professional development sessions, which possibly encouraged the newcomers' interest and participation in the group activities.

In addition to describing specific activities and how they related to the goals of the group, the old-timers also shared their valuations of the collaborative nature of the group, its non-threatening culture, and highlighted the aspects that helped them to open up their practices to other teachers.

Wesley: And every one of us has a style in doing things and that has never been a conflict with what we are talking about. You know, nobody is trying to tell you how to teach. But what we do do is get together, a lot of times we have a group of student work to talk about. And everybody has presented it [task] in different way, and handled in a different way, but what we do, we analyze what we've looked at. And some people do it whole class, some people do it pull out, there is no "you've got to teach this way, or in this process, or these steps." It's intellectual conversation about teaching.

…

Marci: I guess we are all comfortable with each other, and not just that, but comfortable with having people to come in and not criticize you based on what you taught, not on what their idea of teaching math is. For example, it is different when [the researchers] and you all are coming in knowing that we are all looking for the same thing. It is different from when the administrators may come in or even for new teachers, when a mentor is coming to observe. Because you feel that you are looking for something in particular to criticize their way or their method of teaching mathematics.

Wesley: It's been completely non-threatening observations.

Teachers: Yeah.

Amy: It's more like, you know, when she brings something in, what am I gonna learn? What did she do with her kids that I did not do with mine? I wanna *know*, is there something else that I can get? *I* want to know. Can I improve? It's not like somebody coming in to observe your teaching performance.

Marci: Not only that but you get to do what so many of us want to do with their colleagues, and that is to get new ideas. You know, "I never thought about it this way." … It is support.

Jane (N): It sounds like these meetings are gonna be like a rejuvenation… [Jane compared the sessions to conferences where she would go to get new ideas and energy.]

Amy: They [the researchers] have provided us with a [soundboard]. If we had something we wanted to talk about, like at the very beginning, I knew we were supposed to do statistics, [a researcher] came to the first one, and we were sitting there for four hours and she listened to us complain about every single solitary thing that ever crossed our minds as we've been teaching. And I was "when is she going to tell us to shut up, that that's not what we are here for?" And she never did. So they've always sat around and listen. They wanna know what is important to us whether it is on their agenda or not. And the second thing is, they put their money where their mouth is. Everything they do is top notch. They treated us very very well. I even get desert when I want it [teachers laugh]. You know, we get treated like I think we ought to be treated.

Wesley: Which includes good food and good information. Our brains get fed and our mouth get fed.

(Year Three, Informal Meeting, Aug 2002)

During the introductory meeting and the initial two sessions in year three, each of the six old-timers indicated that they positively valued the sessions because they happened in an intellectually demanding and collegial environment of the professional teaching community. Overall, they portrayed the professional development sessions as intellectually valuable and emotionally therapeutic experience.

Out of the five newcomers that were present during the informal meeting, three stated that they were interested in the group because of the “sharing with colleagues” aspect, one hoped to learn to teach mathematics that was different from what she learned in school, and one wanted to learn new things and challenge herself mathematically.

Norms of Institutional Reasoning

As the prior illustrations suggest, negotiations of norms of institutional reasoning played important role in the initial sessions with the newcomers. This was the case because the ways that the newcomers were used to interacting with each other in their schools (e.g., whether and when was it appropriate to ask questions or ask for help, offer an encouragement, volunteer a critique) differed considerably from the norms of mutual engagement that had been established in the professional teaching community by the end of year two. Many of these differences related to the private nature of instructional practices in the teachers’ schools and the district.

By year three, deprivatized collaboration and openness about their own instruction was so ingrained in professional development practices of the old-timers that they commented in surprise on the private nature of the practices of their school colleagues who were not part of the professional development group. For example, Muriel, encouraged by the district mathematics specialist, worked with the sixth-grade teachers in her school. She was genuinely astonished when she realized that her colleagues did not want to let others in their classrooms after she had advocated for the learning opportunities that such collaboration would create. The old-timers, who no longer realized how difficult it was for them to allow others access to their classrooms, thus created opportunities during the initial sessions with the newcomers to discus why some teachers kept their practices private, how this behavior was institutionally shaped, and why teachers’ protective reactions were understandable. One 14-minute discussion of this type occurred during the informal meeting, and one 14-minute discussion during session one. These discussions contributed to the establishment of norms of institutional reasoning in the group. At the same time, they served as a means to communicate expectations, and clarify the purposes for observing each other’s instruction and for examining students’ work. Most importantly, the old-timers and the researchers communicated their views of instructional improvement as a collective responsibility of the group and shared their valuations of deprivatized collaboration as a means to understand and improve instruction.

Besides talking about their valuations, the old-timers also demonstrated the deprivatized nature of their instructional practices by bringing their students' work and classroom video to sessions (for session two and later), and by talking openly about difficulties that they faced in their instruction. Two situations in which the old-timers commented on their classroom difficulties spontaneously occurred in session one, one in session two, and others occurred with a similar rate throughout the year. The following example from session one illustrates the kinds of instructional issues that the old-timers raised in these comments.

Wesley: And just something that clicked with me, yesterday when I did [the statistics task from which] I got my student work. I thought like I had to bully them in doing the work. And I did. But it then just occurred to me that I could have done the same thing by, [if I would] do the launch with them.

Researcher: [There was] no data creation process?

Wesley: No data creation process. [I might have included] Talking about where the numbers came from, why they are important, why we care. And so without that data creation process, it was a bunch of meaningless numbers that can kids [saw] just like ok … "do it, I'll do it." And so they went back to [do it]. These kids that would have got it if I had given them that five minute "Where these numbers come from?"

(Year Three, Session One, Sep 2002)

The accounts of instructional difficulties were constituted in the group as complex problems that required understanding and experimentation in the classroom, not as evidence of poor instruction or incompetence on the part of the teacher. The old-timers' sharing, ways that these situations were constituted in the sessions, the ways that the old-timers attempted to ease out the newcomers' concerns and uncertainties, and their references to working *together* all indicated that collective responsibility for instructional improvement have become a part of what the old-timers valued and worked to re-establish as normative in the whole group.

Norms of General Participation

Importantly, while the old-timers described the non-threatening nature of the collaboration, they all agreed with the researchers' clarification that different members of the group often had very different opinions on specific issues, and that disagreements served as a means to examine such issues more deeply.

Researcher 1: One shift what you all do now … is that you have disagreements. … But what's really important is that's not personal. It's not who's got it right, or who's got it wrong. You know, we are talking always what can we learn. Which is a very different orientation.

…

Researcher 2: And it's [the group is] also different from the conferences. At conferences everyone agrees and everyone is being nice to each other.

Researcher 1: We are not nice to each other.

Researcher 2: [At the same time with Researcher 1] Well, we are not mean to each other but

Amy: Honest.

Researcher 2: you know, we have some very different opinions about how you go about things and how students think and the thing is, challenging each other on that to make us stop and think twice about how we are viewing things, or make sense of things. And that's the way of learning and growing versus everyone [having you pat] on the back.

(Year Three, Informal Meeting, Aug 2002)

During the initial meeting, three situations occurred in which group members expressed disagreement, and eight such situations occurred during session one. Five of these instances involved a newcomer, and three involved a researcher. All of these encounters were constituted as legitimate ways to participate in professional development activities.

From the beginning, the newcomers actively attempted to make sense during the discussions. Four out of the five newcomers present during the informal meeting asked clarifying questions, and every newcomer asked at least one question or contributed a comment in the whole group setting by the end of session one. In addition, the newcomers shared their experiences from prior professional development workshops, worries, and concerns. For example, Erin – a newcomer – shared how the job of teaching mathematics in their district is perceived in ways that make it difficult for teachers to admit that they make mistakes and need to learn.

Erin (N): I think there is a fear, even in study groups, to admit that you don't know something, that you do not understand something. You are the *teacher*, so we are the *experts*, so we should know it. … I think that's a big fear. And you are talking about the planning is so task oriented, we become task masters. … We've got these things we got to cover.

(Year Three, Informal Meeting, Aug 2002)

Hazel and Helen, who were both elementary-certified teachers and now taught middle school mathematics, shared their concerns about possible limitations of their mathematical knowledge, and that they were overwhelmed by statistical activities during session one.

Hazel (N): I think Helen and I feel probably the same because we are both elementary education majors and not math majors and we just worked that out this morning that we were not getting the same thing out of [the activity] that everybody else was. And it was just good when we got to lunch that I realized that I wasn't the only person feeling [like "What I was gonna say?"]. I was looking at the data but I wouldn't sit here and see everything that everybody else was seeing necessarily. I thought on a much smaller scale, I was just like please don't call on me, don't call on me, I don't see [teachers laugh]. So I felt very [small] you know? …

Lisa: You may think that we are all seeing the same, we are not. We do not see the same thing.

Helen (N): But when we started to talk about mean and median..

Lisa: ..Ah, OK..

Helen (N): ..and all that stuff that I like, to me, eight out of ten [cars] stopped fast, and that's what my students would tell me. I was just like "where are we going?"

(Year Three, Session One, Sep 2002)

This newcomers' openness and willingness to admit their own limitations suggested that the group succeeded in initially creating a risk-free and inviting environment, in which the goal was to learn. The old-timers responded to newcomers' concerns by providing encouragement and sharing stories of their own prior struggles. The researchers followed up on situations like the previous one to negotiate norms of general participation in the group.

Lisa: [Responds to Helen] But you know why we went there though? Because we've done it for two years and that's what we always do.

Researcher: Can I say something? What would be good, in future, is when that happens just call us on it immediately.

Helen (N): Ah, I was just like, "Waw!"

Researcher: [Jumps in] No, you need to call people on it.

Lisa: Yeah! [Muriel nodding]

Wesley: But it's also a growing vocabulary that you will, you know, even though I've been doing this for two years, my vocabulary continues to grow and my knowledge – and my ability to discus mathematics is growing. And I think your elementary school background is very beneficial to us, because we are basically making all these good things from elementary school are being borrowed at the middle school.

Researcher: I think the point is, there is some sort of inner group language … Seriously, I hope you would feel comfortable in future to say it: "Look, you are talking about stuff, it's got a history beyond but we weren't here then, and you can't expect us to understand what you are talking about." And that's a pretty fair point, isn't it?

(Year Three, Session One, Sep 2002)

During session one, the researchers explicitly initiated the negotiation of obligations for teachers to listen and monitor own understanding, explain and justify their reasoning, and "play devil's advocate" when others explain their reasoning, especially when sharing statistical analyses. The group also explicitly discussed the purposes for analyzing student work after Erin, a newcomer, expressed concerns about how her students' work would be perceived, given that she taught students who had scored low on the state tests.

Frequency of Newcomers' Participation

Each professional development session involved several (usually four or more) distinct activities, such as working on a statistics task with a computer tool, discussing a video of a classroom statistics instruction, discussing how to organize statistics instruction in a classroom, analyzing students' written work, planning activities to engage principals, or reflecting on what the teachers had learned in the session or how the session could have supported their learning more effectively. Thus far, I have presented the newcomers' participation in the informal meeting and session one across different activities, because the data did not reveal significant activity-specific differences in the newcomers' participation. In this section, I pay attention to the types of activities, in which the patterns of the newcomers' participation were significantly different. Specifically, in two activities, the newcomers appeared not to contribute to the whole group discussions.

From session one, it was typical that the newcomers frequently contributed to the group discussions. For example, during a statistical activity, where the teachers participated as students (session one), 47 contributions to the discussions came from the 6 present old-timers, and 53 contributions came from the 6 newcomers (7.8 : 8.8). During the pedagogical reflections on the statistical activity, 52 contributions to the discussions came from the 6 present old-timers, and 35 contributions came from the 6 newcomers (8.7 : 5.8). While this observation of relatively small differences in the old-timers' and the newcomers' frequency of contributions might initially seem surprising, it indicates that the old-timers and the researchers succeeded in providing the newcomers with access to participation in the activities. For this reason, the two cases of the newcomers' limited participation stood out in the data. I conducted closer analysis of these cases in order to identify the specific demands of the activities and conjecture how the newcomers' learning could have been better supported.

In contrast to the similarities in frequencies of contributions, the nature of the newcomers' contributions at times differed significantly from that of the old-timers, indicating that the old-timers developed a community-specific shared repertoire of practices during the initial two years. I discuss differences in nature of the old-timers' and the newcomers' contributions at the beginning of year three when I discuss the newcomers' mathematical and pedagogical reasoning later in this chapter.

Newcomers' Non-Participation, Case 1

The first activity, in which low relative frequency of newcomers' participation was discernible, occurred during session two, when the group engaged in an analysis of a classroom video from one of the teachers' classrooms for the first time. Two old-timers, Naomi and Marci, co-taught a statistics task and video recorded the lesson. As preparation, the teachers had worked on one similar statistical task as students in session one. During session two, the teachers first watched the video and discussed the *data generation* phase of the lesson in which Naomi introduced the task scenario in the classroom. After the discussion of Naomi's task introduction, the teachers analyzed the task in pairs using a computer tool, and shared their analyses. In both these activities, the frequency of the newcomers' contributions was comparable to that of the old-timers. The newcomers shared their observations, asked questions, proposed changes to how the task might be introduced in the classroom, and shared their statistical analyses. Specifically, during the 25-minute discussion of the data generation part of the lesson on video, the old-timers made 21 and the newcomers made 16 contributions (3.5 : 2.6). Similarly, during a 40-minute discussion of the analyses of data, the old-timers made 43 contributions and the newcomers made 47 contributions (7.2 : 7.8). These two activities were representative in terms of the relative frequencies of the teachers' participation.

Afterwards, the teachers returned to the classroom video to watch the *data analysis discussion* part of the lesson, where different students presented their solutions. The teachers' task was to look at what was happening in the classroom from students' point of view, and focus on what the students learned mathematically. During this activity, only one newcomer asked one brief question and shared one comment about 20 minutes into the discussion, while the old-timers offered 20 contributions in 28 minutes (3.3 : 0.3). It is important to clarify that students' presentations in the videoed lesson were often confusing, some students based their solutions on assumptions that did not align with Naomi and Marci's intentions for the task, and Naomi and Marci's comments in the classroom indicated that they sometimes misinterpreted what the students said. Old-timers' contributions in this discussion indicated that they attempted to understand what the students meant, when guided to do so by the researchers. The following exchange is representative in this regard:

> Researcher: What did the students mean when they were focusing on "bigger?" Just that some of the green [bars on the graph, Figure 14] were bigger? Or did they count totals [for the two groups of data, green and pink]?
>
> Naomi: No. The length of the line.

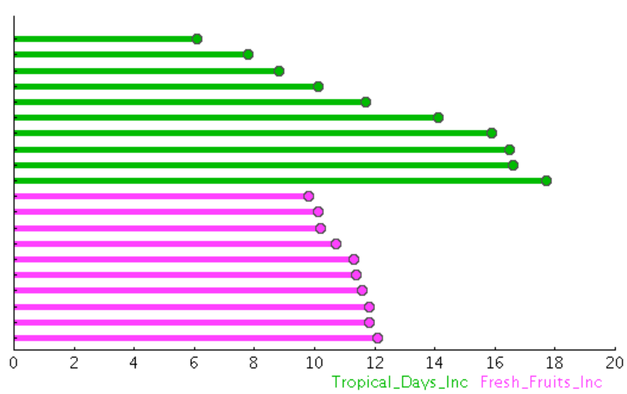

Figure 14. Computer Applet Tool One—Comparison of Two Sets of Data

Researcher: They just focused on a few of them [lines that each represented a data point]? [Naomi nodding] So they weren't looking at the whole data set. What is interesting is, that never came up. Because of how messy the scenario is.

Wesley: That never came up where?

Researcher: In this [classroom] conversation. The kids' questions were not how they looked at or analyzed the data, they were about how they understood the scenario. Because it's not a clean scenario.

Wesley: What I keep hearing is that we are not defining, when they say "bigger" we are not following up with "What do you mean by bigger?"

Researcher: Yeah. Because I have two different interpretations.

(Year Three, Session Two, Oct 2002)

I attempted to understand the reasons for the newcomers' non-participation to illuminate the learning that was involved in becoming a full participant in the professional teaching community. The first conjecture relates to the institutional context in which the newcomers worked, and specifically their lack of opportunities for professional networking. In their analysis of the data generation discussion part of the lesson, the newcomers could critique the task design rather than the video teacher's work. In contrast, the data analysis discussion on video might have required, in the newcomers' view, that they comment on their colleague's practices in a rather problematic classroom situation. The newcomers' silence was consistent with the lack of access to a professional, respectful, and constructive discourse in their work setting that might have served as a resource in expressing disagreement with or critique of others' instruction. It is therefore possible that the newcomers did not speak up because they were learning how to contribute constructively to discussions of others' instruction in a way that would allow them to build trust with their colleagues.

Second, note that the old-timers' contributions were not directed at the video teachers' actions. Rather, they inquired about details of the classroom situation (e.g., number of students in the classroom), and debated both meanings of different words that students used to express themselves and students' understanding of the task. In other words, the old-timers focused on the students, rather than exclusively on the teacher, when they examined this (and other) instructional situations. Given the substantive support provided by the research team to initiate this shift in the old-timers' focus during the first two years of the collaboration, I conjectured that the newcomers also needed to learn how to focus on students in order to participate in similar discussions in more central ways. It is important to point out that the old-timers required rather substantive guidance in this discussion, as indicated by the untypical discussion pattern in which the teachers responded to the researchers' questions or addressed their questions at the researchers for the most part. Specifically, only two teacher questions (one from an old-timer and one from a newcomer) were directed at another teacher. In addition, a teacher spoke after other teacher's comment on only two occasions. This unusual pattern in the old-timers' participation suggests that this was a demanding activity for them as well.

Lastly, it is possible that the newcomers had greater difficulty interpreting the solutions presented by the video students given their understanding of statistics and how students reason statistically. This would suggest that the old-timers developed a specific pedagogical knowledge for teaching statistics (Ball & Bass, 2003) during their previous participation in professional development sessions that now set them apart from the colleagues from their schools. Silver and colleagues (Silver et al., 2007) supported this conjecture when they reported that practice-based professional development programs create ample opportunities for teachers to "*work on* and *learn about* mathematical ideas" (p. 261, italics in original).

I tested the outlined conjectures throughout the subsequent analysis. I analyzed the teachers' participation in subsequent professional development activities in which the teachers watched video of the data analysis discussion part of the lesson. During the second video activity (session four), both the newcomers' and the old-timers' contributions increased. During a 35-minute activity, the 5 present old-timers spoke 73 times and the 6 present newcomers 34 times (14.6 : 5.6). Rapid exchanges between many teachers, like the one captured in the following transcript, occurred four times during this activity. In the task that the students analyzed, the goal was to establish whether to choose supermarkets or community centers as better locations for an emergency blood drive. Each data point in the two data sets that the students analyzed represented total amount of blood collected by a blood-mobile. Ten of the blood-mobiles were positioned in supermarket parking lots, and ten in the parking areas of community centers. Muriel and Helen co-taught the lesson in Muriel's classroom. The group discussion followed after the group watched a video segment, in which a student presented her solution of the task to her classmates.

Kate (N): What was she [the student] drawing? What was the line that she was drawing? [Kate gestures in front of her vertically]

Muriel: She only drew, [she] took a [data] point for just a supermarket-

Helen (N): [jumps in] supermarket-

Muriel: [continues without interruption] and did that on a line graph. Because she wanted to show that it was more consistent-

Helen (N): [jumps in] yeah-

Muriel: [continues without interruption] but she did not compare it to anything.

Researcher: So, her argument was consistency,

Lisa: Yeah, she never said the word.

Muriel: No. But that was her argument.

Lisa: That was her argument.

Researcher: And then this is how she is trying to show that this is more consistent. By drawing that graph-

Muriel: [jumps in] assuming that they've got the other information, "This is the one I want." [Teachers laugh]

Wesley: That's rather sophisticated argument then. Lots of assumptions there.

Muriel: Yes.

Erin (N): You said she did a line, she did a line graph showing?

Muriel: She just put the dots up there and connected them in a line, yeah.

Researcher: So what was the [purpose]?

Muriel: [jumps in] To show they were close together or-

Lisa: [jumps in] But it didn't show that. [Lisa draws the shape of the graph in the air, moving upward right with one hand]

Erin (N): [Together with Lisa's gesture] It showed them going up. Yeah. I guess I'm-

Wesley: [jumps in] I think she did *x* and *y* to equal to each other.

Muriel: I don't know.

Kate (N): This [the graph] is rather straight, it shouldn't be straight.

Jane (N): [Jane spoke at the same time as Kate] It also looked like she was doing a little bit of total.

(Year Three, Session Four, Jan 2003)

Importantly, the focus of both the old-timers and the newcomers in this discussion was on interpreting a solution that one of the students in the videoed lesson presented to her classmates. While the group discussion focused on several additional issues (e.g., what is a good statistical argument, why should students listen to others' presentations, and how to elicit different student responses), these did not include evaluations of Muriel's competence or performance. It appears that as the newcomers began to understand the purposes for video analyses of their colleagues' instruction, and developed the means to conduct statistical analyses, they started to participate more centrally in analyses of classroom videos in terms of both frequency and nature of their contributions. Even though the counts for the newcomers' contributions were lower than for the old-timers', the newcomers engaged in the discussions in substantial ways. The remaining data from year three is consistent with this observation. Specifically, during similar 25-minute activity in session five, the old-timers spoke 24 times and the newcomers 20 times (6 : 5), and during a 40-minute activity in session six, the six old-timers spoke 53 times and the four present newcomers 31 times (8.8 : 7.8). In both cases, the newcomers' contributions were substantial.

Newcomers' Non-Participation, Case 2.

The second case of the newcomers' noticeably low participation occurred in session three. The activity was a continuation of the group efforts to support school principals in developing a more sophisticated view of mathematics instruction, in order to gain access to resources, such as common time for co-planning and collaboration. Esther, the district mathematics coordinator, had been promised an hour of the principals' time during their monthly meeting and hoped that three of the old-timers could join her at the meeting and engage with the principals in an activity that would both manifest teachers' professionalism, and help the principals start developing a sense of the complexity of the work of teaching mathematics. As I discussed previously, Cobb and colleagues (2003) documented that the school leaders in the district viewed mathematics teaching as a routine activity rather than a highly complex and demanding activity that required specialized knowledge. Esther's proposal that the teachers should join her at the principals' meeting built on the group discussions that took place during the year two summer session. There, the group had produced a conjectured trajectory for supporting the school leaders' development of deeper understandings of mathematics teaching, which among "intermediate steps" for working with the school leaders included (a) sensitizing school leaders to what mathematics teaching and learning is or should be about, and (b) helping them to understand and value focus on issues of student reasoning.

In response to the district coordinator's announcement that the meeting with the principals could take place relatively soon, part of session three was devoted to planning a specific activity for the principals' meeting. During this time, the old-timers drew heavily on the group history and the pool of relatively complex activities that they perceived as transformative in their own learning, but which the newcomers had not experienced. In one such activity in year two, the old-timers had conducted informal student interviews, using a series of fraction tasks (see Appendix C), in which they asked students to explain their solutions. At that time, the surprising students' responses motivated the teachers to examine student solutions beyond their correctness and the methods students used. During the year two activity, the teachers generated evidence of the kinds of understanding of fractions that students developed when they received instruction that was driven by coverage of objectives. They concluded that even students who performed adequately on state tests often did not reason adequately in situations that involved fractions. The old-timers conjectured that if the school leaders could engage in the same fraction interview activity with students, they too would find students' reasoning about fractions problematic, and might begin to problematize their views of high-quality mathematics instruction that foregrounded coverage rather than learning.

During the nearly 110-minute activity in year three, session three, the eight old-timers offered over 120 contributions, while the five newcomers who were present only spoke up 26 times (15 : 5.2). The newcomers asked questions and occasionally voiced their disagreement. However, they were not encouraged to participate more fully in this activity by other group members and had limited means to do so on their own accord. Possibly, among the major reasons for the newcomers' lack of participation in this activity were the time constraints and the unequal distribution of responsibility for the activity with the school leaders in the teacher group: It was understood that three old-timers, rather than newcomers, would go to the meeting with the principals to conduct the activity. Engaging newcomers in a brief version of the fraction student interview activity that the old-timers referred to might have been beneficial not only for the newcomers. It might also have elicited more reflective contributions from the old-timers.

When the group revisited the fraction interview protocol[23] during session six in year three, the newcomers participated more centrally. At that time, school leaders were invited to join the professional development session and the newcomers were going to be present during the activity. During a 65-minute planning activity, the six old-timers contributed 68 times and the four present newcomers 51 times (11.3 : 12.8). At this time, the newcomers asked numerous specific questions about conducting interviews as well as about individual fraction tasks, and they actively monitored their understanding. The following excerpts are illustrative of the nature of the group discussions in session six:

Brian (N): Do you let them [students] know [during the interview] that they missed it [the problem] or did not miss it?

Researcher: That's not important. What is important is even if they got it right, even if they did not miss it, you still want to probe and to know how they did the problem. Does that make sense?

Brian (N): Yeah.

Researcher: Not just when they miss it. On *every* problem. The goal is just one thing: understand how they think about fractions.

Erin (N): "How did you do it," "Why did you do it in that way"?

Researcher: Yes. "Why do you think that way?"

Wesley: "Can you give me a counterexample? Can you, does this rule always work? What is the rule? Can you write it in a different way?"

…

Erin (N): How would you respond if they [students] ask: "But they didn't gave us the distance"? [Erin referred to the problem one, Appendix C, where the task was to compare which of the three students walked longest distance: Lauren who walked 1/3, Michael who walked 1/2, and Lawrence who walked 1/4 of the distance between Raleigh and Durham]

[Several teachers nodded in agreement and said that such complaint is very likely]

---

[23] The plans to join principals' meeting had to be changed and instead, several principals and vice-principals joined the group during session six to participate in fraction interviews. The reported counts are from the discussion, during which the group prepared for the fraction interview activity with the principals in session six.

Researcher: You say: "I don't know exactly."

Muriel: Yeah.

Erin (N): And then if they say: "Well we can't work this problem cause we don't have enough information"? That's always their way out.

Researcher: So you push on that, first of all. Cause that's interesting. That is data. If they are saying: "I can't do this problem unless I have exact distances," so you say: "Is there any way that you can do it if you don't know how far it is? Which would be more of a distance? Who would walk farthest?" …

Lisa: I would also stress I'm not asking *how far* they went. I just want to know *who went farther*.

(Year Three, Session Six, Mar 2003)

The newcomers' active attempts to understand and be able to conduct interviews with students were apparent throughout the activity. It is possible that the fact that they were also going to be present during the activity with school leaders was responsible for the newcomers' increased interest in fraction interviews. In addition, the planning activity was more structured at this time, as the group had already determined the content of the activity with principals, and the focus of the planning discussions was on the details of conducting the interviews. Lastly, the old-timers' and the researchers' participation suggest that there was also a difference in communal goals pursued during this planning activity when compared with the one conducted in session three. In contrast to the session three planning, supporting the newcomers' learning was a persistent focus at this time. While intonation of Erin's questions "How did you do it," "Why did you do it in that way" in the excerpt above suggest that she was checking whether these would be a reasonable follow up questions, Wesley (an old-timer) offered a collection of questions that he considered had been useful in past. Like Wesley, five of the six old-timers who were present shared their experiences from the fraction interviews that they conducted during year two. They both answered the newcomers' numerous questions and volunteered their advice on issues they found important. For example, supported by all six participating old-timers, Amy twice brought up the distinction between knowing the rules and understanding what fractions meant quantitatively—the key distinction with respect to documenting students understanding:

Amy: The interesting thing when we did these [fraction tasks] we did them as, we started out doing them at the end of last year. But we would bring in a kid, give them [the tasks] and just interview [the child and] videotape ourselves.

> OK? And it was amazing. Cause I was doing all level fours [the students who performed highest on the state tests]. OK? And they are supposed to "know this stuff" [Amy makes quotation marks in the air] though they know rules. But do they really understand the meaning of a fraction? And could they sit there and draw pictures? *No!*

(Year Three, Session Six, Mar 2003)

As the old-timers participated in this activity, they adopted mentoring roles that benefited the entire group. While the nature of participation differed significantly for the old-timers and the newcomers, I take the comparability of the participation frequencies of the two groups as evidence that the way this activity was organized provided the newcomers with access to meaningful participation. I will revisit this activity when I document evolution of norms for institutional reasoning in the analysis of learning of the professional teaching community in Chapter VII.

General Participation and Institutional Reasoning Summary

The development of norms for general participation and deprivatization of teachers' practices required a long time and significant researchers' support in the initial two years of the collaboration. In contrast, the old-timers' participation in the professional development sessions in year three made those norms overwhelmingly present in all professional development interactions. Consequently, the newcomers not only had a chance to listen to the researchers' justifications for why particular ways of interacting were beneficial, they could see their colleagues genuinely and persistently participate in the advocated ways.

Along with examining institutionally shaped aspects of their work, the newcomers' access to views of instructional improvement as a collective endeavor was especially important in deprivatization of their practices. On the one hand, the focus on institutional aspects enabled the newcomers' to gain insights into some of the limitations of their instruction and how they were institutionally shaped, while construing instructional difficulties as a matter for understanding rather than blame. On the other hand, professional development activities in which instructional improvement was not constituted as a responsibility of individual teachers, but which instead highlighted its collective and systemic character, suggested a direction for improvement that reduced the newcomers' uncertainty and vulnerability. These activities helped the newcomers position their classroom instruction in the context of larger, collective learning efforts. As a result, while some newcomers were initially cautious to open up their classrooms (e.g., recall Erin's concerns about how her students' performance would be viewed by other teachers), they nevertheless did so for purposes of the communal learning. The newcomers brought their students' work to sessions three and later, and agreed to co-teach and video record statistics lessons with the old-timers. Helen co-taught a lesson for session four, Brian for session five, and Kate for session six. They also started to share instructional difficulties and stories from their classrooms. Four newcomers shared difficulties that they encountered in their classrooms during session three, when the group discussed issues of planning and assessment.

Hazel (N): I had some, people from [local university] that visit our class and I've got to the point now when we have people come in, I still sort of tell them [students] tune them [visitors] out, and you know I wasn't gonna get stressed, and. But the kids bombed. None of them had a clue when they left, they just did not. And that was because I assumed that the objective beforehand they covered well and they understood it and they didn't. So, I needed to step back, but because there were people in my room, I was afraid to do what I would normally do, and kick the exercise, and just go back: "You pull out your paper and pencil and let's go back, you know you did not understand it."

…

Helen (N): In my assessment, and Esther knows how I feel about fractions, last year, you remember it? Same thing happened this year. But I've just said: "I am not going back." My kids burned out on it. So it's a matter of you have to decide what is important to whether or not to keep their focus, like … "Let's do something else and we'll come back." And I found that that's important because you just can't sit there and do that to them. You know, you gotta make it fun for them, and yeah, they didn't get it, they really didn't get fractions, but if I move on and do other things, they might, you know, be able to put something with their fractions, when we go back to it later.

(Year Three, Session Three, Oct 2002)

These newcomers' contributions suggested that they considered it safe to share their instructional difficulties within the group, and used them to illustrate both institutional affordances and constraints on their instruction and issues related to pedagogical decision making.

While the general norms of participation were evidently established in some activities from the very beginning of year three (e.g., during activities in which the teachers analyzed statistical data), they had stabilized across all types of professional development activities by session four. By that time, the newcomers were actively making sense during the group discussions, built on their colleagues' arguments, and voiced their disagreement. Along with the deprivatization of their practices that significantly progressed by session four, this was strong evidence that the newcomers were inducted in the professional teaching community in which they participated in progressively central ways.

Nature of Newcomers' Participation

Newcomers' Mathematical Reasoning

To prepare for the analysis of the shifts in mathematical and pedagogical reasoning that took place in the group, it was also necessary to understand whether and how the newcomers' forms of statistical and pedagogical reasoning differed from those of the old-timers. From session one, we engaged the group in statistical activities in which they participated as students and analyzed statistical data in order to address a question at hand. To facilitate the newcomers' participation, we modeled and explicitly negotiated both (a) obligations to listen to others' solutions and monitor own understanding, and (b) what was considered a convincing statistical argument. We paired the newcomers with the old-timers when they first analyzed statistical data using computer tools. During the analysis, the old-timers adopted a role of mentors: they explained options on the tool, answered numerous newcomers' questions, and let the newcomers propose how to organize data.

While much of the newcomers' initial contributions to the whole group discussions of data analysis revolved around what they did on the computer tool, they soon started to provide justifications for their statistical arguments. Data analysis discussion in session one lasted 35 minutes; the old-timers spoke 31 times and the newcomers 28 times (5.2 : 4.7). Two data analysis discussions in session two lasted 60 minutes total; the old-timers spoke 89 times and the newcomers 112 times (14.8 : 18.7). The relative frequency of the newcomers' participation was representative of all activities of this type during year three. With respect to the nature of the newcomers' contributions, all newcomers started to base some of their statistical arguments in the context of the problem scenario, at least when supported by the researchers, by session two. This was important given that, building on George Cobb and David Moore (1997), we hoped to support the teachers to develop an understanding of statistics that goes beyond identification of patterns among numbers, and attends to significance of those patterns in terms of the question at hand. The following excerpts come from session two where the group analyzed data on longevity of two brands of batteries in order to decide which brand is better to buy. Old-timers Muriel, Wesley, and Marci taught this activity in their classrooms prior to the session, and we asked them to lead this activity with the teacher group.

Jane (N): [Concludes her explanation] Our overwhelming idea was we would probably go with Always Ready [batteries] if it is the same price [as Tough Cell brand].

Muriel: Because you were basing it mostly on total hours [that the batteries last]?

Jane (N): Yeah.

Researcher 1: And why would you justify a focus on total hours, over all the other options? I mean, because you are right, you can move that bar [data separator] and you can find a place where it [the argument] goes one way or the other.

Hazel (N): Because we are consumer, not the company. …

Researcher 2: Why is it important for you for it to be total?

Researcher 1: Why is it a good statistics as a consumer?

Hazel (N): Because now you just wanna have more [battery life] for your buck.

…

Wesley: Brian, you were saying something about being on a boat.

Brian (N): Yeah, there are definitely some situations where greater consistency [in battery life] would make a difference. So, if you are on a boat, and you need a transistor radio to stay in touch with the outside world,

Lisa: Yeah,

Wesley: The perfect storm is coming,

Brian (N): The perfect storm is coming; you wanna have good batteries in that radio, that you are very confident will give you at least the minimum level of performance. Even if it is just 80 hours. But it might be a disaster for you if the battery that you had that time was one of the duds. And so you would sort of buy insurance in a sense, by making sure that you had Tough Cell batteries cause you know that there aren't any duds in Tough Cells.

Wesley: How does what your battery use is change how you look at the numbers?

Amy: You mean what you are using the battery for?

Wesley: What you're using the battery for. I mean that the situation, I think that's a very valid thing to say [gestures towards Brian], because when we are talking about what's the best battery, we don't really say the best battery for what? What's the best battery for being on a boat, what's the best battery for being in a house?

(Year Three, Session Two, Oct 2002)

It is important to note that at the time, neither the newcomers, nor the old-timers were basing their arguments in the problem scenarios consistently. Moreover, all the teachers had difficulties in reflecting on the importance of doing so, and most of them treated scenario stories to be somewhat external to actually doing statistics. I will return to this discussion in Chapter VII when I analyze learning of the community.

The two elementary-trained newcomers who initially expressed concerns about their statistical knowledge reported improvement in how they were able to participate in statistical activities in session two. The remaining four newcomers' participation in statistical activities was virtually indistinguishable from that of the old-timers. While there were differences in terms of individual teachers' statistical reasoning, these differences did not coincide with the teachers' newcomer and old-timer status in the group.

Newcomers' Pedagogical Reasoning

In terms of the newcomers' contributions to discussions that concerned pedagogical arguments, several exchanges between the newcomers and the old-timers evidenced that during the first two years, the professional teaching community developed a professional discourse that was not immediately transparent to and shared by the newcomers to the group. In other words, the newcomers and the old-timers initially constructed different meanings while they sometimes used the same words to talk about specific pedagogical situations. I documented two episodes in which differences of this kind became evident during year three. Both of these episodes occurred during session three.

The first episode involved the meaning of *joint planning*. It is important to note that three of the old-timers, Muriel, Amy, and Marci, found their insights into joint planning and how their understandings evolved important enough to bring this issue up when they explained the history and purpose of the group to the newcomers during the informal meeting.

Wesley: We probably were all there before. We all were independent – I just didn't have time to work with other teachers. And so now what we've done over two years is figured out our mistake and we wanna be part of- … First thing I hit you [to Brian, a newcomer from his school] with when I met you was "we are planning together."

Muriel: Now, planning is different.

Wesley: Well, OK?

Muriel: No, what I mean, the way I think of planning is different now. And when I say to my other teachers, we need to plan together. Well, they've already run out their worksheets, they know what page they are doing tomorrow, and just getting them to understand what I mean, you know, about looking at what the kids are doing, looking at what we've taught at the same time and what did their kids get through this, where can we go from this?

(Year Three, Informal Meeting, Aug 2002)

In her description of joint planning, Muriel (as well as Amy and Marci in episode cited earlier in this chapter) drew on the group history of activities in which they learned about Japanese lesson planning. She also suggested that this understanding of planning was not shared by other teachers in her school.

During session three, we asked the teachers to share what their opportunities were to talk to other teachers about mathematics instruction in their schools. A significant majority of the teachers, including three of the newcomers maintained that it was not easy or at all possible for them to coordinate their planning time with other mathematics teachers in their schools. This was due to scheduling conflicts and the fact that the school and team meetings never focused on instructional issues in mathematics. However, three newcomers stated that they talked regularly to other mathematics teachers and jointly planned their lessons. Two of these newcomers, Jane and Hazel, came from a school (Hawthorn) that was newly included to take part in our professional development program. During the discussion, it transpired that there were two different views of valuable joint planning in the group. One view was communicated by the three newcomers, and another by three old-timers: Muriel, Amy, and Marci.

Hazel (N): And we plan everything together, homework is the same in every class, and we do that so that if a child switches the teams that they would never be off the text. Same page, homework is always the same.

Jane (N): Sixth and our 8th grade, the two teachers, we sit down and everything is the same throughout. So we plan, and we talk daily

Researcher 1: [jumps in] OK,

Jane (N): [continues] within. But to talk to other teachers about math, I would say, never happens.

…

Researcher 1: What about Sinclair [school]? You were saying it is very hard to find time to even talk to each other.

Naomi: Yeah. [Marci also nods]

Researcher 2: Do you have a joint planning?

Marci: I know that 6th grade teachers …, I know they plan and just like you all, they have everything exactly the same. I am not sure about 7th grade, I do know that they communicate, and they talk a lot, but as far as the planning, I am not sure. But everything in 8th grade is so busy and so hectic that we have really had no chance to sit down even talk mathematics. We know that we are somewhere around the same area of teaching the same things, *but as far as sitting down and actually talking about what worked or may not work*, you know, just the planning together, we have not had a chance to do that.

Jane (N): And … it's actually decreased my workload, having someone like that. Because we use the, we have inner sessions, so we have three workdays you know, right back to back. Instead of working on report cards and stuff like that during our workdays, we sit down and we plan, so the other 8th grade teacher and I, we planned for a whole quarter at a time.

Helen (N): That's how we do it.

Jane (N): And so when we do that, we are like, ok, well, we need this, we need a test for this, we need a homework for, you know, and it's like, well I'll do this half, and you'll do this half and it really decreases the amount of workload that I have to do at my planning because you know she is doing some of it and I am doing some of it.

…

Amy: Can I ask, I am thinking, [talks to Jane and Hazel] you did not see this but Marci did, and the rest of people did, bearing in mind the Japanese version of planning, that we saw on the tape, OK? How much of what you do is purely math teacher administrative planning and how much is discussion of and analysis of the actual presentation of the lessons, the points we want to cover, could we have done this better? You know just bearing in mind that whole Japanese planning. And how they talk about all the activities and everything.

…

Muriel: That's the part that I see that's missing.

Kate (N): What part?

Muriel: We don't, even if we get together and know what we are doing, there is no time for getting together, it's like the students' work isn't part of the planning.

Kate (N): I can't even get together.

Muriel: I hear from other 7th grade teachers, "ah, my kids did terrible on the last test" or "Ah, my kids aren't doing the homework" or whatever. But, there is never time where we sit and look at their work and say, what did they get, what did they not get, and what can I do with *that*? You know, our emphasis on planning is "OK, let's store up stuff for the next whatever." But-

Amy: Like math without the proof.

Muriel: But, once you've done that, once you've done that, where is the time to come back and say, all right, we did this, what did they learn, what do we need to change, as we are going to the next step? From what we know they got or didn't get. That's what's missing.

(Year Three, Session Three, Nov 2002)

As it became clearer in the discussion, for the newcomers, joint planning included situations where they split an instructional unit with a colleague and only had to prepare instructional materials for a half of the lessons. As a consequence, they did not view joint planning as a time demanding activity, which was difficult to include in their work schedules. In contrary, it saved them some preparation time and made the job of teaching more manageable.

Amy, an old-timer, noticed an important difference in the newcomers' and her interpretation of joint planning and proposed that while some part of planning work is an "administrative planning," she was more interested in planning where she could discuss and analyze "the actual presentation of the lessons, the points we want to cover, could we have done this better." Similarly, Marci described joint planning as "sitting down and actually talking about what worked or may not work." While the newcomers mostly described what Amy referred to as "administrative planning," Amy, Marci, and Muriel all found it important to include assessment of the students' prior learning in joint planning. They also suggested, that such joint planning would be both time consuming and intellectually demanding. Wesley's comments (who was an old-timer) about analysis of student work later during the same activity suggested that he shared the three old-timers' views of joint planning.

The second episode took place later during the same session, when the group discussed reasons for students' failures to learn fractions. In that context, one of the researchers said that re-teaching the same lesson is unlikely to help, but it is often the strategy that the principals might propose, given their view of mathematics instruction that foregrounds coverage of the state objectives. In the discussion that followed, three old-timers agreed aloud that re-teaching is not going to be helpful. Later, three newcomers shared that they found it necessary to re-teach fractions in their classrooms every year. In this discussion, implications of students' not learning fractions were picked up differently by the two groups of teachers. The newcomers shared that they could not proceed with instruction unless their students caught up on the topics that they were supposed to learn in the earlier grades. In this context, students' not learning oriented the newcomers to talk about how they had to re-teach fractions.

Helen (N): When you get to the beginning of the school year, are they not teaching it in 7th grade again?

Marci: Are they not teaching what?

Helen (N): Fractions. Cause when I teach fractions, and I am telling you guys, I have spent my first year teaching, I spent three months on fractions.

Marci: Well, I've got a phone call this year [presumably from a parent] that it was inappropriate for me to teach percent of change when children have not seen fractions the entire time in middle school, which I knew was not true because the 6th grade teachers give some fraction-related concept on daily basis.

Helen (N): And I do.

Marci: It was not a fear for me to teach percent of change, since I haven't taught fractions before that.

Helen (N): See but that's what I am curious about. Like I am curious where they're missing, like if they are getting fractions every year, why are they still not, you know, -

Amy: [Fractions] May be called rational numbers in the 8th grade.

Muriel: Ok, this is 5th grade, 5th grade is [reads from grade objectives] "compare and order fractions with same numerators or same denominators and explain the solution" and "add and subtract fractions of like denominators and multiply a fraction by whole number, use models and pictures to add and subtract fractions and mixed numbers with unlike denominators." That's a 5th grade. "Record solutions."

Helen (N): And see, I re-teach that [in 6th grade].

Kate (N): We re-teach that too [in 6th grade].

Helen (N): I re-teach that in 6th grade, the whole thing.

Erin (N): And I re-teach that in 8th grade.

Kate (N): My 6th graders have a problem actually understanding what 1/2 means.

Researcher: The assumption is-

Helen (N): [Helen jumps in] Where are they loosing it?

[Many teachers talking]

Researcher: The point is that they've been taught this before.

Helen (N): Right,

Researcher: And then you get the kids, and-

Esther: [Esther jumps in] Probably someone has to re-teach it after you.

Helen (N): Right, and see, that upsets me, because I know how much time I spent on it, and so that goes back to the point of this whole discussion of why are they not understanding.

(Year Three, Session Three, Nov 2002)

In contrast, several old-timers acknowledged that teaching the same content in the same way again was not a sensible strategy. To them, students' not learning was an evidence of less then optimal instructional approach. Instead of proposing that she would have to re-teach, Amy suggested that she would have to learn and try a different approach in order to better support student learning.

Researcher: These are kids who were in your algebra class, it's 8th grade, and you've figured out that this is how they think about fractions. In some very strange and non-standard ways, shall we say. What does that mean for you as a teacher?

…

Amy: First thing is, if I were a teacher, I would personally think that I need some re-training. In how to go about assessing student learning. It's not just the numbers. I need to come up with some way [to understand where the problem is].

Researcher: Would it be adequate for you just to re-teach this stuff?

Muriel: No, see, that's what we need to show them [the principals].

Amy: [Responds to researcher's question] I've already been teaching it. [Intonation suggesting, "What would be a use of doing it over again?"]

Researcher: Why not? Why don't you just re-teach this stuff?

Amy: No.

Researcher: Why? Why is that not a productive way?

Naomi: Because you [quietly together with researcher's repeated question]

Muriel: And that's what we're wasting time every year, teaching the same stuff over and over again.

(Year Three, Session Three, Nov 2002)

While the newcomers focused on pragmatic solutions they used in their classrooms when addressing problems of students not learning, several old-timers constituted this discussion as an opportunity to point out the problematic nature of strategies that they typically resort to and to highlight need for further learning. The issue of principals proposing to re-teach in situations when students are not learning briefly surfaced one more time, during session six, when the group planned the fraction interview activity to engage the principals.

Helen (N): But I re-teach. I mean, and so I don't understand what the problem with that is even. Maybe I am just being oversensitive, but it's just my feeling.

...

Wesley: I think when you say you re-teach, nobody said you are doing the wrong thing by re-teaching because when you re-teach, you use your skill as a teacher to analyze and understand what they missed.

Helen (N): Right,

Wesley: And go back and teach it again. And there is a difference between that and "Do the worksheet 7 again"

Helen (N): Right.

Wesley: ... What I learned from interviews, I got better at asking the kids questions. You know, as I was walking around the room and saying, "Why did you put it on that paper that way" or "Draw me a picture that shows me that." And so the standard re-teaching that we talk about as kind of a bad thing is doing it blindly. ... I am much more likely to continue a conversation about fractions as I go to equations. Whereas somebody else might say "You know, let's spend 10 minutes again on fractions" or "Let's stop for two days and do fractions." So re-teaching [for you] is a whole different [idea than for the principals], that's not a bad thing.

(Year Three, Session Six, Mar 2003)

Helen's expressed confusion provided an opportunity for Wesley to contrast a view of re-teaching as a simple process of going over the same textbook pages again and the complex work of teaching that is involved in situations when students experience difficulties to learn. It was this complexity that the group hoped to help make visible for the principals.

These episodes illustrate that even when the frequency of newcomers' contributions was most of the time comparable to that of the old-timers, there were discernable differences in the meanings that the two groups created when discussing pedagogical issues, as well as in purposes they pursued in the discussions. The differences in the nature of participation between the groups of the old-timers and the newcomers evidenced both the newcomers' limited opportunities to develop collective pedagogical practices and ways to talk about them in their school settings, and the old-timers' prior learning that took place within the professional teaching community.

It was in discussions of this kind that the meanings were explicitly negotiated, and the purposes of the discussion could align. In this way, the newcomers were continually being inducted in the professional discourse as they participated in the whole group discussions. At the same time, the professional discourse of the community continued to develop as the group engaged in these negotiations. I further document developments in the professional discourse of the community in Chapter VII.

The developments that I have documented so far suggest that the newcomers' inclusion did not result in discontinuation of the professional teaching community. Rather, it initiated a process in which the researchers and the old-timers jointly worked to support the newcomers' induction to the community. In this process, the newcomers' started to adopt ways to interact within the community that were at the time normative among the old-timers, deprivatized their practices for purposes of communal learning, and participated in constituting professional discourse and in shaping shared purposes of the community.

Year Four Analysis

The detailed analysis of initial participation of teachers recruited to join the professional teaching community in year four revealed patterns remarkably consistent with those found in year three. At the beginning of year four, eight teachers remained in the group[24] and four new teachers were recruited to join. Two of the year-four newcomers[25], Dorothy and Ben, came from schools where one of the old-timers taught and remaining two, Bruno and Josh, came from a school that did not previously take part in the professional development collaboration, but was regarded highly within the district for quality of students' mathematical learning.

The selection of the new recruits illustrates how the district mathematics leaders had come to view the professional teaching community. The district leaders planned to adopt a new NSF-funded textbook series to replace the NSF-funded series that the district had been using for the past seven years. They hoped that the new textbook series and the resources provided to the teachers during the adoption process would encourage more mathematics teachers to use it as their primary instructional material. This was a priority for the district leaders because many middle school mathematics teachers continued to use a traditional textbook series as their primary material. The district leaders planned to pilot units of the new textbook series in several teachers' classrooms during year four of our collaboration. Following this pilot, they planned a district-wide implementation during year five. In relation to the planned adoption process, the district leaders viewed the members of the professional teaching community as future mathematics teacher leaders in the district. They therefore pushed for selecting the year-four newcomers from among the most accomplished mathematics teachers in the district. In contrast to year-three newcomers, where two teachers were elementary-certified and concerned about their mathemati-

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[24] Out of the 12 teachers that participated during year three, one (an old-timer) accepted an administrative position in the district and continued to participate in the professional teaching community only occasionally as a peripheral member, two teachers (year-three newcomers) moved out of the district, and one (a year-three newcomer) stayed home with a newborn child (see also Appendix A).

[25] Further on, if not specified otherwise, I use "newcomers" to refer to the most recently recruited group of teachers and "old-timers" to the rest of the teacher members of the community. As I documented earlier, year-three newcomers adopted the normative ways of participation before beginning of year four, and thus participated like old-timers during years four and five.

cal competence, all of the year-four newcomers were viewed as mathematically competent, and were confident in their abilities to learn more mathematics.

I provide an abbreviated account of the findings on continuation of the community at the beginning of year four, given that the analysis that produced these findings closely followed analysis of year 3. My intent is to illustrate the developments in the group interactions that were consistent with those documented in year three.

Norms for General Participation and Norms of Institutional Reasoning

As was the case in year three, the continuing teachers and the district mathematics leaders organized an informal meeting with the newcomers at the district mathematics specialist's house. A graduate student who attended as a note-taker observed that the teachers already knew each other and that the newcomers were eager to become part of the group. They asked pragmatic questions about their participation in the work sessions and listened to the old-timers' descriptions of what typical sessions were like. During the informal meeting and the initial professional development sessions, like in year three, the old-timers provided explanations, elaborations, routinely built on other participants' arguments, and indicated disagreement in non-threatening ways. They also shared their instructional difficulties and positive valuations of open collaboration. In addition, several old-timers shared how they worked to improve their instruction based on the work session activities. In doing so, they portrayed the purposes of the professional development sessions as being closely related to their classroom instruction. As a result, the old-timers established a safe and inviting environment, and portrayed the sessions as worth the newcomers' time and effort.

Newcomers' participation patterns were also similar to the year three newcomers. They started to ask questions, build on others' arguments, and voice disagreement during session one and systematically did so across all types of activities except for the first activity that involved a summary of the statistics instructional sequence in session two, and the first activity that involved the analysis of classroom video in session four. In addition, the newcomers were relatively open about their instruction from the outset: Two newcomers shared problems that they experienced in instruction during session one. All newcomers willingly agreed to bring their students' work to sessions three and later, and two newcomers agreed to co-teach and videotape their statistics lesson after they had engaged in video analysis activity for the first time in session four. Nevertheless, Dorothy—a newcomer—shared that she felt better about bringing in her students' work after she could see that her students' performance was "normal," that is, similar to performance of students of other participants.

I conjecture that the newcomers' rapid deprivatization of their classroom practices had to do both with the perceived competence of the recruited teachers (i.e., those, who were chosen to potentially become teacher leaders in the textbook adoption process) and with the fact that they constituted a smaller proportion of the community in which it was normative to describe one's own instructional problems in order to make progress collectively. Choice of the initial professional development activities also contributed to inclusion of the newcomers. In year four, the initial activities built on issues of student motivation that the community had begun to explore during the summer workshop. These activities did not create expert-novice situations as much as the activities with long history in the community that were used at the beginning of year three (e.g., statistical activities, video analyses). Instead, the focus on student motivation allowed several newcomers to participate as experts in the initial debates. For example, in session one, we asked the teachers to imagine themselves in two situations. First, we asked them to imagine that they were stopped by a policeman for speeding and elected to go to a Traffic School to avoid an increase in the cost of their auto insurance. The teachers first reflected individually, and then discussed (a) what they thought were some things that they might find useful by attending the Traffic School class, and (b) whether they would be interested in engaging in Traffic School activities. We then asked the teachers to think about their specific hobbies (teachers chose sports such as basketball, running, dancing) and reflect on (a) why they expended time and effort to engage in those activities and to improve their performance, and (b) describe how they behaved during such activities. One of our goals in this activity was to support the teachers in imagining themselves in a situation where they would not be motivated to participate and then in a different situation where their motivation to participate was strong. As I elaborate in Chapter VII, our purpose was to challenge the teachers' idea that motivation and lack of motivation are inherent characteristics of students, and to encourage them to adopt a student's perspective when examining instructional events. This activity, where the teachers' life experiences were the point of reflection, enabled the newcomers to fully participate from the outset. In fact, because he had recently received a speeding ticket, Josh, one of the newcomers, played an important role in helping others understand what happens in Traffic School classes.

Cases of newcomers' non-participation. The first activity in which the relative frequency of newcomers' participation was low was a 34-minute discussion of *big ideas* of statistics instructional sequence that took place at the very beginning of session two. The old-timers contributed 21 times and raised a number of important, specific, and rather complex issues related to the statistics sequence, such as how to get students in the "game" of analyzing data when they want to be told what should they calculate, and the challenges involved in helping students who organize data into three groups with the same number of datapoints in each to instead start using four equal groups tool—a precursor to box plots. In contrast, only one newcomer contributed, agreeing that it was important for students to understand *why* something holds but that her students were often not used to think that way. The newcomers' lack of contributions to this very specific, short activity was understandable, given that they only engaged in one statistics instructional activity as students up to this point.

Although we postponed the first video analysis activity until session four[26], it was again problematic in terms of the relative frequency of newcomers' participation. Specifically, during 55-minute activity, the seven present old-timers contributed 60 comments whereas the three present new-timers only spoke up 5 times[27]. That is, instead of proportional 30% of contributions, the newcomers only accounted for 7.6%. Because the newcomers had demonstrated competence in interpreting students' statistical solutions during two analyses of students' written work, the conjecture that their non-participation in video-analysis was because they were yet to learn how to interpret students' statistical solutions (developed in year 3 analysis) does not seem plausible. This suggests that while relatively solid mathematical knowledge for teaching statistics is key to central participation in analyses of classroom videos, it is not in its own right sufficient. As the year 4 newcomers' practices were not private at time of the first video analysis, this case of non-participation also indicates that deprivatization, while critical in pedagogical discussions that could be consequential to teachers' practices, also does not guarantee teachers' successful participation in analyses of classroom video. The remaining conjectures developed from the year 3 analysis regarding newcomers' non-participation remained plausible, specifically that they were learning (a) how to participate in respectful pedagogical discussions which were not typical in their school settings, and (b) what to focus on in analyses of classroom video.

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[26] In year three, we used first video activity in session two.
[27] One old-timer and one newcomer from the same school could not be released to participate in professional development session on that day.

In addition to the old-timers' experience with analyzing classroom videos, it is important that 5 of them met before the session for a "movie night" at the district mathematics specialist's house to preview the video and prepare for the session discussion. The movie night was open to all members of the community. While some of the old-timers valued this opportunity to prepare for the session, none of the newcomers seemed to consider it important. All the teachers had access to the classroom video ahead of time, but watching it alone was not sufficiently supportive for the newcomers to participate in the session discussions. A movie night was also scheduled before the second video discussion, but only 3 old-timers met. Despite this, the newcomers' frequency of contributions in the second video activity (in session 5) was higher than proportionally expected (49% compared with expected 40% of contributions[28]), suggesting that they developed ways of contributing that were constituted as appropriate in the community.

Video-analysis of classroom instruction was a highly specialized pedagogical activity with a long communal history. Given ways that we structured this activity in the professional development sessions, observation and intent listening appeared to be the only ways in which the newcomers could participate initially.

Newcomers' Mathematical and Pedagogical Reasoning

In comparison with year 3, the ways of reasoning about statistics that were constituted as normative in the community evolved. As I document in Chapter VII, it became normative for the old-timers to justify statistical arguments in the context of problem scenario. In addition, their view of the purpose of the task scenario now extended beyond enticing students to do mathematics, and several old-timers considered it important that the students come to see the problem they were to address as significant.

Interestingly, the newcomers justified their statistical arguments in terms of the situation at hand from the outset, asked sharp questions about data generation process when they engaged in statistical activities as students, and one of them, Bruno, demonstrated a deep understanding of bivariate data distributions when he talked about an instructional activity that he used in his classroom. The data did not reveal systematic differences between the old-timers and the newcomers in their statistical reasoning.

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[28] Four newcomers and 6 old-timers participated in Session 5 of Year 4. Two old-timers did not participate for school related reasons.

In contrast, there were clearly documented differences in the newcomers' and old-timers' pedagogical reasoning. I describe two episodes that illustrate how the newcomers initially constructed different meanings while using the same words as the old-timers to describe and make sense of specific pedagogical situations. The induction of the newcomers in the community therefore included the explicit negotiation of the meaning of these terms. The first episode took place during session one, when Bruno, a newcomer, attempted to understand Wesley's comments on how he is getting better at assessing what students understand and why they might not understand some ideas while they are analyzing data in small groups. Bruno asked if learning what to focus on was a matter of *experience*. Three most vocal old-timers indicated that they know "experienced teachers" who "cannot do it," and who are "gonna stand in front of their classroom and talk." It became clear that these old-timers came to value understanding students' work, and they knew that they have not developed this capability through experience alone. In the old-timers' view, the notion of an "experienced teacher" was not a useful in describing what a teacher needed to learn about students' solutions during a lesson.

The second illustrative episode took place in session five, when Bruno and Josh, two newcomers, brought in their classroom video. After the teachers watched the data generation phase of the lesson, Naomi, an old-timer, pointed out that Bruno and Josh did not organize that part of the lesson as a *classroom discussion*. Instead, Bruno asked a series of well-focused questions that students answered briefly, and attempted to draw the students' attention to a number of facts that that he considered significant within the problem scenario. Naomi explained that the students did not talk enough and some of them misinterpreted the task when they began to analyze the data. She also clarified that when students do not talk enough they do not have sufficient time to develop questions. Bruno and Josh' approach of asking specific, known-answer questions and soliciting students' responses revealed their view of a classroom discussion. However, the long pause after Naomi's contribution, and the fact that none of the old-timers offered a comment that would soften her critique indicated that they found Naomi's concern valid. The group then discussed the importance of students developing a sense of how to conduct an experiment that would result in relevant data, and what should be measured. Bruno and Josh agreed that their data generation did not support students' development of these insights.

The differences in the old-timers' and the newcomers' pedagogical reasoning further supported the conjecture formulated during analysis of year 3, that the professional teaching community had developed normative ways of reasoning pedagogically that differed from those of most other teachers in the district. These differences led to the explicit negotiation of meanings in the course of which the old-timers attempted to justify their views, and the normative ways of reasoning of the professional teaching community were contested. In all such situations that I documented in the data from years 3-5, the newcomers accepted the old-timers' and researchers' justifications and gradually began to participate in activities in ways compatible with those of the old-timers. However, it would be an oversimplification to conclude that they had no other option but to do so. It was important that the norms of general participation within the community obliged the teachers to scrutinize others' claims and justifications. Pseudo-agreement was not an acceptable way to participate. The manner in which the newcomers' participated across different types of professional development activities indicates that the old-timers succeeded in supporting the newcomers in coming to view their justifications as both reasonable and potentially more useful when attempting to improve instruction.

Conclusions

The analysis of the newcomers' participation in professional development sessions provides insights that are relevant to both (a) documenting actual learning of the teacher group and to (b) making design modifications that might be beneficial for future efforts to support the learning of professional teaching communities. First, the analysis supports the conjecture that it is reasonable to speak of the professional teaching community continuing across the years and changes in membership. It is therefore justifiable to document shifts in practices of the single professional teaching community across years 3-5 of the collaboration.

Second, the analysis provides initial evidence of substantial communal learning. This was especially apparent in the negotiations of meaning between the old-timers and the newcomers. These negotiations indicate that the old-timers' pedagogical reasoning differed from the pedagogical reasoning that most of their colleagues had developed while working in the district.

Third, this analysis reveals cases in which the newcomers' participation was not sufficiently supported. For instance, in the initial video-analysis activities in both year 3 and year 4, the newcomers in the group did not initially have a way in which to meaningfully contribute to the group discussions. It is worth exploring whether small group discussions of the classroom video episodes, where the newcomers and the old-timers would come together in a less public setting, could provide the newcomers with better access to forms of participation in video-analysis activities that are constituted as adequate in the group.

Last, it appears that the group benefited from introductory professional development activities in which the resources on which old-timers' and the newcomers' could draw were more balanced, such as in the case of the *Traffic School and Hobbies* activity. Activities that do not draw heavily on the old-timers' history of participation in the community appear to provide more opportunities for the newcomers' participation, and therefore also more opportunities for the newcomers to uphold and contest the general norms of group participation.

CHAPTER VII

LEARNING OF THE PROFESSIONAL TEACHING COMMUNITY

In this chapter, I analyze the realized learning trajectory of the professional teaching community in years three through five of the professional development collaboration. In Chapter VI, I discussed the norms of general participation that were established in the professional teaching community, as well as norms for institutional reasoning that related to deprivatization of the teachers' practices. These normative ways of participating provide background for the account of the successive forms of institutional, mathematical and pedagogical reasoning that became normative as the community engaged in professional development activities. I first attend to several activities during year three that were designed to support the teachers in gaining access to resources that were controlled by the school leaders. I also explain why the community did not pursue this strand of activities in years four and five. I then discuss the subsequent forms of mathematical and pedagogical reasoning that became normative in the community. In doing so, I focus on pedagogical developments and show that the teachers' pedagogical interests and concerns were the driving force behind their attempts to deepen their statistical and mathematical understandings.

Throughout the analysis, I also attend to the activity of the research team by documenting our ongoing decision making. The resulting record of the process of testing and revising conjectures about supporting learning of the community will serve as a basis for formulation of a revised trajectory for supporting learning of a professional teaching community that synthesizes what we learned.

In Chapter II, I introduced the notion of a conjectured learning trajectory that guides researchers' local design decisions about how to support the further learning of a professional teaching community and is itself revised as these decisions are made. It is important to clarify that the conjectured learning trajectory that oriented our decisions about how to support learning of the community in year three was already a modification of the initial conjectured learning trajectory that was formulated at the outset of the design experiment. In Chapter V, I outlined the starting points for the communal learning at the beginning of year three. As I proceed through the different strands of the analysis, I summarize the goals that oriented our design decisions and explain how we conjectured these would contribute in proceeding towards the overarching goals for the learning of the community. I also outline the conjectured means of supporting the communal learning.

Norms of Institutional Reasoning

Goals and Conjectured Means of Support

During year three, we planned to further support the teachers in gaining access to institutionally controlled instructional resources (e.g., time for teacher collaboration) and in helping their school leaders to become more effective instructional leaders in mathematics. We viewed this strand of the professional development effort as a means of supporting sustainability of the professional teaching community beyond our collaboration with the teachers. We therefore planned to build on the professional development activities from the year two summer session, during which the teachers had outlined a conjectured trajectory for supporting the school leaders' development of deeper understandings of mathematics teaching (for an outline of the trajectory, see Appendix D). In particular, the teachers had conjectured that it would be useful to problematize the school leaders' views of mathematics instruction[29]. They had suggested inviting the school leaders to observe how middle school students performed on conceptual fraction interviews (for interview tasks, see Appendix C).

We intended to explore the possibilities for the teachers to support the school leaders' learning. We also planned to devote time in the sessions for updates on institutional changes in the teachers' schools with respect to lesson time, scheduling of teachers' planning periods, and teachers' actual opportunities for collaboration on instructional issues in mathematics. At the time, we considered such updates to be important both for planning the interventions with principals, and for supporting the teachers in coming to view the professional development activities as directly relevant to their work in schools. Our ongoing analysis of the purposes that these activities have accomplished contributed to the formulation of the bi-directional interplay orientation to professional development (Zhao, 2007).

[29] The reader will recall that the analysis of the institutional context conducted during year two pointed out that the school leaders viewed mathematics instruction as a simple and straightforward activity, in which content coverage results in students' learning.

The major resources that we considered when designing the activities within this strand were the teachers' understanding of the institutional context of their schools, the old-timers' histories of participation in activities with fraction interviews and their subsequent insights into students' thinking, and a draft of a conjectured trajectory for supporting the school leaders' learning. In addition, we benefited from the support and brokering skills of Ruth and Esther, the district mathematics leaders who had become members of the professional teaching community, in arranging meetings with the school leaders. Ruth attended every session in years three through five and participated as a full member. Esther fully participated in summers three and four, and attended half of the school year sessions in those two years. In addition she joined the group for lunch during most of the sessions which she could not attend.

Realized Trajectory

From the teachers' perspectives, focus on the institutional contexts in which they worked was an important part of the professional development at the beginning of year three. The old-timers discussed this focus as one of the four major issues when they introduced the newcomers to activities and goals of the group during the informal meeting prior to year three sessions (see Chapter VI). Specifically, they described the need to support school leaders' understandings of what high quality mathematics instruction involved, and why this was important to the group's attempts to modify classroom instruction.

During each session of school year three, the group discussed issues related to the institutional context of the teachers' schools. Ruth and Esther actively participated in these activities. In sessions one, two, and three, we asked the teachers for updates on their interactions with school principals regarding resources for mathematics instruction in their schools. In a 50-minute activity in session one, most of the teachers (7 out of 13, from 4 out of the 5 participating schools) reported that their principals' agenda was managerial rather than instructional, and that the principals remained primarily interested in test scores.

Although the teachers were not satisfied with the situation in their schools, they did not merely share their complaints. Instead, they asked or conjectured what they could do in order to change their principals' views. The following excerpts illustrate the nature of these discussions:

Muriel: Do you think if we bring out more how much they [principals] don't know about math education or math in general, you know [would they start to listen to our concerns]?

[Many teachers speak at the same time]

Researcher 1: [to Muriel: You mean, bring out those issues] With principals?

Muriel: Yes.

Naomi: She [Naomi's principal] already said "I don't know [math]."

Muriel: But if we bring that out enough, maybe they'll see the need that we need to plan together, because we are not getting it [how to improve] from them. My principal can't tell me how to teach math. He couldn't teach it themselves. You know?

Marci: Yes,

Muriel: And maybe that will help back up that we have to get it from somewhere else. Instead of them.

Naomi: I am not sure they are ready to accept the help from the math teachers. But this is my school, "All you have to do is make sure that the scores are up."

Muriel: Uhm [I see].

Naomi: "I don't want to hear what your suggestions are." Believe me, I've tried some. "I don't want to hear [not understandable] but I don't like the subject, I am not good at it." She [the principal] don't even come to observe my classes, ok?

Muriel: Uhm [yes].

Naomi: "But you make sure that you get those scores up because," because of whatever reason, ok? What can we do to help them accept that we need some reform? This is my question. What would it take? What can we do?

…

Pat: One of the things that we might want to look at is how do we recognize student understanding. How can we show that our students are really learning. We have to be able to recognize it ourselves to feel confident that we are seeing it, too. Because the principals are only relying on these test scores.

Researcher 2: That's right, they are. Exactly.

Pat: And we want to look at it another way. How can we recognize it? … How do we know that these students are understanding?

Researcher 2: And maybe another step further, to show if you try to look at what students are understanding, how much better off you are than if you just look at test scores.

(Year Three, Session One, Sep 2002)

While the teachers shared their frustrations about the institutional context of their schools, their contributions also reflected the assumption that they needed to find ways to actively work on changing the sources of these frustrations. None of the teachers, old-timers and newcomers, questioned the need to change aspects of their institutional contexts or the possibility to do so. As the teachers participated in the group activities in year three, they collectively regenerated the norm of viewing the institutional context as something they could affect. Their participation in the subsequent sessions supported this conjecture.

One of the teachers, Muriel, often reported on changes in her school that she tried to negotiate. In a 12-minute update in session two, Muriel reported that her principal agreed to change the lesson schedule to a "block schedule" in which the time of periods would be doubled. Other, more profound changes that she attempted to support, such as systematically working with other mathematics teachers in her school on instructional issues, were not successful. Muriel's colleagues kept their practices private and were not willing to let others in their classrooms. Muriel thus faced similar institutionally shaped issues that the research team faced at the beginning of the collaboration, and illustrated that these cannot be easily re-shaped by a single teacher. While teachers from the remaining four schools mostly continued to report on their problems, for example that they had no time to meet with their colleagues to collaborate on instructional issues, changes to institutional context were not constituted as merely a topic for academic debate. The group started to discuss plans to invite the school leaders to join one of the upcoming professional development sessions for two hours.

During a 90-minute activity in session three, the group first re-constructed with the newcomers the grounding activity in which the old-timers engaged during the year two summer session. Specifically, the group discussed notes that the teachers had created during the summer session: (a) lists of the major weaknesses and the supports available in their district, and (b) the learning trajectory for school leaders (see Appendices E and D). It was our goal to focus the teachers' attention on the resources that they collectively saw as most critical for effective instruction that builds on students' reasoning. In this context, all the old-timers reported on their further attempts to negotiate planning time and time for instruction with their principals and all the newcomers shared their views of opportunities, or the lack thereof, to collaborate with other mathematics teachers in their schools. In addition to highlighting the key instructional resources, this discussion was also productive in that it provided opportunities for the newcomers and old-timers to negotiate the meaning of terms used when reasoning about instruction. As I reported in Chapter VI, differences in meanings of *joint planning* became the topic of explicit discussion and clarification.

Later during session three, the group planned the fraction interview activity for school leaders. I discussed this planning in Chapter VI with respect to the newcomers' lack of participation. The reader might recall that the newcomers were not able to participate substantially in this planning activity as the activity for school leaders had to be prepared relatively quickly and required that the old-timers built on the group history, which was not made easily accessible for the newcomers. With respect to present analysis, it is only important to note that the ways in which the old-timers contributed during this activity clearly and consistently suggested that their intent was to support the school leaders in understandings that teaching mathematics is a complex and demanding endeavor.

The teachers' exchanges during the updates on the institutional context of their schools supported the conjecture that the communal norm of treating the institutional setting as something the teachers could and should actively influence were being regenerated. The old-timers consistently promoted the goal of challenging the school leaders' views of mathematics teaching and learning and through this obtaining control over essential resources needed to teach mathematics with a focus on student reasoning. At the same time, the newcomers did not question legitimacy or validity of such goals and over time themselves increasingly contributed to the discussions.

Leading activity. It is important to highlight that the updates on the institutional context of the teachers' schools served as one of the means to negotiate the primary motivation for the teachers' participation in the professional development sessions. As the excerpt above illustrates, these discussions often led the group members to explicitly relate the focus on student reasoning both to their institutionally shaped instructional practices, and to access to key instructional resources. The data supports the claim that the overarching leading activity for the teachers was one of influencing the institutional contexts in which they worked, rather than merely complying during the activities that we assigned in the professional development sessions. The teachers contributed to planning some of the professional development activities (e.g., fraction interviews) and their contributions suggested that the goals they were pursuing were goals-for-themselves rather than goals-for-others. I further tested this conjecture throughout the analysis of the remaining data.

Role of the district mathematics leaders. Esther and Pat, the district mathematics leaders, repeatedly supported the groups' efforts. For example, when the Washington Hill[30] school district extended an invitation to the professional teaching community to attend their workshop related to an NSF-funded curriculum that Jackson Heights contemplated adopting, Esther agreed to cover part of the travel expenses from the district grant. As a result, a team composed of Pat, Muriel, Amy, and Muriel's principal traveled to the Washington Hill district, observed mathematics instruction in several classrooms, discussed the ways that the mathematics instruction was organized in the two districts, and participated at the curriculum workshop. During session four, all four team members reported on their Washington Hill visit to the entire group. In doing so, they focused on the differences in how the two districts organized for mathematics instruction. This 50-minute discussion included considerations of organizational features (e.g., mathematics coaches) that Jackson Heights district might need to adopt if they decided to implement the same curriculum.

In addition to providing material resources, the district mathematics leaders also attempted to broker between the professional teaching community and the school leadership communities. On the one hand, Ruth and Esther's contributions during the professional development sessions helped the teachers to better understand the views and rationales that guided decisions of the school leaders. On the other hand, Ruth and Esther worked to create opportunities for the school leaders to learn about mathematics instruction. Specifically, Esther invited a small team of three teachers to join her during one of her monthly meetings with the school leaders. The goal was to engage the leaders in some of the activities intended to support the leaders in coming to view mathematics teaching as a complex endeavor. Because of last minute meeting changes, the teachers were unable to participate, but Esther actively communicated her positive

[30] Washington Hill was a second district involved in a parallel professional development design study. The district was also located in a state with a high stakes assessment program. It was selected as a contrasting case to Jackson Heights for the differences in institutional context of schools and the district.

valuations of the professional teaching community and arranged for the school leaders' participation in the professional development session six.

School Leaders' Visit: Fraction Interviews

The school leaders' visit in the session six was viewed as an important step in introducing changes in the institutional context of the teachers' schools and gaining access to needed resources. In line with the earlier group discussions, the professional teaching community members' major goal during the session was to help the school leaders' problematize their views of mathematics teaching as a straightforward and simple endeavor. Our conjecture was that what the school leaders would see in fraction interviews would not fit their assumption that students either "get it" or not. Instead, they would realize that students frequently "get something different." The group planned to help the school leaders realize that despite repeated re-teaching of fractions in prior grades, the interviewed students' understandings of fractions often do not align with conventional mathematical understanding. To make a real advance in terms of students' understanding, a straightforward practice of re-teaching fractions to the students again would likely not be effective.

At the beginning of the session, before the school leaders joined the group, the group revisited the goals for the planned activity and discussed the teachers' roles during the student interviews. In particular, the researchers clarified that the goal was to understand what the students knew and understood about fractions, not to teach them to solve the problems. The group also discussed how the activity would be organized.

Wesley: So, I try to recap here, there is launch: small group [discussion of school leaders and teachers], gather the information from all the groups quickly,

Researcher: Yeah,

Wesley: that will go on to chart paper or the notebook paper on to

Researcher: Yeah, somebody take notes and they can write it up afterwards.

Wesley: Then we've got to do the interviews, and then we – do I get to sort of debrief with [the school leader in my group]? And then we come back?

Researcher: Yeah, they got 45 minutes.

Wesley: To interview or to debrief?

Researcher: To interview and do, if you want to do an individual debrief. Because we need 45 minutes to come back. Because there is a number of groups, and hopefully some issues would come up out of that conversation.

Erin (N): When you say debrief, what do you mean? Like,

Wesley: You've got to have time to detox of what you see. It's just like "Oh my gosh, did you see that happen?" That's debriefing.

Erin (N): About student? Ok.

Wesley: Yeah.

Erin (N): I did not know if you meant like debrief about what principal have said.

Wesley: No, we'll do that after they leave.

Erin (N): Yeah, ok. [All laugh]

(Year Three, Session Six, Mar 2003)

The activity with school leaders was planned to be "launched" in groups of four—two school leaders and two teachers—addressing the following questions:

1. What should be the goals for teaching fractions?
2. What do you anticipate as the main challenges in teaching fractions?
3. Suppose you are a middle school teacher and find most of your students have not mastered fractions. What would you do?

The major goal during the launch was to make the school leaders' current knowledge about fractions explicit. Each school leader would then pair up with a teacher to interview one student on a series of fraction tasks (Appendix C) and briefly debrief on the interview before joining the other participants for the final discussion.

When talking through the different phases of the activity, the teachers considered the school leaders' participation from the school leaders' perspective. Four teachers explicitly raised issues about how to put the school leaders in groups with the teachers so that the school leaders would be comfortable sharing their opinions, and would not feel evaluated. A fifth teacher raised concerns about the school leaders' backgrounds in mathematics and the possibility that they might feel offended during the activity. These issues were considered legitimate within the group and the suggestions for changes were incorporated during planning (e.g., placing two school leaders in each small group together rather than just one).

Participating students. As part of planning, the group discussed both logistics and details of conducting the student interviews. The old timers shared their experiences from the fraction interviews that they had conducted during year two while the newcomers exhibited an active interest in specific details. I discussed the dynamics of participation in this activity in more detail in Chapter VI. The following discussion of the students who were to be interviewed is especially relevant to my analysis of the activity with the school leaders:

Erin: Now, the kids we are interviewing to— that understand their—? Your upper level?

Wesley: I was kind of [going to bring] more middle [performing kids].

Erin: Middle. The kids who would have passed the [state tests], right?

Researcher: We looked at level three, we asked for the level threes. [State tests evaluated students as performing at one of four levels, with level four being the highest.]

Wesley: Yeah. You know, I am trying to balance articulate as well as, well enough to [not understandable]. Because this is a pretty [important situation], they've got to be *on*.

Erin: Right.

Wesley: You can't not be on. But yeah, I mean level threes. But you'd be amazed what level four can't do.

Erin: Yeah.

Wesley: Or can't articulate.

Erin: That's what I think is important too, for the principals to hear it

Wesley: [Jumps in] But these will all be level threes. Threes and fours.

Erin: that these are level three and four kids. You know, when you say this kid is level three and level four, I know [my principal] would say: "That's level three and that's level four, therefore they can do the math. That's a good math student."

[Teachers nod]

Erin: Well yeah, *but*.

(Year Three, Session Six, Mar 2003)

As this exchange indicates, the teachers considered it important that the school leaders would view the students they interviewed as good mathematics students. At the same time, the goal was to help the school leaders understand that state test evaluation did not necessarily indicate deep mathematical understanding. Therefore, it was important that the selected students would be relatively typical for the teachers' classrooms, which meant not fluent in explaining quantitative relations involved in the fraction tasks. Wesley's comment "But you'd be amazed what level four can't do," suggests that he used these criteria in selecting the students. At the same time, he found it important that the selected students would be "on," possibly meaning on task, or be willing to complete assigned tasks to the best of their ability.

Participating school leaders. Four school leaders from four out of the five participating teachers' schools joined the professional development session at the beginning of the activity. The school leader from the school which hosted the professional development session had to leave but his colleague joined the group while the students were interviewed. In the initial small group discussion, the school leaders identified issues important in teaching fractions that went beyond students' proficiency in calculations. All four school leaders mentioned that students needed to be able to both understand why fractions are important and use them in real-life situations. Two school leaders with a background in social studies discussed the need for use of fractions outside mathematics, in other disciplines, as illustrated in the following excerpt:

Wesley: The objective we have is when the students come in here, you will pair up with a teacher and watch them do math. That's when we really start to work. And so topic that we will be talking about today is fractions. So. We are [now] gonna take some notes about teaching fractions mathematically, and share that up with the other two groups … just to get us started about fractions. That's really what we are doing here. So what have your experiences been? As a student, as a teacher? … What is your background? Education? Were you in a math classroom?

Mrs. Jonash: My educational background was social studies. And special education.

Wesley: So you experienced talking about 70%— three tenths of world is water.

Mrs. Jonash: Fractions will come into place with anything. You can work it in pretty much with any other issue. Or discipline. I mean kids, I think that more that we let the kids realize that the math can be worked in with any other discipline, they will want to do math more. Because when you take it out of context and just teach math, and something else, and something else, then they really don't see the importance of having to do fractions.

Wesley: [taking notes] So I've got a multidisciplinary, don't take it out of…

Mrs. Jonash: Don't teach it just as "That's math," we need to relate it to real life.

(Year Three, Session Six, Mar 2003)

When the teachers shared summaries of the discussions in their small groups, they did not mention operations and algorithms that students needed to learn. After we asked the groups about the relevance of adding and subtracting fractions, all participants agreed that students needed to learn how to perform operations with fractions, and expressed that these "basic skills" need to be in place in order for students to "tackle the more complex problem solving." It appeared that, given their expectations for students' learning about fractions, the school leaders would view the contextualized interview problems as reasonable, and would expect middle school students to be able to solve them.

We expected that the students would struggle with explaining their solutions in conceptual terms and that some of them would use mathematically incorrect algorithms for addition and subtraction. We planned to capitalize on the anticipated mismatch between the school leaders' expectations and students' performance to point out that (a) the students developed these kinds of understandings as a result of instruction and that (b) teachers need to study and understand how students reason in order to support their learning more effectively.

Fraction interviews. Each of the four school leaders paired up with a teacher for student interviews, and three more pairs of remaining teachers interviewed additional three students. I include the following discussion of the pair reports as evidence that the teachers and the school leaders engaged in a relatively deep analysis of students' understanding of fractions that went beyond the "getting it or not" views of student performance.

All but one pair reported examples of their respective student's reasoning which did not align with conventional ways of working with fractions. The student interviewed by the remaining pair, Marci and Mrs. Jonash, had an exceptional understanding of proportional relationships involved, solved all the problems, could reason about adding and subtracting fractional amounts, and drew appropriate representations. Two teacher pairs reported that their students had some proportional understanding because they could explain that when a circle gets spilt into more parts, the parts would be smaller. Both these pairs shared additional examples of their students' reasoning that they saw as concerning, such as neglecting numerators all together, and the inability to interpret 16/15 as a result of addition because in the student's view, the numerator could not be larger than the denominator (see Figure 15).

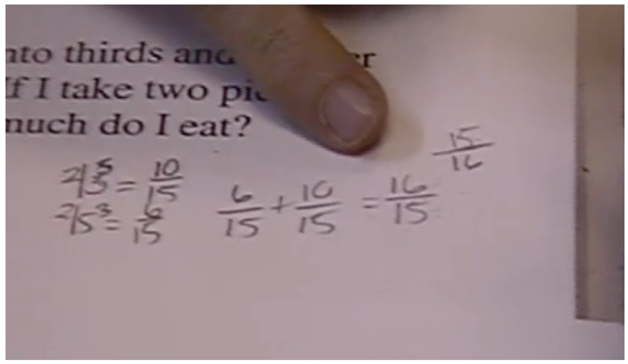

Figure 15. Student Work—Fraction Interview. A student struggling to interpret 16/15 as a meaningful result of the calculation.

The remaining four reports focused solely on what seemed concerning about the students' performance. To illustrate, I include the reports by two school leaders, Ms. Presley and Mr. Kelly:

Ms. Presley: Our student just used the denominator rule to compare everything, to compare anything and all. She just changed everything to common denominator. She was not very good either with pictures. And that was interesting to me because most of this I just figure out in my head just visualizing it. So it was interesting for me to see that she just had these rules, stuck in this rule mode. And when she got one wrong, because she forgot to do something out of the problem, she just had no clue how to go back and check it. Or how to fix it.

Researcher: OK.

Amy: The one she had the most difficulty with, she could do the candy bars where she compared how much before this child had and how much this child had. But when is it how much do they have all together, that was the end of it.

Researcher: And why do you think that was?

Amy: I think her common denominators got into way [Amy gives details of how the girl calculated.]

…

Mr. Kelly: Two things we noticed that stood out to me—cause I am not a math teacher—were that our student, when she was visualizing, she was always making each piece have the right number of parts, but the parts were not equal. And so she may say that one fraction was larger than another, when actually it wasn't. But the way she drew it was. Because the parts were not equal. And then the other piece was that when she was trying to add and subtract the fractions she was not changing the denominators. So she did not seem to have an understanding of that ruling piece.

Wesley: Something that I noticed too was that she was very much, she was much more comfortable with circles than with rectangles. Even when talking about the candy bar, she wanted to make that candy bar round.

Marci: Wow!

Mr. Kelly: Or the swimming pool.

Wesley: Yeah, the swimming pool was round. [see also Figure 16]

(Year Three, Session Six, Mar 2003)

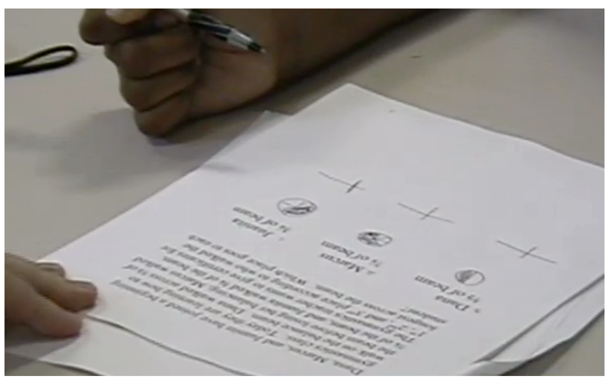

Figure 16. Student Work—Fraction Interview. Work of the student interviewed by Wesley and Mr. Kelly. The student drew circle to compare the distances that three kids walked on a gymnastics beam (1/2, 5/8, and 3/4). When prompted if she could illustrate the distances on a straight line segment as if that was the beam, the student used the circle graphs to gauge where to place tick marks on the line. The placement of tick marks did not reflect appropriate proportional relationships.

The school leaders and the teachers jointly examined the students' performance critically and made substantiated claims about ways that the students reasoned. They did not merely check whether the students did or did not "get it." A number of the episodes they reported indicated that the students' understanding was not necessarily what they would expect it to be after so many years of fraction instruction. This, in the researchers' view, opened up opportunities for the teachers to focus the group discussions on the problematic aspects of students' reasoning, the insufficiency of addressing the problem by re-teaching fractions one more time, and the need for the teachers to learn more about the nature of students' difficulties and ways to address them.

However, these were *not* the issues that came to the fore as the discussion proceeded. It appears that the teachers were working to accomplish a competing agenda throughout the activity. This is not to imply that the teachers consciously worked to undermine the activity. As I will illustrate, their contributions were genuine and, from the teachers' perspectives, could plausibly fit with the overarching goal of understanding students' reasoning. However, as a result of some of the teachers' contributions, the problematic nature of students' reasoning about fractions was down played and therefore did not emerge as the issue that needed to be addressed and resolved.

The first contributions of this type stemmed from the fact that reasoning of one of the students was relatively sophisticated. Marci and Mrs. Jonash were very positively surprised, and as a result, eager to share everything they learned about their student's skills and knowledge. While the reports of four pairs, which focused on problematic students' reasoning, averaged 1:29 minutes (AAD 0:22 min), Marci and Mrs. Jonash's report lasted almost 3:20 minutes. The remaining two pairs, which reported both their positive and problematic findings, averaged 2:40 minutes (AAD 0:10 min). These time differences indicate that the participants were more inclined to discuss positive findings about students' understanding. This was the case despite the collectively developed plan to focus on problematic aspects of the students' reasoning during the discussions in this activity with the school leaders. It is important to note that the overall sophistication of the interviewed students' reasoning was more advanced than that of the students the old-timers had interviewed in year two. Therefore, it is possible that the teachers focused on positive findings because they did not anticipate them based on the old-timers' accounts of their prior interview experience.

The second type of contribution that appeared to direct the group's focus away from instructional problems that needed to be resolved occurred when a teacher, usually Wesley or Amy whose students were being interviewed, felt that the reports did not take in consideration the reasons for the student's performance. For example, both Wesley and Amy attempted to rationalize Lisa and Naomi's report of the performance of the student they had interviewed as follows:

Lisa: He had just mentioned he studied percentages. Then he changed all these fractions to percentages. And he did very well with it, but I don't think that he could model that fraction for fraction. It had to be a percentage. When he started drawing, it really did not make any sense to him.

Naomi: When he was comparing fractions, he was able to do it as fractions. But what was interesting to me, when he was adding and subtracting fractions, he was still adding denominators, or subtracting denominators.

Researcher: What would be an example?

Naomi: On the last page, it was 2/5 - 2/7, he had 0/-2. And he knew it wasn't right, but he thought that that's what you should get. Until he changed it to percents.

Wesley: Now, what level kid? Sixth grader?

Naomi: Yes.

Amy: He is a sixth grader. But so far all we've got to this year has been conversion between fractions, decimals, and percents. Not starting [curriculum unit name] yet, where they actually do the multiplication.

Naomi: Yeah. You can tell that.

Amy: Now, he should have had it before he got to me.

Lisa: [Jumps in] Yeah.

Amy: But.

Wesley: I think it is great that he got negative 2.

[Teachers laugh]

Researcher: So what you were saying, he did not have a way to figure things out by thinking about fraction quantities.

Lisa: Right.

(Year Three, Session Six, Mar 2003)

After what he seemed to perceive as a critique of the student's reasoning, Wesley found it important to emphasize that the student was only in the sixth grade. Amy, the student's teacher, reinterpreted the boy's performance as adequate in relation to what she had focused on in the class (i.e., the conversion between fractions and percents). In doing so, she also positioned herself as a successful teacher by attributing possible gaps in the boy's reasoning to instruction in prior grades. Wesley then went on to further highlight positive arithmetic skills of the student, suggesting that these could not be taken for granted with all students in their schools. The teachers recognized what Wesley was trying to suggest and laughed in agreement. Even though the researcher attempted to bring the group's attention back to Lisa and Naomi's main claim and the students' understanding of fractions, it no longer appeared to be a significant problem. The participants could now assume that when Amy teaches the next curriculum unit, the boy's reasoning will improve significantly.

A similar situation occurred when we, the researchers, asked the group to think about how to organize the different kinds of students' reasoning that they identified from the least to most sophisticated. Specifically, we provided the participants with a poster listing the kinds of reasoning that the participants had reported. Then we asked them to "figure out what are some of the main distinctions in ways kids are thinking about fractions." Our intent was to examine what were some of the strengths that a teacher could build on with these students in subsequent instruction. As I will discuss later when I document norms of pedagogical reasoning, contrary to our conjectures, the teachers at this time interpreted this type of task as one of creating a grading rubric. The following excerpt illustrates this teachers' interpretation:

Wesley: I have a question. I don't wanna say it, cause I'm gonna have to [be] trashing Asasha. Cause I really like Asasha, she is a great kid.

Researcher: It's not about Asasha.

Wesley: I know, so I am trying to separate that out. I'll celebrate Asasha. She was, you know, drawing correct number of parts with unequal sections. If I am scoring a test, and I see that on there and I am doing it on a wonderful rubric,

Mr. Kelly: [behind Wesley's back, shows one on his hand suggesting low grading]

Wesley: I will give that a one.

[Teachers and leaders nod in agreement]

Wesley: And yet she knew so much. And I am also saying, "Numerator Not Important" [this was a label for another type of reasoning written on the poster]. I'm like, "ah that's awful, how could," you know. And yet, that would be one as well.

(Year Three, Session Six, Mar 2003)

Wesley contrasted Asasha's reasoning, which was based on unequal size parts of a whole, with the reasoning of another student who intentionally ignored numerators when completing the tasks. Wesley argued that a rubric that would place both these students in the same category is not useful, because in his view, Asasha already knew so much. Wesley did not clarify why he considered Asasha's reasoning to be of higher sophistication than that of a student who ignored numerators but at the same time, constructed unitary fractions by splitting a whole into a number of equal size parts—something that Asasha's solutions did not stress. It is possible that Wesley protested assigning a low grade to a student from his classroom because this assessment could also be viewed as an assessment of his instruction. It is important to consider this possibility given that the school leaders in the district used student test scores as a primary means of teacher assessment.

Wesley's interpretation of the task as creating a grading rubric and other teachers' nodding in agreement suggested to us that continuing this discussion would not be productive. At the time, supporting teachers to generate categories of students' reasoning that would be useful for instructional planning was an open problem within our agenda for teachers' pedagogical learning and we decided not to pursue this agenda during the activity with the school leaders. For this reason, one of the researchers summarized different types of students' reasoning that the teachers documented, then illustrated how some of those could be build upon, and concluded the activity.

Discussion. Overall, even though the participants critically analyzed the students' understandings of fractions, some of the teachers highlighted positive aspects of the students' performance, and others down played its problematic aspects. As a consequence, it was unclear how the school leaders understood both the overall purpose of this activity and what they were supposed to take away from it. It was plausible for the school leaders to assume that the teachers merely wanted to illustrate for them the kinds of discussions that they have engaged in during the professional development time. The professional teaching community thus did not progress with the agenda of supporting the school leaders in examining their ideas about effectively helping students learn mathematics.

It is important to point out that the teachers' participation in the activity with the school leaders was not consistent with the normative practices that had been established within the professional teaching community. During other year-three activities, the old timers did not make contributions to defend their students and themselves. Defensive contributions were more typical both during the first year of the collaboration, and for the newcomers at the beginning of year three when they still kept their practices private to some extent. It is significant that Amy's and Wesley's contributions were not questioned by other teachers. Instead, these and similar contributions were consistently constituted as legitimate (e.g., notice Naomi's and Lisa's supportive comments during discussion about the sixth grader).

In retrospect, it is clear that when we planned the activity, we failed to realize that the presence of the school leaders in the session would significantly alter the participation of the members of the professional teaching community. We conjectured that because the teachers' practices were already deprivatized, they would be able to focus on the communal agenda, that of critically analyzing students' understandings of fractions, during this activity. It is understandable that the teachers—especially those whose students were interviewed—were concerned that their school leader might develop a particular view of their competence as teachers as a consequence of this activity. To some extent, they constructed the activity as an evaluative situation, similar to their other interactions with the school leaders in their school (e.g., drop-in classroom observations).

The analysis of this activity can inform future attempts to perturb school leaders' views of teaching mathematics. One important lesson is that presence of outsiders in activities of a professional teaching community might alter established normative practices. Another lesson is that students' performance is often interpreted in terms of the quality of their teacher, especially in environments where administrators view their primary role as instructional leaders to be that of monitoring and regulating teachers' instructional practices. It is possible that if the interviewed students had not come from some of the participating teachers' classrooms, the teachers would have focused less on how the administrators were evaluating their competence, and more on the nature of students' reasoning about fractions documented during the interviews.

Community's Changed Perception of Institutional Context in Years 4 and 5

As it will become clear from further analysis, the group discontinued its attempts to work with the school leaders in the remaining years, choosing instead to focus on issues related to student learning. Among the major reasons for this decision on the part of the research team were the relative lack of success of the activity with the school leaders and the continuing difficulty in gaining the leaders' time commitment.

Our lack of progress in perturbing principals' views of mathematics instruction illustrates that we were not effective in bringing about substantive changes in the institutional setting.

Nonetheless, for the teachers, the institutional setting did not remain the major impediment in years four and five. Their views of the setting and, in particular, of their principals changed significantly. The teachers came to view their principals as someone who does not know enough about improving teaching and learning of mathematics and as a result, they discounted their principals' drop in visits in their classrooms, and ceased bringing up concerns about what was happening in their schools. Instead, the old-timers seemed to have developed a sense that it is both possible and meaningful to attempt for instructional improvement even within the current institutional context, provided that they have the support of the group.

The strongest evidence that this was the case comes from session 1 in year 5. During one of the activities, we had introduced a list of resources that the group developed during the first 4 years. As we read through the list, one of the researchers elaborated on the activities during which different resources were created. One of these was the Dots Activity that the group used in the summer workshop at the end of year 2. In this activity, the teachers identified weaknesses and strengths of their school district and discussed whether and how could they change some of the weaknesses and capitalize on the strengths. When the researchers briefly reminded the group about this activity, Muriel spoke up:

Muriel: I think that [Dots activity] would be helpful to look at because ... when I really think back, about the things that we did like those first couple of years, it's like we had to work through just being able to do the [statistical] activities [in classrooms]. Period. Because there was so much baggage in every school.

Researcher 1: Had the schools changed?

Muriel: No [Naomi, Dorothy, shake heads, agree with Muriel]. But I think the way we work around it is different.

Researcher 1: So it's been very interesting from our point of view to see how people's orientations and attitudes have changed.

Erin: We just do it [Erin chuckles, others laugh, some nod].

Muriel: We just do it.

Researcher 1: It's been amazing.

Muriel: Yeah, and I think that's part of the group.

Researcher 2: Muriel, can you say more about it? You said the school is pretty much the same but the way you guys are working around it is changing.

Muriel: Yeah. I think it's part of the power of being a group. I think we are questioning and challenging things more. And finding ways- I guess the importance of it maybe is what's different. The reasons that we see. It's not just activities that we are doing, it's activities we're doing for a reason. And maybe that's what's changed.

(Year Five, Session One, Sep 2004)

The teachers who were present in the session supported Muriel's contribution by frequent nods and laughter. They all agreed that they now do not see obstacles in their schools that would prevent them from experimenting with statistical activities in their classrooms. Yet, at the same time, the teachers did not attribute the difference to changes in their schools. They instead came to view both their new insights and the professional teaching community as counteracting the weaknesses of the institutional context of their schools.

Importantly, the district mathematics leaders' actions during years 4 and 5 continued to support the teachers' views that instructional change in mathematics in their district is feasible. Both Esther and Ruth continued to participate as peripheral members of the professional teaching community. Ruth participated in all sessions during these years, and Esther joined the group during most of the lunch breaks and occasionally stayed for half of the professional development day. Their positive valuation of the community's work was especially evident in the fall of year 4 when Esther purchased laptop computers and data projectors for all the teacher members of the professional teaching community from her middle-school grant. This decision was shaped by Ruth and Esther's participation in the summer workshop at the end of year 3, during which they came to appreciate how projections of computer tools during classroom discussions enhanced opportunities for students' learning.

The fact that Ruth and Esther's plans for a district-wide adoption of a new reform textbook series included the community members as potential teacher leaders also made it evident that the professional teaching community had achieved a central standing within the district improvement efforts. Ruth had already recruited some of the teachers to organize district-wide study groups for their colleagues in year 3. In year 4, she has recruited additional teachers to pilot several units from the textbook series considered for adoption, and Esther planned to use the professional teaching community as a setting for preparation of teacher leaders that would coordinate the implementation of the new curriculum. Ruth's contributions to session discussions during years 4 and 5 consistently indicated that she considered the collective notes that were generated during the sessions as a useful resource for other teachers in the district.

I conjecture that the district leaders' participation in the sessions as well as their plans for the mathematics instruction in the district provided the teachers with an important connection between their engagement in the professional development sessions and its relevance to their classroom instruction. Indeed, the teachers' adoption of a leadership position within the district became increasingly a part of the group interactions. For instance, in session 6 of year 4, Brian (who had become a chair of the mathematics department in his school) and Kate pressed for creating an instructional unit in statistics based on the activities of the group. They argued that such a unit would be useful both for their own use and for the teachers who were not members of the group. During the same session, Dorothy proposed to hold future sessions in schools rather than in the district office, to make it possible for more teachers to drop in. The teachers indicated that membership in the professional teaching community was now recognized by the group outsiders and requested informational brochures that they could hand out in their schools. As a result, the newcomers who joined the group as part of the reform textbook implementation plans saw the community as a means of instructional change from the outset and never brought up institutional constraints as an insurmountable obstacle to changing their instruction.

Additional reason that we did not further target the teachers' institutional reasoning was the teachers' dramatic increase of interest in issues related to students' classroom participation that occurred during and after the summer workshop at the end of year 3. I illustrate how this interest emerged and why the teachers came to view examining students' classroom participation as a meaningful activity in the professional development sessions when I discuss the shifts in the teachers' pedagogical and mathematical reasoning later in this chapter.

Institutional Reasoning Summary

Dean's (2005) analysis of the learning of this teacher group in years 1 and 2 indicated that the teachers' developing understanding of how the institutional setting influenced their instruction made possible the deprivatization of instructional practice. Similarly, the focus on bringing about changes in the institutional setting in year 3 contributed to re-negotiation of the shared enterprise within the group and supported the inception of the newcomers to the professional teaching community. In addition, this focus supported the teachers in coming to actively pursue the goal of changing how mathematics is taught in their schools.

Throughout summer session two and the sessions conducted in year three, we attempted to support the professional teaching community's engagement with the school leadership communities in bottom up manner. These efforts resulted in several important shifts in the teachers' views of their institutional settings. The teachers developed insights into the school leaders' limited capacity to be effective instructional leaders in mathematics and started to view themselves as a possible resource for the school leaders' learning. They outlined a trajectory for supporting the leaders' learning and took initial steps to realize this trajectory. In the process, the communal norms of treating the institutional setting as something the teachers could and should actively influence were firmly established in the professional teaching community.

Norms of Pedagogical and Mathematical Reasoning

Professional Development Goals and Starting Points

Goals for the Teachers' Learning

Our intent in the professional development sessions was to engage the teachers in activities that would enable them to reconstruct the rationale for the instructional sequence[31], and thus facilitate the teachers' adaptation of the sequence to the contingencies of their classrooms. The key aspect of the teachers' pedagogical learning that we intended to support included coming to view students' reasoning as an instructional resource and learning to build on it in instruction. This required that we support the teachers in developing ways to (a) anticipate and monitor the diverse forms of student reasoning that arise during instruction, (b) decide *which* forms of reasoning need to be further supported with respect to the big ideas of the domain, and (c) envision *how* can students' reasoning be best supported in a classroom (McClain, 2002; Stein, Engle, Smith, & Hughes, 2008).

Coming to view statistical activity as inherently involving numbers with context was central to the mathematical learning of the community that we planned to support. Following Cobb

[31] In the professional development sessions, the community members referred to the two consecutive statistics instructional sequences (i.e., sequences for supporting students' reasoning about (a) univariate and (b) bivariate data) as a single statistics instructional sequence. In this joint sequence, the univariate sequence was directly followed by the bivariate sequence.

and Moore (1997), we viewed "context"—or the scenario of a problem introduced in a classroom—as a means of creating a situation in which students can come to see analyzing data sets as a reasonable way to solve a problem that they consider significant. It is more than just a story to help students see that statistical tools can be applied in "real world" situations. Similarly, "numbers" are more than numerical values for performing calculational procedures or for creating graphs; they are measures of a relevant aspect of the phenomenon under investigation.

Current Normative Practices

While the teachers already paid attention to their students' thinking at the end of year two, their focus was exclusively retrospective. They attempted to understand what the students did and sometimes hypothesized why they did it. The view of statistical activity that had become normative was that it involved calculations and the use of statistical tools, that the numbers used in these calculations must be measures of something, and that the calculations produced as a part of data analysis must be produced for a reason. The view of problem scenarios that became normative was that they were a precursor to rather than an integral part of statistical activity and could be useful in eliciting student engagement (Dean, 2005).

The teachers became increasingly proficient in conducting genuine statistical data analysis themselves during the first two years of our collaboration with them. By the end of the second year, the nature of teachers' participation in the sessions allowed for genuine discussions of the pedagogical problems that they encountered in their classrooms. Their practices were deprivatized to the extent that the teachers were comfortable collecting their students' written work, videotaping their teaching of statistics, and sharing and discussing these records of their practice in the professional development sessions.

Planned Means of Supporting the Learning of Professional Teaching Community

During the third and fourth years of the collaboration, the professional development work typically followed the pattern of teachers (a) solving a selected task from the statistics sequence during the work session, (b) discussing possible ways to organize classroom activities when using this task in their classrooms, (c) teaching the task to their students, and (d) bringing their students' written work to the following work session for group discussion. In addition, two of the teachers co-taught and video-recorded the lesson, and the group used the recording as a focus for discussion in the following professional development session. The designed professional development activities, the statistics instructional sequence, and records of classroom practice—student work and classroom video—were the key means of supporting the learning of the professional teaching community.

Realized Trajectory

In the following sections, I document how the group's normative ways of mathematical and pedagogical reasoning evolved during years 3 through 5. I lead the discussion with the pedagogical issue that was in the center of our design and research efforts, that of supporting teachers in coming to view students' reasoning as a resource in their instructional decision making. I first document how our research-grounded efforts at supporting teachers to focus on students' reasoning were repeatedly unsuccessful in year 3 and why that might have been the case. I then document how, in year 4, supporting teachers to *decenter* and adopt a student's perspective on classroom events became a key means in promoting shifts in norms of pedagogical reasoning and eventually led to teachers' appreciation and use of students' reasoning in instructional planning. It is noteworthy that progress towards the main goals of our professional development agenda required major modifications in the means of support since our initial design conjectures proved unviable. Building on teachers' current practices and interests was critical in designing modifications that proved effective.

While discussing shifts in normative views of students' reasoning across the years 3-5, I also point out shifts in norms of mathematical reasoning in the group that occurred during the same period of time and pertain to what Ball and Bass (2003) refer to as mathematics for teaching. At this point in the group's learning, important shifts occurred in the teachers' views of statistical activity when they reflected on their own and others' classroom instruction and compared it with their statistical learning experiences in professional development sessions. For instance, I document how normative purposes for the part of a lesson in which a task is introduced changed over time from being merely a precursor to doing statistics to being inherently statistical part of a lesson in which relevant data is generated. The shifts I document had immediate pedagogical consequences and they played an important role in the group's development of pedagogical reasoning. I also foreground the mathematical, or more specifically statistical, aspects of these shifts to highlight the extent to which teachers' instructional decisions were shaped by the meanings they constructed of what is a *statistical activity*.

Initial Struggles to Support the Teachers in Focusing on Students' Reasoning

In year 3, we attempted to orient the teachers *directly* to their students' reasoning and how it could be used in instructional planning. In doing so, we built on findings of professional development research that had succeeded in supporting teachers to focus on their students' reasoning in a relatively straightforward manner (Carpenter et al., 2000; Fennema et al., 1996). This research indicated that use of records of practice, in particular students' work and classroom videos, became a resource in making students' reasoning a focus for teachers.

Students' Work as a Means of Supporting Teachers' Focus on Students' Reasoning

In pursuing the pedagogical goal of making students' reasoning central to instructional decision making, we asked the teachers to use statistics tasks in their classrooms and bring their students' written work to professional development sessions. We designed activities in which the teachers were to (a) categorize the solutions evident in their students' written work and (b) propose how could they use these solutions when planning for subsequent instruction. We conjectured that the teachers would come to appreciate the richness and diversity of ideas in their students' approaches and would become curious about how could they capitalize on these ideas in instruction (cf. Fennema et al., 1996).

We remained largely unsuccessful in these efforts throughout year three. In all three activities that involved students' work (sessions 3, 5, and 6), the teachers grouped solutions according to the mathematical methods and procedures that the students used (or did not use) rather than by ways in which students reasoned statistically. In addition, the teachers did not see the activity of examining students' work as useful in planning further instruction.

Focus on solution methods. To examine the categories the teachers generated, consider the Blood Drive task. The goal was to decide whether supermarkets or community centers are better locations for an emergency blood drive, based on data on amounts of blood collected at those two types of location in the past (Figure 17).

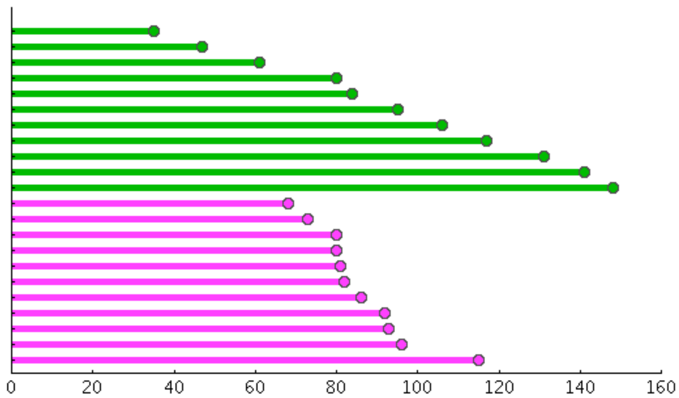

Figure 17. Computer Tool One—Blood Drive. Each pink bar represents amount of blood in liters collected in one day by a bloodmobile placed in front of a supermarket. Each green bar represents amount of blood in liters collected in one day by a bloodmobile placed in front of a community center.

During the discussion of student work from this task, a group composed of Wesley, Lisa, Kate, and Jane presented the following categories (Figure 18):

1. No Math, decision not based on data (For instance: Supermarket parking lot is not clean, therefore blood drive should be conducted at community centers.)
2. No math reasoning but a statement of “most” or “more”
3. Comparisons of maximum or minimum data value (not in terms of range)
4. Totals, Averages (Recommended Community Centers)
5. Totals, Averages, Ranges, Consistency (Recommended Supermarkets)

Figure 18. Categories of Student Work Generated in Year Three, Session Three, Nov 2002. The categories are ranked from 1—least to 5—most sophisticated.

The remaining two teacher groups also included “Total or Average,” “Range or Consistency,” and “Comparison of extreme values” among their categories. The teachers categorized solutions based on what students *did* (i.e., solutions methods they used) but did not consider students’ interpretations of the problem, their rationales for using specific methods, or their intentions in doing so (i.e., what they hoped to show about relative effectiveness of blood drives by comparing maximum data values). The focus on what students did was obvious in all discussions of student work. In the following excerpt, the teachers discussed one student group’s solution of the Blood

Drive task[32]. A girl described that her group thought the supermarkets were better because neither too few nor too many people came to give blood, and the data were "in the middle." To support the argument, the group produced a "line graph" (Figure 19) which depicted the data from blood collections at supermarket parking lots.

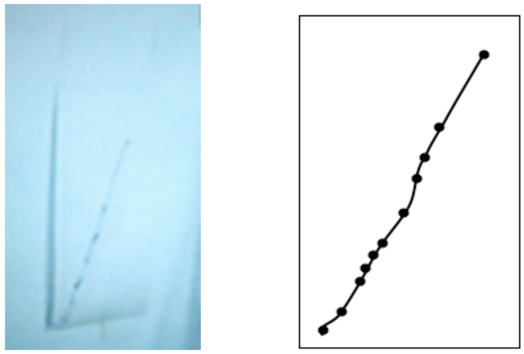

Figure 19. Student Work—Blood Drive: Line graph of supermarket data, discussed in session four, Jan 2002.

The following teachers' discussion was representative of the level of detail on which they examined and categorized students' work:

Kate:	What was she drawing? What was the line that she was drawing?
Muriel:	She only drew, [she] took a point for just a supermarket-
Helen:	[jumps in] supermarket
Muriel:	[together with Helen] and did that on a line graph. Cause she wanted to show that it was more consistent [Helen jumps in: Yeah] but she did not compare it to anything.
Researcher:	So, her argument was consistency?
Lisa:	Yeah, she never said the word.
Muriel:	No. But that was her argument.
Lisa:	That was her argument.
…	[The teachers discussed how the line graph did not illustrate consistency well because it showed increasing tendency.]

[32] This example comes from the discussion of student work that took place as part of a video activity in session four. The group discussed student work on Blood Drive in session three and the teachers continued that discussion when they watched students share their solutions on the video, one session later. Because all the teachers could see the student's solution, this discussion was less cryptic than similar discussions in student work activities.

Jane: It also looked like she was doing a little bit of total.

(Year Three, Session Four, Jan 2003)

"Consistency" as a solution method typically involved examining which of two datasets had less variability or a smaller range, but it did not involve reasoning about why more consistent data were (or were not) desirable in a specific situation. There is no indication that teachers realized that when students look for greater consistency, they are focusing on the entire datasets, rather than on some of the data points. Similarly, "Total" as a solution method involved comparing which of two datasets had a greater sum of all its data values. The teachers continued to look for different solution methods in students' solutions and categorize students' work according to these methods throughout year three.

Students' work and instructional planning. With a single exception in session five, all the questions that the researchers posed about how instruction could build on students' solutions were followed by silence on part of the teachers or by general comments about having students learn from each other. This was the case despite the researchers' concerted support, which included more than ten illustrations of follow up discussions, and rephrasing of questions and prompts.

Moreover, the teachers visibly struggled with the idea of using student work to plan subsequent instruction. It is apparent in the following exchange that Wesley's spontaneous reflection captured the group's concerns:

Wesley: I am very uncomfortable.

Researcher: With what?

Wesley: … I am very uncomfortable with my lack of understanding of my answer to the question "What is there that I can build on?" … I am just trying to make a statement that I am not sure I've got it. … But I am just not sure, I feel.

Researcher: And I think you're not alone. For what I can make out. Is that fair? [Muriel, Erin, and Kate who faced the researcher nod.]

Lisa: The kids were good and they've done it [the task]. That's good isn't it?

Wesley: I mean I feel good about what my kids did and I would be confidently walking to the classroom the next day and not feel like I was creating educational malpractice.

Researcher: No, let me put this in perspective. … We're talking about something that is *way* out there.

Muriel: And we aren't there yet either. [Teachers laugh.]

(Year Three, Session Five, Feb 2003)

Overall, we continued to be unsuccessful in supporting teachers' use of their students' work as a means for examining their statistical reasoning during year 3.

Classroom Videos as a means of Supporting Teachers' Focus on Students' Reasoning

The means that we used to support the teachers in focusing on students' reasoning included classroom videos of statistics lessons recorded by pairs of teachers. All the teachers had received a copy of the video prior to the session and were expected to watch it and keep notes on issues they wanted to discuss in the session. During a professional development activity, we played several sections of the video in 5-10 minute segments. The teachers rarely raised questions and, almost invariably, a researcher stopped the tape and asked the teachers to comment on the viewed part of instruction.

Instruction captured on 5 videos produced by different pairs of teachers in year 3, as well as the discussions in the four professional development sessions in which the teachers analyzed these classroom videos, provided additional evidence for both (a) teachers' focus on specific solution methods that they hoped their students to "get" (rather than on developing new forms of statistical reasoning), and (b) the irrelevance of student work, in teachers' views, to instructional planning.

Focus on solution methods. The teachers' comments and classroom videos indicate that solution methods—in particular "Consistency" and "Total"—were constituted in the teacher group as what students were supposed to "get" as a result of statistics instruction. Six teachers shared how they helped their students to use a greater variety of solution methods, and others appreciated the shared strategies:

Lisa: Kate had a good point as divide the class and say: "OK, you [one half] have to argue for supermarket, and you [second half] have to argue for community center." Have them do their argument and then say: "OK, who would you go for now? Or where would you go now?" And see if any of them would move.

Kate: Because once my kids stopped at community centers, they never ever gave even a look to see [if there was an argument for supermarkets], because I told them that you could justify either answer. But they've stopped as soon as they found the community center. I only had one group who actually looked to see if they could find the reason to justify the supermarkets.

…

Marci: [I] just to try to get them to think beyond totals as being the answer to everything that may seem as though total is the best answer.

Researcher: Yeah.

Marci: To try to get them to explore some of the other measures of central tendency maybe, other things that they may not have tried, like we talked about the range, and how there is probably some kid, and they reason for choosing that large range versus the smaller range was probably based on what they were picturing as total. Because all of the papers that we looked at pretty much most of the papers we looked at, it didn't matter what they chose as their reasoning, they all went back to [solution] being the total.

(Year Three, Session Three, Nov 2002)

Lisa, Kate, and Marci all demonstrate that it was their goal to help their students learn how to use new, different solution methods.

The teachers enacted the described interventions while their students worked in small groups, as I illustrate in the following section. Consequently, the teachers viewed students' written work as a final product of students' learning, rather than as a starting point for orchestrating classroom discussions. To clarify this contrast, I first outline our—the researchers'—view of students' work as an instructional resource.

Based on experiences from classroom design experiments, we viewed students' work to be instructionally useful in planning and orchestrating the concluding part of a statistic lesson: data analysis discussion (P. Cobb, 1999; P. Cobb, McClain, & Gravemeijer, 2003; McClain & Cobb, 2001). During the discussion, different solutions that students generated in small groups along with their justifications were compared and contrasted, making the ways of reasoning that underlay these solutions[33] accessible to the listening students in the classroom. We therefore viewed the teacher's understanding of the types of reasoning present in the classroom to be criti-

[33] The underlying reasoning might have included, for instance, usefulness of comparing data sets proportionally rather than comparing absolute numbers of datapoints; or justifying solutions in terms of the problem at hand.

cal in planning for specific comparisons that both built on students' reasoning and furthered the teacher's instructional agenda (Ball, 1993). The small group analysis time provided opportunities for the teacher to survey the range of ways in which their students were reasoning and to plan for building on these in the concluding discussion. The specific organization of classroom activities (that proved effective in classroom design experiments) facilitated the teacher's use of students' reasoning as an instructional resource. I now document that the organization of classroom activities adopted by the teachers in year 3 was of a different nature.

Irrelevance of students' work to the teachers' instructional planning. Throughout year three, the normative view within the teacher group was that student learning occurred primarily when they completed instructional activities in small groups but not when they participated in the subsequent whole class discussion of their solutions. This was apparent in the teachers' frequent interventions during small group work as students conducted their analyses and their minimal interventions during the concluding class discussions. The teachers' instruction captured on all 5 classroom videos of statistics lessons collected in year 3, as well as the teachers' descriptions of their instruction in all four professional development activities in which classroom videos were discussed, reflected this regularity in the teachers' instruction.

Lisa's instructional strategies were representative of those enacted by other teachers in the group. In session four, Lisa reported that when she taught the Blood Drive lesson, she instructed the groups to come up with "reasons for both sides"—one in support of community centers and one in support of supermarkets as the best places to conduct a blood drive[34]. She then monitored the students as they worked in small groups, discussed and challenged their emerging arguments, reminded students in each group to produce arguments for both sides, and when needed, helped the students improve their solution methods. More than half of the teachers indicated that they also intervened while students produced their solutions in small groups. These instructional strategies were established as reasonable within the group.

In contrast to the teachers' frequent interventions during small group time, their interventions during the concluding part of a lesson were minimal. If class time permitted, the teachers concluded the lesson by having students *present* their solutions. In all five video-recorded lessons in year three, the format of student presentations was the same. A student briefly shared

[34] Lisa's asking her students to produce "reasons for both sides" again illustrated her goal of teaching both "Consistency" and "Total" methods. In the statistics tasks the teachers used in their classrooms, one of the datasets was typically more consistent while the other one had a greater total.

what his or her group did (typically standing in front of the classroom), displayed a poster of their group work (if groups produced posters), and was succeeded by the next presenter while the classmates clapped their hands. The teachers limited their own role during presentations to selecting the order of the presenters, occasionally asking clarifying questions, and managing turn-taking and student behavior.

Throughout the professional development discussions, all the teachers indicated that their understanding of the whole class conclusion of a statistics lesson was that student groups present their analyses at front of the classroom, much like the students on classroom videos. The normative justifications for devoting instructional time to student presentations included that (a) students find it easier to learn from their peers than from the teacher (stated in discussions on 4 occasions), and (b) presentations help students improve their social skills, support the development of their self-esteem, and provide a rationale for producing solutions (discussed twice). Justifications did not include pursuing the instructional agenda by making specific types of solutions a topic for the whole class discussions.

The ways in which the teachers organized their statistics instruction support the conjecture that, in the teachers' views, small group time—not the concluding presentations—was the part of the lesson in which *they* could influence students' learning and pursue their instructional agenda. This agenda consisted of making sure that the students "get" the specific solution methods needed in analyzing data. Within the teachers' instructional practices, the analysis of students' work had minimal practical implications for how they intervened in small groups, as well as for how they orchestrated students' presentations. Students' reasoning (in contrast to mastering solution methods) was not part of the teachers' goals or instructional strategies, and as such it was to a great extent invisible to the teachers. I further illustrate the last point in the following section in which I discuss how the teachers seamlessly incorporated our questions about students' reasoning in their discussions about instructional strategies.

The Invisibility of Students' Reasoning

One of the issues that we found problematic in the teachers' adaptations of statistics lessons and on which we pressed consistently (or so we thought) was the organization of the concluding part of the lesson: students' presentations. We found presentations problematic because they did not provide the listening students with adequate opportunities to make sense of their classmates' arguments. The listening students did not ask questions, did not raise comments, and often appeared to focus on preparing their own *performance* rather than on understanding the presenting groups' reports.

We attempted to make the impact of presentations on listening students' learning a topic of discussion in professional development sessions. We therefore pressed the teachers to consider "What sense did the listening students make of the presentations?", "What did the listening students *hear*?", and "What was there that was worth to listen to?" Throughout all four video analyses in year three, the teachers' interpretations of our orienting questions differed significantly from our intent in that they did not revolve around the sense that students were making of others' explanations, but instead centred on *teaching strategies* that were or could have been used by the teacher in the video-recorded lessons. The teachers' responses addressed issues such as (a) what helped the students in understanding presentations (e.g., use of color printouts, teacher questions), (b) what they could do to understand the students' presentations (e.g., what specific questions could the teacher ask during small group analysis to understand a group's solution), and (c) why it was beneficial for the students to engage in the presentations in general and in the video-recorded lessons in particular (e.g., learning from each other).

These repeated miscommunications indicated to us that from the teachers' perspectives, our questions and comments fit well with their discussions of teaching strategies. Only rarely did these miscommunications lead some of the teachers to realize that they were not sure what we meant, or how could our questions be answered (e.g., Wesley in the context of building on students' solutions). We made successive (unsuccessful) revisions to the ways in which we spoke about instruction and about students' learning in the sessions, while the teachers kept hearing and answering the same questions about teaching strategies. Our attempts at reorienting the teachers' focus in 3 analyses of students' work and in 4 analyses of classroom videos resulted in teacher dissatisfaction with the repetitive nature of the professional development activities and in complaints that we have already "done the same thing" many times.

It became obvious that we could not continue to press directly for a focus on students' reasoning, at least not in the ways that we tried during the six whole-day sessions in year three. At the same time, through the elaboration of our attempts at orienting the teachers to think about classroom instruction from a listening student's perspective, we became convinced that the ability to *decenter* and examine instruction from a student's perspective was central to pro-actively supporting students' learning in classrooms.

Why Did the Teachers Keep Coming?

Given our lack of success in enabling the teachers to focus on students' reasoning, it is appropriate to ask: Why did the teachers keep coming to professional development sessions in year three, despite being dissatisfied with repetitiveness of the activities and despite seeing little relevance of the focal issues to their instruction? In this section, I document that the discussions of students' work and students' presentations contrasted with explorations and discussions of the opening part of the statistics lesson, to which the teachers referred as *launch*[35].

The opening part of the statistics activities, organized as a discussion in which the teacher and students talked about the purposes for generating data and the ways in which data could be generated, proved effective during classroom design experiments (P. Cobb, 1999; P. Cobb, McClain, & Gravemeijer, 2003; McClain & Cobb, 2001). We viewed these *data generation discussions* as occasions in which the teachers could renegotiate the nature of classroom statistical activity by providing their students with both reasons and means for engaging in genuine data analysis. We also viewed these discussions as inherently statistical in that the students were developing insights into processes of data generation that would allow them to address realistic problems by means of statistics.

When the teachers planned for and introduced the statistical activities in their classrooms during year three, it was evident that they both built on their understanding of and experience with launching instructional tasks, and that they worked enthusiastically on improving their launching strategies. In the following section I illustrate that the teachers viewed the launch as the primary means (a) to engage their students and (b) to help their students understand the task—issues that are both of paramount importance in making teaching more manageable. Their launches focused on the scenario of the problem and most of the time did not include a discussion of the data generation process. I discuss evidence which indicated to us that from the teachers' perspectives, the introduction of the task was a precursor to doing statistics and was not

[35] *Launch* was the term used for the opening phase of classroom activities in the mathematics textbook series used in the teachers' schools.

itself a part of a statistical activity. As I will document, the teachers' interest in facilitating productive launches later provided leverage for their subsequent learning.

Launch as a Means of Engaging Students

The following discussion of Naomi and Marci's classroom video of the Watermelon activity in session two illustrates both (a) the level of teachers' involvement in the discussions and (b) the normative view in the group of problem scenarios as a means of engaging students. In the Watermelon activity, students were to compare data on weight of watermelons from two different fruit distributors with the goal of advising a restaurant owner about the company from which she should purchase watermelons for making juice, given that price per watermelon was the same. The data comprised the weights of 20 watermelons, 10 from each of the two fruit distributors. Two old-timers, Naomi and Marci, co-taught and video recorded their lesson. All the old-timers had analyzed the Watermelon data previously, at the beginning of year two. The newcomers learned about the Watermelon scenario as Naomi introduced the task in the video-recorded lesson. The following discussion ensued after the video-recording was stopped for the first time as Marci was about to distribute data printouts[36] to students.

Muriel: How did you decide to ask them about the different kinds of juice they like?

Marci: [laughs] Just to get them interested and to kind of get them to buy into what we were going to do that day. So we thought that would be a good launch because we did not have a scenario [laughs]. So we just went with that.

Naomi: We had the scenario, but we wanted to do it even more interesting for the students as Marci said, with them buying into the whole activity first.

Muriel: [Noted that her question was like what researchers would ask and laughed].

Marci: Because we just did not want to go in there and say "Look, we are gonna do something, and I am giving you some information, and here is what it's supposed to represent." So we wanted them to have some type of ownership. ... I've just felt that it would give them a [not audible].

Researcher: OK, so that's your intention. Now if you look at it from the kids' point of view, that first part when you asked them at the beginning what was their favorite type of juice, was that the question?

[36] Given that some of the teachers did not have access to computers in their classrooms, they used color printouts of Computer Tool screen shots for the students to conduct their analyses.

Marci:	Mhm. [Many teachers nod.]
Researcher:	And you were in the class, and maybe you said what your favorite type of juice is, and you hear some other people. What's going on from your point of view as a student?
Amy:	[After 3 second pause.] I am getting involved in what's going on.
Researcher:	Why? How?
Amy:	Because any time somebody asks me my opinion and if I am a kid, I've got one and nobody ever asks me my opinion. So.
Researcher:	I mean, do other people agree with that? [Teachers nod.]
Muriel:	Yes, you start to think about what kind [of juice] do you like.
Researcher:	So you are actually thinking about juice you drunk, you know, what you know about juice, and that sort of thing. [Muriel nods.]
Hazel (N):	I was thinking they would probably wanna taste it [the juice].
Erin (N):	[Laughing.] I did say, I also wanted them to be able to taste it.
Hazel (N):	The teacher said "What do you like?" and then they have thought there was gonna be a taste test at some point in class.
Researcher:	OK. So what were the main issues that came, when you've asked them [about] juice and then you said there's this restaurant and there's these two companies, right? And what were the main themes or issues that came up then? From the kids.
Muriel:	Does anybody like watermelon juice?
…	[The researcher then repeatedly pushed the teachers to elaborate on what attributes of juice came up as important in the classroom discussion. Marci and Jane named quality of juice and its price.]
Researcher:	[To Naomi and Marci] Did you feel that the kids were buying into this scenario? Is that your experience as you were doing it with them?
Naomi:	Yes. [General agreement in the room.]

(Year Three, Session Two, Oct 2002)

During this episode that (unabbreviated) lasted 3.5 minutes, 7 teachers spoke up, raised questions, built on each other's comments, and volunteered responses to researchers' prompts. This pattern of interaction was typical across the 4 discussions of video-recorded launches in year three (sessions 2, 4, 5, 6[37]), and it indicated to us that most of the teachers were interested in improving this part of their lessons[38].

According to Marci and Naomi's explanation, their primary goal for discussing the scenario was to foster the students' interest. The goal of getting the students to "buy in" was constituted as legitimate within the group, as indicated by Amy's contributions and by supporting nods and agreements from most of the teachers. The teachers speculated that the students would buy in as a result of discussing juices they liked because the discussion would give them a sense of being included and able to share their opinions. All the teachers agreed that the students in the videotaped classroom were buying into the scenario, and in that sense, the group constituted Marci and Naomi's instruction as effective.

The teachers did not propose additional expectations for the launch and waited to see, as Brian had put it, "where was the math gonna be in all of this." Brian's comment reflected that he, as well as other newcomers, did not know beforehand what kind of data the students would analyze in the Watermelon activity. Brian's comment also suggested that he did not expect the launch to help him (and his students) understand what data he needed to analyze and why. It was constituted as acceptable in the group that the problem to be addressed and the data would only be introduced and explained after the students have already "bought into" the general scenario (e.g., of drinking fruit juice), as was the case in Marci and Naomi's lesson.

The three other discussions of classroom video that took place in year three (in sessions 4, 5, and 6) are consistent with the claim that in the teachers' view, buying into the story was the key precursor to mathematical activity. Two out of the three video-recorded lessons resembled the Watermelon lesson in that the teachers first guided a relatively lengthy (7-8 min) discussion of the broad topic of the scenario, which preceded a significantly shorter (3 min) introduction by the teacher of data and the problem at hand[39]. The decision to discuss the broad topic with students was met with approval of the teachers present in the sessions who argued that this was a

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[37] In year three, 5 pairs of teachers video-recorded their statistics lessons, but only first 4 of these recordings were used in sessions. The last lesson was video-recorded after session 6. I analyzed the 5 recordings to examine how the teachers conducted launches in their classrooms.

[38] In contrast, the discussions of students' analyses and presentations had slower pace, and towards session 6 also a noticeable lack of teachers' enthusiasm.

[39] The third video-recorded lesson, taught by Wesley and Brian, started by 5 min discussion of the broad topic of the task intended to get students involved. However, it also included 5 min discussion of problem at hand, in which students were asked to propose what to measure and how. One more lesson was video-recorded by Erin and Kate after session 6 and never discussed in the group. This lesson included 7 min discussion of the broad topic of the task, 18 min discussion of the specific problem to analyze, and 5 min discussion of what to measure and how.

good way to get students involved. At the same time, six teachers[40] commented that the students engaged in these discussions in order to intentionally delay doing mathematics, which they knew was eventually coming. The clear distinction that the teachers made between mathematics and launch suggested that they valued launch as a means to elicit students' engagement and viewed it as an external prerequisite to mathematical (and statistical) activity.

Buying into the story: Good problem scenario is enjoyable. The ways that the teachers participated in discussions of launch and the time they devoted to launch in their instruction both indicate the significance they attributed to fostering students' engagement in mathematical activity. Student engagement continued to be significant throughout the remainder of our collaboration with the teachers, while the normative ways of reasoning about student engagement evolved along with the criteria for assessing the effectiveness of a launch. These shifts were indicated in part by changes in the topics of scenarios that the teachers viewed as instructionally effective at different points in time. In year three, the teachers brought up three scenario topics that they believed would be effective in their classrooms: basketball, a popular talk show, and an animated TV series.

Wesley: I think they care about the basketball question because basketball for some of them is right in their face. They are very interested in basketball. So perhaps it's a bad example [of a scenario where the teacher would need to discuss significance of the problem with students] because they're gonna be interested in basketball, period.

…

Amy: What I am getting at is if you set up the model where you can change your data in and out. So that in two years when the kids don't care about SpongeBob SquarePants anymore, you can have it [the scenario] on something else.

(Year Three, Session Five, Feb 2003)

The assumption behind the teachers' choices was that because students already enjoyed or were interested in particular topics, they would be more likely to engage in a mathematical activity if it was grounded in these topics. An assumption of this type also oriented Michele and Naomi's choice of an introduction in Watermelon activity that targeted juice flavors that students *liked*.

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[40] Teachers with these suggestions were Jane, Kate, and Muriel in session 4, and Brian, Lisa, and Amy in session 5. There were no disagreements raised on this issue suggesting that this view was acceptable in the group.

From the researchers' perspective, the teachers' assumption was problematic. Research findings (e.g., Eisenhart & Edwards, 2001; Hodge, Visnovska, Zhao, & Cobb, 2007) indicate that students' enjoyment of or interest in general topics (e.g., sports, music, pop culture, cosmetics) do not guarantee that they will be willing to engage in mathematical or scientific inquiries related to these topics. Hodge and colleagues' (Hodge et al., 2007) analysis of problem scenarios used in the statistics design experiments revealed that tasks where students have developed an awareness of the *specific question* to be investigated and came to view this question as *significant* were instructionally effective, whereas tasks where students only developed interest in the general topic but not in the specific problem at hand were not effective.

Launch as a Means to Help Students Understand the Task

Realizing that the teachers viewed the launch as a "non-mathematical" precursor to, rather than an integral part of statistical activity, the researchers pressed them to justify the instructional time they devoted to it. The teachers argued that in addition to engaging students, the launch was useful in both addressing and preempting students' questions about data. Three teachers suggested early in year three that they viewed launch as a way to save instructional time. These suggestions were based on a claim that when launch is done effectively, the students do not have to ask many follow up questions when they receive the data sheets and could instead proceed to analyze the data directly. Erin's comment captures this perspective:

Researcher: Suppose you're doing [a] launch in statistics, what are you trying to do, or achieve, or clarify? Because it takes time, right? Time when you could be teaching other stuff. How do you justify that? What's the goal of doing this what you call launch?

Erin: I think if you do launch and take a few minutes to explain "Here is what they did to get the numbers, here is the experiment they set up," and now you don't have to run [around small groups explaining]. … It will eliminate—what we were talking about—the questions you get: "What is this?" "Tadada, tadada." So in the five minutes maybe that you do launch, you save the ten minutes of running around and answering questions.

(Year Three, Session One, Sep 2002)

As I reported earlier, three of the 4 pairs of teachers whose launch was discussed in sessions in year three each spent only 3 minutes handing out the data sheets, explaining what the data meant, and introducing the question that students needed to answer. This part of task introduction did not have a discussion format. Even though clarifying questions were encouraged, the teacher was the authority on telling students what was measured and how, and why the measurements were relevant to addressing the question at hand. This indicates that while the teachers hoped to introduce data in ways that would help students understand the task, having students buy into the topic was their primary goal during a launch.

The Launch as a Context for Teachers' Learning

It is significant that the teachers' initial motivations for holding discussions of problem scenarios in their classrooms were based on pragmatic considerations that involved making teaching more manageable. The teachers sought to improve their prospects for getting through a lesson smoothly and for using instructional time more effectively. This is a further indication that the teachers participated in the professional development activities with a goal of improving their classroom instruction. This goal, in turn, provided a rationale that was valid from the teachers' viewpoint to examine and discuss classroom videos of launches.

Importantly, during these discussions, it became clear that the questions that researchers posed routinely had became questions that at least some of the teachers started to ask themselves while teaching and while reflecting back on their teaching during the session discussions. The evidence for this claim comes from the spontaneous unsolicited reflections that the teachers shared during the discussions[41] and from the fact that the teachers started to bring up the same topics the researchers had previously raised. I was able to identify 1 or 2 comments in each session in which the teachers shared recent insights. Most of these comments related to the opening part of the statistics lesson.

Wesley and Kate's comments from session four illustrate the nature of the insights that the teachers found worth sharing. In this session, the group discussed classroom video-recording of the Blood Drive task co-taught by Muriel and Helen. During the discussion, Wesley indicated that he still has a problem with "the data collection or creation term." The researchers systematically used "data creation-", "data collection-", and "data generation discussion" when referring to the opening part of the lesson. Wesley clarified: "I mean I can talk about and lead the kids

[41] Starting from summer session at the end of year 3, we collected teachers' written reflections on key themes at the end of the sessions. These reflections enabled me to establish more accurate counts of the teachers who reflected on their instruction in specific ways. Even though lower in count, the insights that the teachers volunteered in year 3 indicate that these were revelations that they saw as worth sharing.

through the big discussion about blood. But then how do you collect the information that we are looking at [on the data printout]?" About five minutes later, during which the discussion focused on students' need to understand the problem situation, Erin spoke:

Erin: Something that was just said here that I never even thought about discussing with the kids was how data would be collected. Where this stuff [holding a data printout in her hand] came from. I talked about a blood drive [the scenario topic], what it was, and giving blood, and lot of the same things [as were discussed] here and how they would bring in the trailers, and that type of thing. And then I said, "Now, here is some data." And I just threw it at them. At no point we would talk about, you know, what would *you* do. I mean how would *you* collect the data.

Researcher 1: Why are you raising that issue?

Erin: Because I think that you would talk about the blood drive and you would turn around and say: "Now here is data on it being collected" and then, that's why I think my kids got lost.

(Year Three, Session Four, Jan 2003)

Wesley and Erin both realized that describing to students what the data printouts represented after handing them out was different from what the researchers proposed when they pressed for discussing the data creation process with students. Erin's comments indicate that she believed it would be valuable to have her *students* propose how to generate data that would help the class to address the problem at hand. The data printout would then be introduced as an outcome of a process that was similar to the one proposed by the students. Cobb and colleagues (P. Cobb, 1999; P. Cobb, McClain, & Gravemeijer, 2003) reported that as a result of introducing data in this way during the classroom design experiments, students came to view data as grounded in the situation from which they were generated. Importantly, when students consider how to generate data (e.g., when they discuss whether comparing amount of blood collected from one supermarket and from one community center blood drive is enough to decide which type of site is better for emergency blood collections), they are already participating in a statistical activity. A launch is then no longer just a preparation for the subsequent statistical activity, and its purpose reaches beyond eliciting students' engagement and describing what the data printouts depict. It becomes a means of supporting students' understanding of why particular decisions matter in generating data.

Erin stated that by discussing the topic but not the data generation process in her launch, her students "got lost" when she handed out the data sheets. From the subsequent discussion in this session, it became apparent that many teachers did not find the distinction between explaining "what the dots meant" (i.e., what was measured and where the data came from) and discussing the data generation process (i.e., how to design an experiment that would generate useful data) to be significant. I conjecture that for those teachers whose major goal was to get through the lesson smoothly, a thorough explanation of data sheets might have appeared adequate for enabling their students to conduct the analyses as intended. Nevertheless, Erin and Wesley both attempted to facilitate data generation discussions in the video-recorded lessons[42] in which they were one of the teachers. While this view of purposes for launch did not become normative in the group in year three, both the teachers' interest in launch and the emerging diversity in perspectives in the group on what a productive launch entailed became resources for subsequent work within the group.

Two Broad Shifts in Norms of Pedagogical Reasoning

I now proceed to describe two broad shifts in the normative pedagogical practices of the teacher group and explicate how each of these shifts made possible several subsequent developments in the teachers' reasoning within the resulting perspective. The first of these shifts occurred early in year four, when the teachers started to systematically adopt a student's perspective *in the context of examining students' interest and engagement* in classroom activities. I illustrate how the teachers anticipated what would "grab" the students' interest when they made planning decisions. As they reflected on data from their own and other teachers' classrooms, their views of which aspects of instructional activities were critical in promoting students' interests in analyzing data changed over time.

The second shift became evident much later, during summer session at the end of year four, when the teachers systematically adopted a student's perspective *in the context of examining students' statistical reasoning*.

I organize the discussion in following way: After documenting the first shift, I address the means by which it was supported, especially the professional development activities intro-

[42] Both these lessons were recorded after session four, year three. So was a lesson by Lisa & Kate, which did not include a data generation discussion, indicating that the distinction was meaningful only to some of the teachers.

duced during the summer workshop at the end of year three. I point out that these activities did not yet support the teachers' adoption of a student's perspective when they considered students' mathematical reasoning. Instead, the teachers continued to focus on teaching strategies. I then document how the second shift occurred later on and discuss how it was facilitated by prior developments in the group and required additional means of support.

Shift 1: Adopting a Student's Perspective in the Context of Students' Interests and Engagement

As illustrated in the excerpt from the Watermelon lesson co-taught by Marci and Naomi (in year 3, session 2), the researchers systematically pressed the teachers to adopt a perspective of a student during discussions of launch. Although some of the teachers adopted this perspective when supported by the researchers' direct questions and prompts, most focused on what they needed to do during a launch (e.g., what questions to ask, how to debate what a blood drive is; Erin's comments in additional excerpts from year 3 are representative in this regard). In contrast, from the beginning of year 4, most of the teachers started to adopt a student's perspective during discussions of a launch, and concerns about what to do as a teacher remained in the background. The teachers adopted a student's view as they attempted to envision which types of problem scenarios were likely to capture students' interests. In the three following sections, my goal is to document two issues concurrently. First and foremost, in all three sections I provide evidence that it indeed became normative in the group to adopt the perspective of a student when considering whether a problem scenario was suitable for use in the teachers' classrooms. Second, I document that the normative view of the aspects of scenarios and launches that contributed to students' interests in analyzing data evolved over time.

Good Problem Scenarios are Aligned with Students' Prior Personal Experiences

The teachers who occasionally adopted the perspective of a student in year 3 conjectured that enjoyable or "fun" scenarios had the best chance of engaging students in classroom instruction. In contrast, at the beginning of year 4, all but two teachers proposed that in order for an instructional activity to be of interest, students needed to have a *prior personal experience* with the topic at hand[43]. The teachers' view of personal experience as key to students' interests was evident during session two of year 4 when the teachers participated in two data analyses as students. In the first activity, they analyzed data on a teacher's morning driving time from home to school in order to provide advice whether using highways or back roads was a better option. In the second activity, the teachers analyzed data on the braking distances of cars and trucks in order to advise a police department whether highway speed limits should be lower for trucks than for cars. The second scenario was introduced as a problem of driver safety when the highway traffic stops abruptly. After they had completed the activities, we asked the teachers to reflect on why were they engaged and whether these problem scenarios would be useful in their classrooms.

Brian: I didn't think either of these were great examples in terms of engaging the kids. It made me aware … of this sort of tension between creating these examples that have real consequences for society—these are important issues—and coming up with the examples that are interesting to the kids. I mean, this is sort of an adult, kind of grown up issue. That's real, and that's important, but it just doesn't, you know, it's not going to grab the kids.

Researcher: … Why was it that you [wanted] to participate in this activity?

Ben: We drive. We drive to work 5 days a week, and so getting to work on time is an issue. We drive on a highway and we see trucks rolling by and so that is a real issue for us whereas a kid could care less about a route they take. I mean, I've driven kids home, they don't even know the best way to get back to their house. … They follow the bus route, you know? [Laughter.] And so kids are not all hang up on the best route in a car. And they are not worried about truck speed limits. You know? That's the last thing on their minds. Whereas it's important to me because I am thinking about safety. 13 year old is not worried about the trucks on a highway.

…

Wesley: I think I'd have a hard time selling both of them [in class]. [Dorothy, Naomi: Yes.]

Josh: Now, if it was [instead of driving to school] about getting to class, in between the bells- [multiple teachers nodding, yes]

Wesley: But that would have to take note passing time and things like that.

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[43] The remaining two teachers (Muriel and Pat, a district math specialist) proposed that students could also become interested in a topic if they could see it as important to someone else, not necessarily being an issue from their personal experience or important to themselves.

[Several teachers share brief comments at the same time.]

Muriel: So we are assuming that they can only do a task that they get to make the decision about themselves. That they cannot analyze something for somebody else.

Erin: What if you started the launch with what he [Josh] is talking about?

Josh: I think they can do that but not now.

Muriel: So you don't think they would care about you and think about your morning and what you do and how soon you get to school or not? [Naomi, Josh, and some others say "No" and shake heads.]

Wesley: Not readily. I'm not saying it's impossible, I'm just saying if I have a choice of things to do I think there're things that would be much more accessible.

Josh: I am looking at engaging, [Teachers: Yeah] you know, engaging activity. Presenting before-

Amy: [jumps in] What about taking kids to soccer practice?

Josh: [continues without interruption] school board and I mean that's engaging. This right here [driving scenarios], they would do the math, but would that be engaging? Would they really-

Muriel: [jumps in] Or, would it be different if it was like time taken to get to their field trip destination? This is how long it took this team, this is how long it took this team, this is, so which way should we go? [Teachers: hmmm.] Does that have to pertain to them?

(Year Four, Session Two, Nov 2003)

In this episode, the 7 (out of 11) teachers who spoke up all explained their participation in the activity in terms of the personal relevance of the topic at hand and all but Muriel conjectured that since their students do not have experiences similar to theirs, they are not likely to be interested in classroom activities grounded in these scenarios. Throughout the discussion, all the teachers attempted to imagine what might have been a reasonable and engaging task scenario from perspectives of their students.

This episode was representative of the teachers' participation in sessions 1-4 of year 4, where all the teachers except for Muriel and Ruth (a math specialist) expressed concerns about the usefulness of scenarios when their students did not have sufficient prior personal experience, such as experience with braking on a highway. As a result, most of the teachers did not view topics of broader social significance (e.g., highway safety, treating AIDS) as potentially interesting to *their* students. These teachers' views contrasted with the data from the classroom experiments where scenarios of this type proved to be the most productive with a similar group of students (Hodge et al., 2007). The analysis Hodge and colleagues conducted revealed that the students needed to have some basic awareness of—rather than personal experience with—the general phenomenon. However, it was critical that they also had opportunities to develop an adequate understanding of the specific question under investigation and its significance *during the instruction.*

Good Problem Scenarios Should Address a Significant Problem

The subsequent shift concerned teachers' appreciation of significance of the problem and it became apparent in session five of year four. We engaged the teachers as learners in a launch in which we discussed the significance of a problem in detail. Coincidentally, the teacher who brought his classroom-video recording for the group analysis had not addressed the significance of the task in his launch. Several teachers pointed out the contrast in the two launches, and clarified that if students came to see the problem at hand as significant, they might develop a need to address the problem and also a reason to analyze the data. While individual teachers (e.g., Muriel, Amy, Wesley) had previously brought up the issue of problem significance, it had not been picked up by others and did not become a topic of a discussion among the teachers prior to this session. Seven out of the ten teachers who attended session five mentioned the necessity of discussing the significance of the problem at hand with students in their written reflections at the end of the session. The two statistics lessons that were co-taught and video recorded after this session included a discussion of the significance of the problem in their launch. In addition, four out of the 12 teachers present in session six of year four commented that they now actively worked to help their students appreciate the significance of the problem to be addressed during data generation discussions, rather than expecting students to relate to the scenario at the outset.

The group subsequently developed a way of talking about the significance of problems as *the broad issue that helps students develop reasons to engage in the problem* (e.g., driving safety). The teachers specified that this broad issue has to be *narrowed down* in the launch *to a specific question that can be analyzed statistically* (e.g., whether trucks have longer braking distance than cars). This perspective on the significance of the problem was carried through all of the subsequent professional development sessions.

A Good Launch Includes a Discussion of the Data Generation Process

In addition to making the significance of problems a topic of discussions in professional development sessions, we designed additional opportunities for the teachers to notice *how* the influence of a launch could extend beyond engaging students to analyze data. We did so to further support the teachers in coming to view a statistical activity as inherently involving both numbers and context (cf. G. W. Cobb & Moore, 1997). During the classroom design experiments, discussions of the data generation process proved crucial as a means of clarifying both which aspect of situation at hand should be measured, and how to design an experiment that would make it possible to address the question under investigation statistically.

Readers will recall that in year 3, the teachers saw it as important to introduce the data at the end of the launch for reasons of manageability of a lesson. They realized that even the students who were already curious about the problem could only "do it" if they could interpret the data as measures that related to the problem at hand. In order to support students' interpretations, and thus get through the activity more smoothly, the teachers found it reasonable to explain to students in detail what was measured, the meaning of the data points, and the measurement scale. Discussing these details relevant to data did not require justification in year four.

To further support the teachers coming to appreciate the importance of conducting launches that involved data generation discussions, we engaged the teachers as learners in an intentionally inadequate launch in which we discussed the significance of the problem but omitted a discussion of both the aspects of the situation that needed to be measured and how the data might be generated by making measurements. When we introduced the data and asked the teachers to analyze it, they bombarded us with questions about the data generation process. We thus prompted the group to generate a list of questions that needed to be addressed before they could conduct an analysis. Only then did we continue with the data generation discussion, in which the teachers both used and further refined the list of questions. In this conversation, the role of data generation discussions, from a learner's perspective, in both (a) making it possible for students to analyze data and (b) supporting learners' interest and engagement *while they analyzed data* became apparent.

One issue that stood out in the discussion was the distinction between answering questions about data that had already been generated, and conducting a collective thought experiment with students in the classroom in which decisions about what data to collect and how to collect them had to be justified in terms of the question at hand. In the following sixth session, five teachers shared that they had skipped the thought experiment at some point previously, even when they hoped to include it. At this point, the usefulness of a thought experiment of this type was no longer a revelation for the teachers (like it was for Erin in episode from year 3) and the need to include a thought experiment no longer required justification. Rather, the teachers sought a way to remind themselves about the thought experiment when they conducted data generation discussions in their classrooms. Brian's contribution after a researcher had elaborated how this might play out in a classroom illustrates this collective interest:

Brian:	How would you break out our typical presentation? *Launch*, *Explore*, what would you call the last part? *Reporting out* or something?
Lisa:	Yeah.
Brian:	We might wanna call what you're [researcher] talking about [how to design the experiment] a fourth category. Cause what happens with me, I do this launch, and I've got my mind on a clock, you know? And I feel like ok, it's time you get into it. [The teachers—Wesley, Erin, Bruno, Lisa, Naomi, and Dorothy—nod and make quiet comments expressing agreement.] Because this is the natural last thing of the launch. This is the thing that I rush through because I'm starting to feel the time pressures and I wanna get them into the data. Maybe if we called it-
Wesley:	[jumps in] Experimentation
Brian:	[overlaps with Wesley] a separate thing it would make it easier for us to-
Researcher:	[jumps in] Design the experiment?
Brian:	[continues] experimental design or something like that, maybe it will make it easier for us.

(Year Four, Session Six, May 2004)

This is a strong indicator that the teachers had come to see it as important to include a discussion on how to generate data at the beginning of statistics lessons. At this point, the launch was no longer viewed as merely a way to motivate students to do statistics. Instead, planning the part of the lesson that preceded students' analysis now included considerations that were inherently statistical.

By the end of session six, the normative purposes for the opening part of the statistics lesson[44] included clarifying both (a) the significance of the problem at hand and (b) how an experiment could be designed and what aspects of situation should be measured. The issue of the significance of the problem at hand was constituted as a means of *engaging* students in problems worth solving. The discussions of the design of experiments and measurement issues were constituted as a means of making the subsequent analysis and *students' interests in it* possible. Importantly, making decisions about what to include in launch now involved envisioning the consequences of these decisions to students' interests in analyzing data.

Supporting the Teachers' Initial Adoption of a Student's Perspective

Thus far, I have documented the first broad shift that occurred in the ways that the teachers approached pedagogical issues in the professional development sessions, and the subsequent refinements in the teachers' reasoning about launch that this shift in focus made possible. I now back up and examine how the teachers' adoption of a student's perspective in context of students' interests was supported in the first place. Readers will recall that we have attempted, unsuccessfully, to support the teachers in focusing on students' reasoning in year three, and how we came to see *decentering* and examining instruction from a student's perspective to be central to this endeavour. This was because adopting a student's perspective was central to making sense of this student's statistical justifications and rationales. We have also realized that because the students' reasoning was largely invisible within the teachers' current practices, we needed to support them in coming to view students' reasoning as relevant to planning and orchestrating instruction.

In designing the 3-day summer workshop conducted at the end of year 3, we planned to continue supporting the teachers in shifting their focus from the teacher's actions to the sense that students might be making of classroom statistical activities. However, rather than continuing

[44] By this point, the opening part of the lesson was called "launch" by some and "data generation discussion" by other teachers.

to do so in context of students' reasoning, we sought issues that the teachers already considered instructionally important. Students' motivation provided such a focus.

An analysis of teaching sets collected after session six of year three revealed that all the teachers considered student motivation to be a major determinant of both students' engagement in class and their mathematical learning (Zhao et al., 2006). However, the process by which teaching resulted in students' learning was largely a black box for the teachers. Whether students learned or not depended to a great extent on their motivation, which the teachers attributed to societal and economical factors beyond their control. Student motivation and engagement were thus highly problematic issues for the teachers.

Revised Ways and Means of Support: Focus on Issues of Students' Motivation and Engagement

In designing activities that would help us leverage the teachers' concerns about motivating students, we built on an orientation towards students' development of content-related interests that draws heavily on the ideas of John Dewey (1913/1975). Dewey describes interests as something that people develop rather than as inherent aspects of people. From his perspective, students' current interests are starting points for their development of content-specific, disciplinary interests. Dewey used the term *cultivation* to indicate that he regarded it a teacher's responsibility to support the development of students' disciplinary (e.g., mathematical) interests. He argued that disciplinary interests are an inherent aspect of disciplinary literacy, and as such their development should be an instructional goal in their own right.

Importantly, Dewey's perspective on interests also highlights the nature of students' interests. His focus was on students' interests in particular content ideas that could be cultivated *over time*. His view contrasts with the more typical emphasis on enticing students to participate in particular instructional activities in the classroom without necessarily noting what students are becoming interested in over the long-term as they engage in such activities. Dewey's orientation on cultivating mathematical interests reflects a developmental perspective that emphasizes the deeply cultural nature of students' interests. In this way, Dewey anticipated Vygotsky's argument that interests cannot be adequately accounted for by either biological desires or skill acquisition but are culturally developed (cf., Hedegaard, 1998; Vygotsky, 1987).

Building on this orientation, we conjectured that it would be important to support the teachers in (a) coming to view students' motivation as situated and within their control to influence by re-framing this issue as one of students' mathematical *interests*, and in (b) coming to view the cultivation of students' mathematical interests as an important goal of instruction. To accomplish this, we engaged the teachers in investigating how students' classroom experiences contributed to the development of their mathematical interests during the statistics classroom design experiment. We conjectured that in attempting to understand what became interesting for the students in the design experiment class, the teachers would adopt a student's perspective. We further conjectured that this would later allow us to focus on students' reasoning as an aspect of instruction that is relevant in cultivating students' mathematical interests. The means that we conjectured would be effective in supporting the group's learning along this modified trajectory included a series of activities in which the teachers analyzed (a) interview excerpts in which design experiment students described their learning experiences and valuations of those experiences in the statistics class and in an algebra class in which they were enrolled at the same time, and (b) a series of videos from the classroom design experiment (see Appendix F for the sequence of activities in years 3-5).

The Relevance of the Revised Context for the Teachers' Learning

Thus far, I have clarified that the teachers did not consider students' reasoning to be relevant to their instruction. It is also important to document that the teachers considered student motivation to be highly relevant. At various points of the 3-day summer workshop in year 3, all the teachers[45] indicated, in their comments and in written reflections, that they found issues of students' motivation very relevant to their instruction and appreciated this focus. Brian's comment at the end of first day was representative of the views expressed:

Brian: I was very happy that we are talking about students' motivations. Except just what math is there. I think that is big. If I can get students motivated that's more important than anything else.

(Summer 3, Day 1, Jun 2003)

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[45] Seven teachers and two district mathematics leaders (Ruth and Esther) participated in the summer workshop. Unless the distinction is crucial, I refer to all of them as the teachers. Out of the 5 teachers who did not attend, 3 were leaving the group (see Appendix A), and 2 were unable to participate in summer months.

In addition to overall satisfaction with the focus issue, the teachers also expressed that they felt they were learning a great deal. The following teachers' reflections capture this sense of learning and the determination in the group to learn more at the end of day one:

Lisa: [You push] to get us to really figure out what is going on in the classroom. And what is going to motivate these kids we've got. We need to be thinking past just what's on the surface. We've got to. I know it's hard to think that way. [Teachers nod and one researcher speaks together with Lisa: "Yeah, this is not easy."] And I think we've got some ideas out there that I haven't really thought about. I know we have.

Erin: I think, when like Esther said, what we are doing is making the invisible visible. [Several teachers say "Yeah"] … if I look down on what I wrote down on that first [activity today, "Why are some students motivated and others are not?"] I'm going: "Oh, wait a minute." You know, it was [I focused on] more things outside of my control versus [what happens] in the classroom. … Just a lot of stuff that goes on in the classroom, that just happens and I've never thought that I—how much maybe I played into it. Or have a part in it. … Just things I have not thought about that I have to think about.

(Summer 3, Day 1, Jun 2003)

In these comments, the teachers referred to the sequence of activities that took place in day 1, the goal of which was to support the teachers in reconceptualizing where students' motivation comes from and whether it could be shaped in classrooms. We first asked the teachers to describe their initial thoughts about why some students are willing to engage in mathematical activities in the classroom while others are not motivated to do so. In their reports, the teachers identified factors impacting motivation that could be categorized as belonging to one of the following categories:

- Environment, society, and nature (e.g., economic factors, parents' school experience),
- Prior math schooling (e.g., prior math knowledge, fear of failure), and
- Teacher and school (e.g., teacher dedication, student's attitudes towards the teacher).

These initial thoughts aligned with the findings of an analysis of the teaching sets (Zhao et al., 2006), in that they focused on, as Erin later pointed out, "more things outside of my control versus [what happens] in the classroom."

We then asked the teachers to analyze transcribed excerpts from interviews conducted with students who participated in a statistics classroom design experiment. In these interviews, the students were asked to comment on their obligations in the statistics class and in an algebra class in which they were enrolled at the same time. The students were also asked to assess their own and their classmates' competence in the two classes (see Appendix G for interview excerpts). To summarize, the students described their obligations in algebra as following procedures introduced by the teacher and arriving at correct solutions. They named a few students who were good in algebra, based on grades assigned by the algebra teacher. In contrast, the students explained their obligations in statistics class as looking for answers that "make the most sense," being able to explain their solutions, and asking questions when they did not understand. Most of the students stated that they and all the other students were good in statistics because they all had good ideas and could explain them so that others understood.

During the professional development activity, the teachers' task was to investigate whether the students valued their obligations in the two classes and identify possible reasons why the students might have become increasingly interested in analyzing data. This analysis led the teachers to question their prior views that some students were inherently unmotivated. The group concluded that the same students who were unmotivated in the algebra class appeared to become highly motivated and to view themselves as competent in the statistics class. Like Erin and Lisa, all the teachers were fully aware of the change in their views about students' motivation, stating that they realized they needed to be able to influence students' interest. The teachers therefore found it worthwhile to analyze interactions in the statistics class in detail in order to understand how the students became interested in analyzing data and how the development of their interests was supported.

During the following two days, the teachers spent almost 4 hours analyzing selected video episodes from the statistics classroom[46] in small groups. They voluntarily worked during the scheduled lunch break on day 3 to complete their analyses. In the course of these analyses, they re-viewed the episodes several times[47] and discussed in detail what happened in the classroom. The level of the teachers' engagement suggests that they were not merely cooperating with us on the assignments. Rather, they became invested in learning about issues that they considered to be significant.

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[46] The teachers analyzed 3 classroom episodes, each 7-14 minutes long (33 min total). The first episode, Batteries, came from the beginning of the 7th grade design experiment, Speed Trap from the middle of the sequence (one month later), and AIDS from the end of the sequence (two weeks later). All episodes focused on univariate data analysis.

[47] Each teacher had a notebook and earphones available for viewing digitized videos, and transcripts of all episodes. The groups conducted their analyses in separate rooms. Researchers were present but did not steer conversations.

An Example of the Teachers' Emerging Insights Derived from the Analysis of Classroom Videos

To illustrate the depth of the analyses that the teachers produced while they worked in three small groups, consider the emerging insights that pertained to ways of judging students' classroom participation. Lisa and Esther pointed out a distinction that they had noticed during their analysis between participation as the appearance of paying attention and participation as a student's ability to make insightful contributions:

Lisa: We also talked about the little boy that, she [the teacher] had to tell him to get the hood off the head. [Teachers nod and say "Yeah, Derek"] And just the fact that he may not- he didn't seem like he was engaged *but then all of a sudden he took off. Remember?* We talked about how he started all of a sudden-

Esther: [jumps in] Yeah, it bothered me how many kids' heads were on the table, in the different ones [video clips], *but then they would come back.*

Lisa: And I was saying, that-

Esther: [jumps in] So maybe it was a morning class. [People laugh.]

Lisa: when kids have something, when something hits them and they have something to say, that's when they will go off. So that they have something to say. And they can be listening and not having a thing to say, and just kind of [be there]. But with him [Derek] … the discussion sparked, you know, in some way where he had still something to say about it.

(Summer 3, Day 2, Jun 2003)

All three groups of teachers who analyzed classroom videos mentioned the episode where Derek first did not appear to pay attention and later contributed to classroom discussion with an insightful comment. The teachers took Derek's contribution, rather than his appearing unmotivated with his head on the table, as evidence of his classroom participation. While the previous episode is unusual in that most of the remaining teachers' public comments related to the video teacher's actions, it illustrates that the teachers indeed used the classroom videos as a tool for understanding instruction and that their detailed explorations occasionally resulted in findings that surprised them and shaped their developing views of motivation and engagement. The teachers referred back to the episode with Derek whenever they discussed students' participation in years 4 and 5.

The Role of Classroom Video in Teachers' Learning about Students' Interests and Motivation

The teachers' enthusiasm for video analyses would be unlikely to last through the 3-day workshop if they did not find the classroom videos to be a useful tool for their learning. It is therefore important to document that they constituted the videos that they analyzed as examples of *good instruction,* and to examine what contributed to this teachers' view. Amy's comment is representative and illustrates the teachers' valuations of the video teacher's instruction:

Amy: The teacher would set goals, and accountability, and expectations. This was something that [the teacher] did a very good job at the very beginning of each of these things [video clips]. She clearly stated what was going to happen and what she expected to happen and throughout the [lesson] she held students accountable for participating in the discussion. And it was done in very what we consider a positive way: … Instead of telling a kid "Speak up! We cannot hear you!" it was "Use your big voice," or it was done in a way that was not an embarrassing or intimidating experience.

(Summer 3, Day 2, Jun 2003)

The three groups of teachers consistently highlighted positive aspects of video teacher's instruction when they reported their analyses.

It would be a mistake to assume that the teachers' positive interpretations of the video episodes were in any way natural or that they were necessitated by the videos. Indeed, they were in stark contrast to the ways that the group interpreted the same videos two years earlier during summer workshop after the first year of collaboration[48] (Dean, 2005; Visnovska, 2005). As I noted in Chapter III, the group at the time focused on student behavior and classroom management issues and considered the video teacher's instruction ineffective. The same episode in which Derek's head was on the table was at that time constituted as evidence of lack of the teacher's control and students' unwillingness to participate in the lesson.

Several aspects contributed to the teachers' interpretation of statistics design experiment classroom videos as examples of good instruction at the end of year 3. First, the instruction was enacted in a real classroom with a diverse group of students that the teachers considered similar to their own classrooms. Second, the video instruction surpassed the teachers' own efforts to support student learning. The teachers constantly drew comparisons between the video episodes

[48] Lisa, Marci, Wesley, and Muriel participated in both video analyses (Summer 1 and Summer 3) and their participation was each time consistent with that of the rest of the teacher group.

and their own experiences with teaching the tasks from the statistics sequence. For instance, when the three groups of teachers reported their video analysis findings on day 2, ten such comparisons occurred. The following excerpt is representative and illustrates the nature of their observations:

Esther: ... this [whole class discussion] was different than [how] … most of you have done this, in that there was one computer and the projector and the kids had to give directions to the teacher … about how to display the data, or "Put in that line for 80" or whatever. And so we had quite a bit discussion about how that's actually a strategy that made the kids verbalize more what they're doing. … You wanna add to it? [To her group]

Brian: Yeah, it was sort of a jump-start to explaining your ideas and responding to each other. To have to tell out loud to the teacher what you wanted to do with the tool.

Esther: And because I haven't taught this, I wouldn't have reflected on that the way that Lisa and Brian both brought that up. That it made the kids give specific directions and that kind of thing. So we looked at video 1, line 4 as an example of that. The child is asking her [the teacher] to put the range on and [the teacher] saying: "What do you want me to change it to?" and so on.

(Summer 3, Day 2, Jun 2003)

Although the teachers' engagement in video analyses of their own and their colleagues' instruction earlier in year 3 was, in comparison, very low, these activities did help the teachers become aware of how their instruction appeared on video. I conjecture that the teachers' participation in the prior video-based activities helped them to look beyond what they interpreted as problematic students' behavior two summers ago and to appreciate how the video teacher addressed some of the challenges that they also faced in their classrooms.

Lastly, the teachers came to view the video instruction as aligned with the discourse of reform mathematics that was current in their schools. This was apparent from the ways in which they reported their findings from video analyses. All the groups organized their findings according to general categories that they viewed as relevant to "reform math" (e.g., novelty, variety, affiliation, choice, safe haven[49]) and used examples from the video-recorded lessons to illustrate how the video-recorded instruction addressed these pre-chosen categories. This organization of

[49] For instance, Esther, the district mathematics coordinator, proposed to organize her group's reports according to five categories from *Working on the Work*, a school reform initiative adopted by the district. The remaining teacher groups organized their findings in a similar, top-down manner, even thought they proposed new categories.

their findings indicates that the teachers did not adopt a student's perspective during the whole group discussions but instead reported on the instruction from an observer's perspective. I omit specific details of the teachers' reports in this analysis because these details do not help to clarify the subsequent learning of the teacher group.

In the previous paragraphs, I summarized several developments that made it possible for the teachers to both (a) begin using classroom video as a resource in learning about what they saw as effective classroom instruction and (b) view the summer session as productive. I now document how teachers' engagement in subsequent activities in which the issues of interest and motivation were explicitly brought to the fore provided them with opportunities to first reflect on their own interests and to adopt a similar view when envisioning whether specific activities could be of interest to their students.

Supporting the Teachers in Reflecting on Their Own Motivations and Interests

The nature of teachers' participation in the summer session activities indicated to us that the issue of motivation was a viable starting point for engaging the group in collective work on instructional improvement. In the first session of year 4, all the teachers who attended the summer workshop emphasized that they had since made changes to their instruction, and that these changes were aimed at engaging their students in a mathematical activity. Because the shift of the context to issues of motivation did not result in the teachers adopting the perspective of a student during the summer workshop, we designed additional activities in which the teachers reflected on their own interest and motivation.

In the first session of year 4, we engaged the teachers in *Traffic School and Hobbies* activity that I have previously described in Chapter VI. In this activity, we asked them to imagine themselves in two situations: in a Traffic School where they went to avoid a fine after they were stopped for speeding, and in a situation where they pursued one of their hobbies. We asked the teachers to explain whether they would be motivated to participate in activities in these two situations and whether they would be more interested to learn in one context or the other. Our goal was to provide all the teachers (including those who did not attend the summer session) with opportunities to imagine themselves in a situation where they would not be motivated to participate and in a different situation where their motivation to participate was strong. This was intended to further challenge the teachers' initial assumptions that some students are inherently motivated and others are not, or in other words, that motivation is a characteristic of students that is independent of the activity in which they participate. From this point on, we consistently asked the teachers to reflect on their own interests after they engaged in statistical activities as students, specifically asking them why they were interested in participating in the statistical discussions. As I have previously illustrated, the teachers soon (year 4, session 2) started to adopt a perspective of a student when they attempted to envision whether specific task scenarios would be interesting to their students.

Adopting a student's perspective became later normative in the group also in the context of examining students' reasoning during the summer workshop at the end of year 4. In the following two sections I first document that this indeed was the case and then return to discuss the events that took place during year 4 and supported the second shift.

Shift 2: Adopting a Student's Perspective in the Context of Students' Statistical Reasoning

The major goal for the summer workshop at the end of year 4 was to support the teachers in reconstructing the statistics instructional sequence. This involved examining the forms of statistical reasoning that the sequence supported, how these forms of reasoning built on one another, and how successive shifts in students' reasoning were supported at different points in the sequence. Once the teachers had reconstructed the instructional sequence, we planned to focus on how it might be adapted so that the teachers could use it in their schools as part of their regular instruction. This required that the teachers understood which of the State objectives for teaching statistics were already addressed by the sequence, which needed to be added, and how could they be addressed in a coherent manner.

The major means of supporting teachers' activity in the professional development sessions was again a series of classroom video episodes from the statistics design experiments. At this time, the series of videos included 8 edited clips, 4 of which the teachers had analyzed previously (i.e., Braking Distance, Batteries, Speed Trap, and AIDS). Three of the 4 new video clips came from the 8^{th} grade classroom design experiment and involved the analysis of bivariate data analysis.

The teachers worked in small groups and were first asked to list their initial thoughts on the overall goals of the statistics instructional sequence. Second, they analyzed a classroom video from the very end of the sequence to determine the overall instructional goals. Third, they drew on experiences from their own classroom statistics instruction to determine the likely starting points for students' statistical learning. In most of the remaining analyses, the teachers examined classroom videos grouped by the computer tool that the students used[50]. The goal for the teachers was to identify the major shifts that occurred in students' statistical reasoning and how these shifts were supported. After the groups had presented their separate findings in day 1, the researchers synthesized them during a short break and the whole group checked and edited the synthesis.

Throughout these activities, the teachers directly examined students' reasoning from a reasoning student's perspective. The following excerpts are representative of ways that all the teachers reported findings from their analyses during the summer workshop. The excerpts come from the activity in which the teachers, while working in 4 small groups (2 or 3 teachers in each), identified the endpoints for the students' statistical learning based on their analyses of classroom video of the *Education and Salary* activity. In the video clip, the students analyzed data on the salaries of men and women organized by years of completed education to determine whether gender impacted pay (Figure 20). Each graph represented annual incomes of 50 people in each of the six levels of education (completed 8, 10,12, 14, 16, and 18 years).

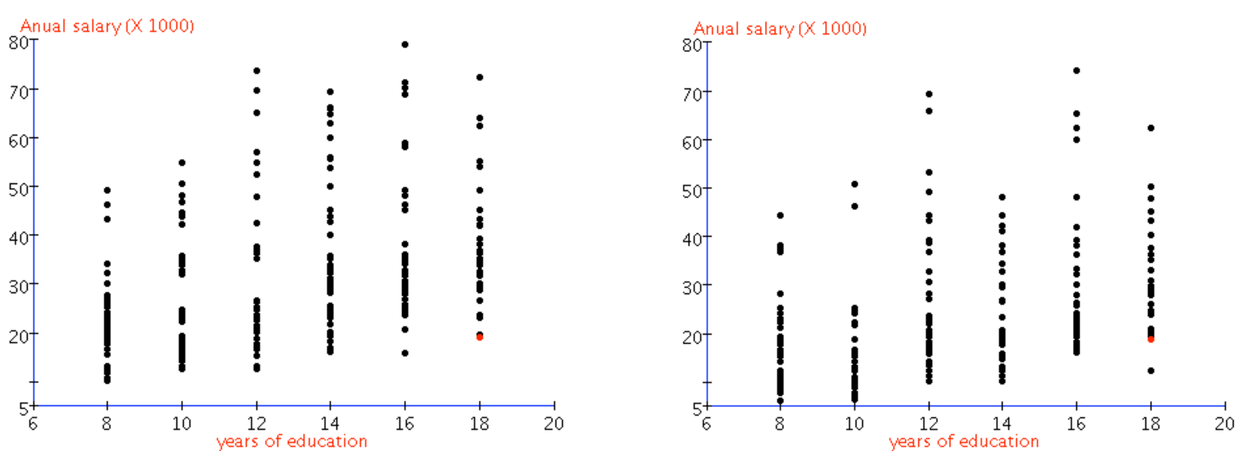

Figure 20. Computer Tool Three—Education and Salary. (a) Data for men. (b) Data for women.

[50] Computer tool 1 was used in comparisons of two univariate data sets with a small, equal numbers of datapoints (10-12). Computer tool 2 was used in comparisons of two univariate datasets with larger number of datapoints (30-300), both equal and unequal. Tool 3 was used in comparisons of bivariate datasets. (For details, see Chapter II.)

When the students analyzed these data, they used different computer tool options, such as "grids" and "4 equal groups" (see Figure 21) to locate what they referred to as the "humps". Given how the data were represented in Computer Tool 3 (see Figure 20), it is important to point out that no "humps" were readily visible in the representation. The students had to be able to read the representations as having a third, invisible dimension, in which each stack of data appeared as a "hill" similar to those in Computer Tool 2. They developed ways to use the options on the computer tool to help them determine the shape of each stack of data in this third dimension. They used the median, as well as the largest number in each vertical column in the grid to indicate the peak of the hill, and used four equal groups (i.e., box plots) to determine where the slopes of the hill were steep and where they were more gradual.

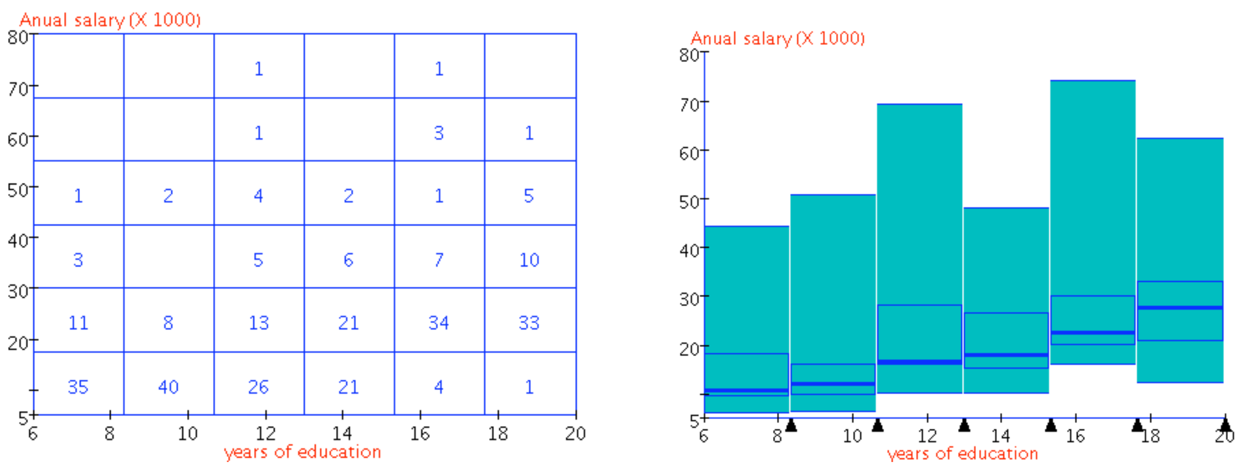

Figure 21. Education and Salary—Computer Tool Three. (a) Grids option, women data. (b) Four equal groups option, women data.

Ben opened the whole group discussion by sharing findings of his small group:

Ben: One kid asked a question: "How do you know what a good salary is?" That is not in itself a specific need for this activity, but it shows that they are thinking about the context of the problem and the real meaning of what we're doing. I'm not just answering some random math questions, there is a context behind it. So you can see that they are aware that the context is important. … They knew what parts of the data they were talking about. When they mentioned extremes they knew what they meant by extremes. And when they said median, and middle, they knew where that was. And the idea that the hump would be kind of in the middle [of the data stack], and that the extremes probably aren't as important. They initially kind of, looked at the—first kid there said that he looked into the extremes, but then later you saw that they got away from the extremes. I forgot what the quote was, [tries to find it in the transcript] “Where all the people,” or “most of the people are,” and that they “go by where the people come together.” That was another line that he said, “where the people come together” [Ben gestures with his hands as if pushing data together]. So you can kind of see the grouping [same gesture], and the distribution coming out. They are seeing how they are distributed, not just a bunch of dots.

… [Another group presented after Ben, before Lisa]

Lisa: We are starting to repeat ourselves though [Lisa carries poster of her group and fastens it to the board]. It is, basically, that they learned to determine the shape. Talked about the shape a lot. On line 18 and 35 [in the transcript]. They were using the median to see the hump. And that the median is separating, we talk again about that they know the median separates it [Lisa quickly shakes her hands horizontally as if cutting a vertical stack of data] in two equal parts. They were using grids to show distribution of the data. Which is seeing the hump as the larger number in the grid. And that was one of the big eye openers for them. And then the use of box plots [four equal groups] to find the hump.

[Summer 4, Day 1, Jun 2004]

Ben, Lisa, and presenters from the remaining two groups all described how students could see how data were distributed. In contrast to their participation in analyses of student work in sessions 3, 5, and 6 of year 3, the teachers did not merely categorize students’ solutions according to the solution methods the students used. Instead, they explained what students attempted to accomplish (e.g., locate where the humps were in the data) by using specific options on the computer tool for organizing data. The computer tool options—instead of becoming solution methods that students mastered, or needed to master—were now seen by the teachers as a means to read how data were distributed. In describing what the students tried to see and which tools they used to do so, the teachers adopted a student’s perspective. They consistently focused on the forms of reasoning that the students developed in different phases of instructional sequence both when analyzing the videos and when sharing their findings with the whole group. The teachers also consistently pointed out the key mathematical ideas on which different students’ solutions were based (e.g., examining all the data, proportionality). In addition, the teachers were highly engaged in these analyses. On the second day of the summer workshop of year 4, they proposed composing a synthesis of different groups’ work themselves, taking over the role adopted by the researchers on the previous day.

The teachers' approach to analyzing the classroom videos also strongly contrasted with their approach during the summer workshop at the end of year 3. The readers will recall that during the summer workshop in year 3, the teachers considered classroom videos to be examples of good instruction from which they could learn. The teachers then reported their findings in general categories derived from the discourse of reform current in their schools (such as *safe haven*, where students are not afraid to speak up, and *real world problems*) and used the data to illustrate these categories. In summer of year 4, the teachers instead derived forms of reasoning that students used from the data and did not determine their categories in advance. Moreover, not only did all the teachers participate in this way, but some of them, Brian in particular, could explicitly articulate why this was a more productive way to analyze video. The following discussion took place during reflections at the end of day one, after the teachers established the instructional endpoints for students' learning, the starting points, and analyzed videos to identify the initial shifts in students' reasoning. Several teachers took this discussion as an opportunity to clarify the workshop goals and propose additional agenda items.

Brian: What we are trying to develop, sort of a conceptual map of how we want during middle grades students reasoning about statistics to progress? Is that accurate?

Researcher: Yeah.

Brian: And then once we've got that conceptual map, then we will start talking about how to make it happen in our classrooms? Using the Minitools and using the [newly adopted reform textbook] materials?

Researcher: Yeah, how to make it happen. And also how to adapt it.

Pat: What do you mean, how to adapt it?

Researcher: We did this with certain ideas in mind. In the certain context, in Nashville, whenever. There are going to be other issues that you are gonna need to address because of [state standards], because you are adopting [new textbook]. That's what we mean by adapt. And there will be other changes no doubt. So it will have to be further adapted later. That's what we mean by adapt.

Muriel: Will we also talk about how to assess? [For instance] how many people in the room do understand what consistency means.

Researcher: Yes. We need to make a note of that. Because the question for me is: Should we address it right next, or come back to it later? But it seems like several people are nodding. Since Muriel has said this assessment issue twice, it is very important, right? [Laughter.]

Brian: Is our main technique for developing this conceptual map gonna be to watch videos and discuss about what shifts in reasoning- [?]

Researcher: [jumps in] That was our plan. Now, what we [researchers] will be doing then—because right now we've had a list of 4 reports [from 4 small groups]—so we see our jobs to try to synthesize that. And then bring it back to you, to say "Does this kind of represent what people were talking about?" In that spirit. And to, at least, have very rough notes on it. That's gonna be a basis for a web page [of resources for teaching statistics].

Brian: I would characterize this approach as data driven, sort of like from the bottom up.

Researcher: Yes. That's right.

Brian: And I'm assuming that the reason you wanna do it this way instead of a more of a from top down process because if we go sort of top down, we are going to be just sort of following the way we've been already taught, or the way it's organized in textbooks.

Researcher: Or not the textbook, but the way we organized it. And to be quite honest, we are not sure what sense that would make.

Brian: So doing this bottom up gives us sort of the fresh way, fresh perspective of thinking about these things?

Researcher: And our hope is it has some reality for you. You view these videotapes, they really happened in classrooms, and because you used some of these activities in your own classroom, making sense of these videotapes is not gonna be quite so tough.

(Summer 4, Day 1, Jun 2004)

The excerpt above illustrates that the teachers were actively making sense of the professional development activities with respect to their own learning and monitored their usefulness to classroom instruction. While the teachers were not equally articulate and assertive in asking their questions, they all asked for clarifications at some point and the group collectively engaged in negotiations similar to the one in the excerpt above in every session in year 4. In his questions, Brian summarized the goals for the workshop, the tools that the group used (and planned to use) to accomplish these goals, and the rationales for using these tools. He both built on ideas that the researchers had shared with the group earlier and created additional opportunities for the teachers to explicitly examine the directions of the collective activity.

The last important contrast with the activities in the summer of year 3 was in the teachers' overall view of the video-recorded instruction. They no longer constituted this instruction as exemplary. The teachers now proposed alternative ways in which instruction could have proceeded and pointed out opportunities for supporting students' learning that the video teachers[51] missed. The following excerpt is representative of the nature of these contributions. Muriel referred to an episode in which some students started to use the four-equal-groups option to organize data, while most of the students could not yet interpret this representation effectively. Specifically, many students interpreted narrow intervals as containing fewer datapoints. Muriel noted a need for clarification in the whole class discussion, which the video teacher did not pursue:

Muriel: You [researcher] let the comment go, about, somebody said, "It's kind of like the range." They said "What did they mean by the group is smaller?" And somebody said "It's kind of like a range of the group." And you said: "Uhm, it's [data's] all scrunched up." And then went right on to the next thing,

Researcher: [jumps in] That's right. [with irony] So I knew it [how to read the graph].

Muriel: [continues] and didn't let it- And everybody else wasn't with you.

Researcher: You're exactly right.

(Summer 4, Day 3, Jun 2004)

To summarize, the teachers approached video analysis in the summer of year 4 by directly examining ways in which students reasoned about statistics. In addition, by looking at classroom instruction from a student's perspective, they noted situations in which students' reasoning was not optimally supported in the video classroom. In the following section, I examine how the emergence of this shift in teachers' participation was supported. I will then come back and discuss aspects of teachers' participation in summer workshop of year 4 that indicated to us limitations in the teachers' pedagogical reasoning and led us to formulate conjectures about additional supports for their learning.

[51] One of the researchers, Paul Cobb, was one of the two teachers that appeared in the video episodes.

Supporting the Teachers' Adoption of a Student's Perspective in Context of Students' Reasoning

The readers will recall that from year 3 on, we pressed the teachers to examine instruction from students' point of view, conjecturing that as the teachers developed this perspective they would come to view students' reasoning as a relevant aspect of instruction. From the beginning of year 4, we attempted to capitalize on the teachers' focus on students' developing interests. We thus designed activities that aimed to help the teachers understand that (a) other aspects of the lesson (beyond launch and data generation discussions) also contributed to students' development of interests in analyzing data, and that (b) unguided students' presentations had limited impact on their students' learning.

From the beginning of year 4, there were initial indications that some of the teachers occasionally adopted a student's perspective in specific discussions about students' presentations and data analyses. Once adopting a student's perspective had become a norm in the group in the context of students' interests (session 4, year 4), most of the teachers started to notice students' participation in data analysis discussions as a source of students' continued interest. I first discuss activities from the beginning of year 4 in which some of the teachers started to adopt a student's perspective. I then document how more sophisticated ways of reasoning about the launch that became normative in the group contributed to the teachers' development of interest in forms of students' statistical reasoning.

Initial Contributions Related to the Use of Students' Reasoning in a Classroom

In session one of year 4, we asked the teachers to share their findings from a statistical task that they solved as students. In this discussion, we intentionally prevented the teachers from raising additional questions and concluded each presentation by thanking the presenters before asking the next group to report. Our goal was for the teachers to experience the contrast between *presentations* as reports of results of learning and *discussions* of analyses, in which all participants are expected to monitor their understanding. After the presentations, we asked the teachers to reflect in writing on what they have learned from the presentations and then discussed the teachers' experiences in the whole group. While many of the teachers focused on the mathematical potential of the task, 9 out of the 13 teachers present in the session also noted that the presentations alone were not very informative, and did not help them learn. This indicated to us that we might, over time, become successful in helping the teachers realize that presentations did not provide sufficient support for their students' learning.

During the first three sessions of year 4, some of the teachers started to make comments about students' reasoning and how it could be built upon. For instance, in session 2, Wesley responded to Muriel's concern that all her students used 3 equal groups, rather than 4 equal groups, to organize data on second computer tool. Muriel's students could have created 3 equal groups by using the "Fixed Group Size" option, or by using "Create your own groups" option and trying to make the number of dots in each group equal (see Figure 22). Students usually did so in order to isolate the middle part of the data (where the hill was) from the rest.

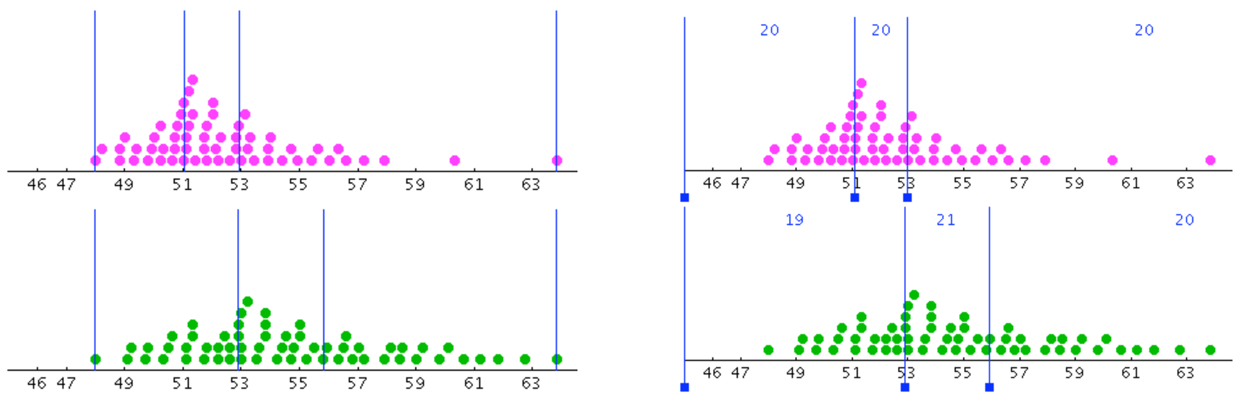

Figure 22. Computer Tool Two—Speed Trap. (a) Fixed group size of 20. (b) Create your own groups option.

Muriel:	My class, when we went to the computer lab, they were dividing it like into thirds. And we did that. Didn't we do that at first? Thirds? [Wesley nodding]
Researcher:	Yeah.
Muriel:	But they, none of them divided into the 4 equal groups. Which did not lead to box and whisker plots at all [Muriel laughs].
Researcher:	No, that's fine. That's actually kind of what we expect, by the way. … [The researcher talks about box plot being a relatively recent development in statistics.] So it's just unreasonable to expect the kids that they are just gonna invent this, they're not. … I think it's just fine to introduce stuff to kids. Not that they just create it out of nothing. But it's at a point when they see a need for it. And it's gonna make sense when they can see it as a useful tool.

Wesley: And the industry standard is to divide it into fourths. [Researcher nods.] And if they're dividing into the thirds, they've got the idea in their head, to divide. And then you'd say, "Well, you know, most people talk about it in terms of fourths," and they're not gonna have a hard time making that leap to there.

(Year Four, Session Two, Nov 2003)

Wesley's comment illustrates that he both recognized important similarities in the two ways of structuring data and considered these two ways to be similar enough from students' point of view that proposing to use 4 equal groups in the classroom would be reasonable at that point.

The last excerpt comes from the students' work discussion in session 3 of year 4, in which at least 3 teachers contributed to a discussion of how students' solutions could be compared and contrasted in a classroom. The students had analyzed data on number of hours in a week that students from one classroom watched TV. Their task was to represent this data graphically and decide whether the students were watching too much TV.

Bruno: Another one [type of solutions] is comparison against some arbitrary standard, like this 1.5 hours, when they just picked what they thought was a reasonable number and then based argument on that.

Researcher: So how would you get at that arbitrariness, by comparing what with what? That's what you have to work with.

Bruno: Well, my group compared, they took the mean and took the total number [of students] above and below the mean. And the mean is sort of arbitrary.

Researcher: Yes. So, was there another type of solution where it's not arbitrary? I guess that's what I'm asking. A point of comparison.

Brian: Well, like if you thought that two 30 minute shows a day is normal, then it is not arbitrary to chose one hour per day as your cut off point. That gets away from arbitrary.

(Year Four, Session Three, Dec 2003)

While the teachers did not propose on their own how would they compare and contrast different students' solutions in their classrooms, some of them were now participating in discussions of such comparisons when guided by the researchers.

It is also noteworthy that from the session 3 of year 4, we started each but one session by discussing an overview of the statistics instructional sequence. We then situated every activity used in the session within the big ideas that the sequence was designed to support. While the first of these overviews (session 3) resembled a 38-minute monologue, in the subsequent sessions[52], 5 to 8 teachers spoke up during these discussions, asking about and commenting on different topics closely related to the big ideas of the sequence.

Students' Reasoning Relevant to Students' Interest in Analyzing Data

After session 3 in year 4 we realized that the teachers' sophistication in producing and sharing analyses of univariate data made their reflections on why they were interested in data analyses discussions and *what made them interested* unproductive. Because they already understood the statistical reasoning involved, the teachers could not point out ways in which the discussions enhanced their own learning and interest. In order to make it possible for them to experience more realistic data analysis discussions, we designed statistical activities directly tailored for the teachers that were not intended for their use in classrooms. We used these new activities that were statistically more challenging in sessions 4, 5, and 6 in year 4. One of them involved bimodal univariate data and two involved bivariate data. In their written reflections from session 4, four of the teachers recognized data analysis discussion as something that supported their interest. In the subsequent sessions, we pressed the teachers to elaborate what held their interest during these discussions.

Most of the teachers began to adopt a student's perspective more systematically in the context of the concluding data analysis discussions when they attempted to determine if students were interested in specific instructional activities. Although it continued to be difficult for the teachers to focus on the listening students, they no longer focused solely on teaching strategies, as was the case throughout year 3. Most of the teachers' interpretations of our press for what listening students understood and why they continued to engage were now aligned with our intentions. In session 4 of year 4, four teachers suggested that if the listening students did not understand their classmates' explanations, they could not gain new insights *and would not remain engaged*. In addition, at different points in sessions 4, 5, and 6, four teachers shared that they had begun to support the listening students in their classrooms during discussions of data analysis. In session four, Ben shared:

[52] The lengths of the subsequent overview discussions were 45 min in session 4 and 23 min in session 6.

Ben: When we discuss, kids would say it [solution] in their own words, and I knew what they meant, but I could tell a lot of the other kids [did not know]: "What are you talking about?"

Researcher: What did you do in that situation?

Ben: I tried to rephrase what they were saying a little bit, I asked them if they could—at first I'd say "What do you mean?" and try and get them to say it. But when they couldn't explain it well, I would then try "Is this what you are saying?" Usually I know what they are saying, but most of the other kids don't. And they [presenters] know what they mean; they just have trouble verbalizing it. My kids wanted to have a debate about this. And I've never done a debate. I thought it was really interesting… But they were having trouble doing that.

(Year Four, Session Four, Feb 2004)

The previous discussion suggests that the assumption that students should be able to make sense of each other's contributions became constituted as relevant in the group. In addition, some of the teachers now found it reasonable to investigate why some students in the videos[53] of lessons that they co-taught could not understand others' explanations. In an analysis of a video of the Batteries activity conducted by Brian and Wesley (session 4, year 4), the teachers first tried to understand the reasoning of a girl who compared single data points in her presentation. We then pressed them to envision what this girl might understand from a presentation of her classmate who reasoned proportionally about groups of data. This question led to a discussion in which 5 out of the 10 teachers who were present in the session spoke up. The teachers attempted to understand what the girl could or could not "see" and used the reasoning apparent in her presentation as a rationale to argue that the girl had no means to make sense of her classmate's solution.

The teachers also engaged in discussions of students' reasoning in the analyses of classroom videos in sessions 5 and 6. In session 6, some of them started to point out how what was discussed in launch impacted the types of solutions students produced and also affected what they talked about in the data analysis discussions. As I have previously illustrated, all teachers started to adopt a student's perspective and found it reasonable to examine students' reasoning in

[53] In sessions 4, 5, and 6 of year 4, the group again analyzed videos of statistics lessons co-taught by pairs of the teachers in their classrooms.

order to understand both students' learning and engagement during the summer workshop at the end of year 4.

Limitations of Pedagogical Reasoning of the Group

In the previous section I discussed the events that preceded the teachers' participation in summer workshop at the end of year 4. I now return to that workshop to discuss two areas in which the teachers' learning required additional support.

Proposing Shifts in Students' Reasoning

The readers will recall that the teachers focused on the diverse forms of reasoning that the students developed in different phases of the instructional sequence. The teachers derived these forms of reasoning in a bottom up manner by watching and analyzing classroom video clips. In addition to describing students' solutions, the teachers also identified the big statistical ideas that underpinned these solutions. Nevertheless, we needed to provide substantive support for teasing out from the teachers' analyses (a) *broad commonalities* in the diverse solutions produced in each phase of the sequence, and (b) the *shifts* in ways that students reasoned in different phases of the sequence.

The following excerpt is representative of the level of support that the researchers needed to provide in the discussions of the major shifts in students' reasoning. The excerpt comes from the second day of the summer workshop in year 4. The teachers had just finished the discussion of posters they produced in 4 small groups where they documented the diversity in the students' reasoning present when the students analyzed univariate data on Minitool 2. The readers will recall that the teachers proposed to synthesize their findings from this video analysis on their own rather than letting the researchers to propose the synthesis. In our view, the big shift that occurred in the video students' reasoning as they engaged in activities with Minitool 2 was between seeing a "hill" in dots on the picture and starting to use the options on the tool (e.g., 4 equal groups) to read where the hill was when data were hidden (see picture 11b in Chapter V). We saw this shift as crucial given that in Minitool 3, the hills were no longer visible and in order to infer how the data were distributed, students needed to be able to read shapes of the data from the tool options. After one of the researchers talked about this shift from seeing to inferring the hills for about two minutes, we asked the teachers to synthesize the findings across their posters.

Researcher: How do we wanna proceed? Do people maybe just wanna few minutes in their pairs or individually? To pull out what you see as cross-cutting issues?

Muriel: What kind of form are we looking towards, [researchers' name]?

Researcher: Let me give some examples. Maybe we should look back at the Challenge 4 when we talked about what the major shifts were on Minitool 1. And there we talked about how you are trying to get kids in the game [of analyzing data]. Would you say the kids you saw there [in Minitool 2 video clips] were in the game, they were actually analyzing data? [Teachers: Yeah.]

Erin: It seems like that is where they picked up and kept going. On the first one [Minitool 1].

Researcher: So we can say these kids [on Minitool 2] were in the game. We didn't have to question are these kids actually dealing with data? [Teachers: nope.] They appeared to be. Even when they talked abstractly, they were saying things like middle 50% of the data, so that tells you that this treatment helped people. This was about figuring out whether this treatment worked or not. Maybe that will help us frame it. Because we are looking for shifts beyond this. So there [in Minitool 1] they are just getting started, and trying to get in the game. They are in the game [in Minitool 2]. What about this issue of having to ground their arguments in the context? Were we seeing that?

Muriel: Yes.

Researcher: And we even saw when it didn't happen, one kid called another kid on it. When Valerie said "Why [did you put a cut point at] 550?"

Bruno: Yeah, you saw it in their choice of partitions. When one partition was the speed limit … [When someone asked] why did she picked that number, she said "That's the speed limit." So that's a context-based partition.

Researcher: That's right. So it indicates that that stuff happened pretty early in the sequence, those sort of goals. And you can see how they pay off down the track. In other words, if those hadn't been the focus earlier, this wouldn't have played out later.

(Summer 4, Day 2, Jun 2004)

While the teachers were quickly able to relate their findings to the big issues that we brought up (such as students being in the game of analyzing data and grounding their arguments in the problem situation) they did not, on their own, see these issues as examples of major shifts that took place in students' learning. Ten minutes later, Wesley asked:

Wesley: I'm still trying to wake up. Can you define shift for me? And I know that's a really basic question, but I'm having a trouble wrapping my head around. Because shifts in understanding- are they, you know-

Lisa: Progression.

Wesley: Progression. You know? They've looked at, they've seen a bunch of dots and now they are seeing something more. That's a shift, right?

(Summer 4, Day 2, Jun 2004)

Wesley's question indicated to us that we needed to talk about the shifts explicitly and we spent the following 20 minutes doing so. In effect, we built from the starting points that the teachers delineated in day 1, and proposed the three shifts on Figure 23. The teachers found this organization of shifts reasonable and used it effectively when they synthesized and reorganized the findings from their small group analyses. Several teachers made sense of the proposed shifts in terms of "progression of difficulty", "conceptual levels", and "Van Hiele categories but not so formalized". However, our press for keeping in mind what the above labels meant in terms of

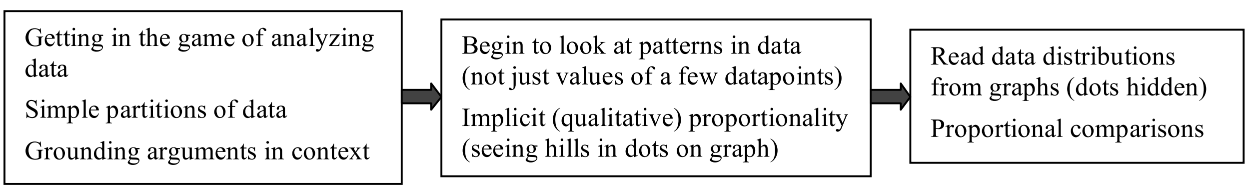

Figure 23. Initial shifts in students' reasoning.

specific forms of students' reasoning was only partially successful. We conjectured that in order to use this framing more productively, the teachers needed opportunities to explore what guidance it could provide in their instruction.

Proposing Ways to Support Students' Understanding

Two more activities from the last day of the summer workshop at the end of year 4 indicated the extent to which a view of classroom instruction as supporting a series of shifts in students' reasoning was problematic in the group. In the first of these activities, we asked the teachers to examine why some of the students did not understand their classmates' argument in the AIDS activity. The students compared data on T-cell counts of AIDS patients who enrolled in two treatments, traditional and experimental, in order to determine whether the experimental treatment was better and should be used in hospitals. While 186 patients who participated in the study received traditional treatment, only 46 participants volunteered to receive the experimental treatment. One of the student groups proposed the following graphic (Figure 24) in support of their argument that the new treatment was better. The students organized data into 4 equal groups and argued that while more than 75% of people in the new treatment had a T-cell count above 525, only about 15% of patients in the old treatment had a T-cell count above 525.

When addressing why some students had difficulties understanding others' 4-equal-groups explanations, the teachers initially focused on students' use of vocabulary. They pointed out that it is tempting to interpret "equal groups" as having to appear equal on a picture (that is, having equal width of the intervals). The group then discussed that the key in understanding the

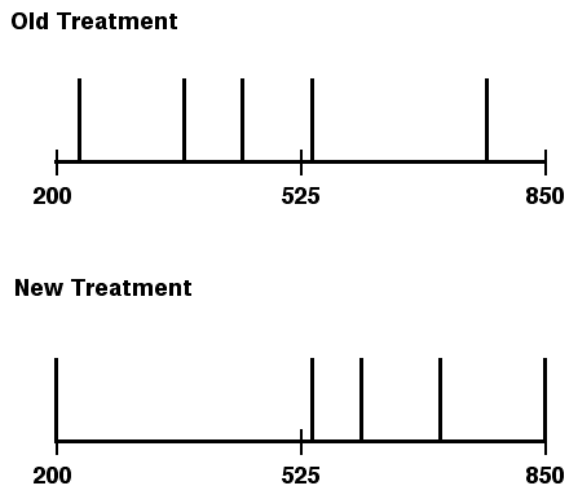

Figure 24: One Group's Report—AIDS. Inscription of data organized into 4 equal groups.

4-equal-groups representation is to see differences in *density* of the data in different intervals. We pointed out that density is not visible on the picture and that students needed to learn to read it from this representation. At least 4 teachers tried to understand what they needed to tell or directly show to students who struggled with making sense of this representation to help them "see" density better. For instance, Brian suggested putting numbers on the picture to indicate how many of the datapoints were contained in each interval, so that the students would see that these numbers were the same. Erin proposed putting additional numbers on the axis to help the students see that the intervals were not the same width. We repeatedly tried to explain that even when students knew that the numbers of the datapoints were the same and widths of the intervals were not, they still experienced difficulties in coordinating these quantities in ways that would be helpful in reading densities of the data.

These teachers' suggestions of ways to support students' understanding suggested to us that most of them adopted a perception-based view of learning (Simon et al., 2000; Tzur et al., 2001). We therefore explicitly discussed the distinction between claims that "students understand what they see" and "students see what they understand" with the teachers. After the discussion, one teacher indicated that he was still puzzled about students seeing only what they understood. Two teachers indicated that they found the discussion helpful and 2 teachers brought up examples from classroom videos that indicated that they considered adopting a conception-based perspective to be useful when thinking about supporting students' learning.

In the second activity that proved problematic, we asked the teachers to examine the role of the video teacher in supporting the shifts in the students' reasoning that the group previously identified. We conjectured that the teachers would find it reasonable to examine classroom videos for specific situations where the teacher's decisions reflected how she attempted to support a specific shift in the students' reasoning. The teachers' responses instead resembled those from the summer workshop at the end of year 3 in that they described general teaching strategies (e.g., the teacher included all students, asked questions) and did not address the details of classroom interactions. Before the end of small group analysis time, Muriel realized that she and the other teachers misinterpreted the activity and discussed this issue with her partner Naomi. At the beginning of the whole group discussion, when the teachers displayed posters created in small groups, Muriel pointed out that all of them had misinterpreted the activity and Naomi elaborated:

> Naomi: I think I thought, as most of you did, that we were supposed to look at the video and describe what we saw the role of the teacher [was]. What was she doing, or what was he doing. But Muriel had a question and apparently that's not what we were supposed to have done. So it was a little misunderstanding and we didn't have time to go back and look at the video and actually do what we were asked about.
>
> Researcher: What do you think the intent was?
>
> Naomi: Just to look at the teacher and see what they[54] were doing in order to facilitate the task in front of them.

[54] There were two teachers interacting with students in the edited video clips.

Researcher: That's interesting because that's what we meant by the role of the teacher, but clearly it means different things to you [Naomi: Yes], so that was our mistake. What would be an example?

Muriel: See, the things that are on the sheets are the things that I … felt like we went over last summer, and things that were taken as normal

Naomi: In this group [Naomi nodding]

Muriel: Right. I thought that we were [now] supposed to get into specifically when a student says … "most of the data are in that group" what do they mean by "most" and what specifically would you [as a teacher] do to bring up that point or clarify that point. Specifically.

Brian: Other than just general teacher strategies like [Researcher & Muriel: Yeah!] "Restate what you said," and "What else have you noticed"

Researcher: [jumps in] It's not-

Brian: [continues] How are you gonna nudge them so that they experience the shift.

Researcher: It's like how are you gonna achieve your mathematical agenda in these discussions with students.

Muriel: Yes.

(Summer 4, Day 3, Jun 2004)

Muriel, Naomi, Brian, and possibly other teachers, developed a sense of the intended goals of the activity. However, because this discussion came at the end of the last day of summer session, the group did not continue with the activity at the time.

During the summer workshop at the end of year 4, the teachers developed a solid grasp of different forms of students' statistical reasoning and of the goals of the statistics instructional sequence. However, the teachers did not start to seamlessly capitalize on these insights in analyzing the video teacher's instruction. We conjectured that if the teachers started to draw on these insights in analysis of work of the teacher, this would provide them with a support in developing ways of drawing on these resources when making instructional decisions in their classrooms. We also conjectured that reasoning about instruction on this level of detail would help the teachers develop a conception-based view of how students' learning can be effectively supported.

Initial Steps Towards Further Shift

In session 1 of year 5, we returned to the Teacher's Role Activity. However, this time we asked the teachers to examine classroom interactions by focusing on issues that became a topic of conversation in the design experiment classroom. The teachers found it sensible to examine what the teacher and the students talked about together, which mathematical topics were sustained in the discussions, and how and why the teacher made sure that these topics were discussed in the classroom. For example, the teachers noticed that the video teacher pushed for more elaboration of some solutions but not others and was selective about which mathematical ideas she rephrased. The teachers' participation in this activity suggested that they started developing imagery of building instruction on students' reasoning, yet continued to guide it with respect to a specific mathematical agenda.

Year 5: Performance Assessment

We conceived of year 5 as a performance assessment[55], the goal of which was to understand what the teachers had learned and the role that the professional development had played in supporting that learning. During sessions 2-6, the teachers reviewed and critiqued two sets of instructional units in statistics. One of these sets came from the NSF-funded textbook series that the district adopted in that year. The second set of units was designed by the teachers in the Washington Hill district professional teaching community[56] and was based on the statistics instructional sequence. During the summer workshop at the end of year 5, the teachers selected tasks from these units that they found the most suitable for their instructional needs and organized them into an instructional sequence. We continued to support the teachers as they participated in the professional development activities in year 5 but we had provided considerably less press for them to align with our professional development agenda.

In this section, I document how the teachers' participation in the activities where they used the resources that built on the statistics instructional sequence[57] provided further evidence

[55] Starting from session 2 in year 5.

[56] The readers will recall that the teachers in the Washington Hill district were part of the parallel professional development design experiment. They used tasks from the statistics instructional sequence in their classrooms for 4 years before producing a series of binders intended as a resource for their and their colleagues statistics instruction.

[57] The teachers' initial participation with (i.e., critique of and planning with) materials that they previously used in their schools rather than in the activities of the professional development proved to require additional support. For this reason, I do not draw on data from sessions 2-4, in which the teachers critiqued instructional units adopted by

that the forms of pedagogical reasoning that I have documented in this chapter were normative in the group throughout year 5. In addition, further evidence indicated that the group came to reason about students' learning in terms of the shifts in students' statistical reasoning. In doing so, the teachers drew both on the notes produced during summer of year 4 and on episodes from the video analyses. In proposing specific ways to support students' learning, some of the teachers' contributions could be characterized as drawing on a perception-based view of learning, while other could be characterized as drawing on a conception-based view.

Adopting a Student's Perspective

The teachers consistently examined instructional tasks from the perspective of a student when they critiqued instructional units that were based on the instructional sequence in statistics in sessions 5 and 6 of year 5. During these two sessions, the teachers critiqued various aspects of the instructional units 38 times. It is significant that these 38 topics were raised by 9 out of the 10 teachers who were present in the sessions. 23 of these critiques pertained to the mathematical purpose of the task and to the significance of the problem at hand. Muriel, Erin and Julie's contributions are representative of these comments. At this point, the teachers expected that a statistical problem must aim to answer a significant question and would not merely be a means to accomplish a specific teaching objective, such as having students draw certain graphs. They consistently pointed out the absence of a significant question as a major flow of the reviewed tasks.

Muriel: It [the activity] doesn't have any external goals- It's just compare and contrast solutions, so it's like, you know how we've talked about how you have to establish what is an adequate solution, an adequate argument, it's like this is trying to establish what's an adequate solution, but that's the only goal it has.

…

Erin: The next one [task] we had major issues with.

Julie: Yes … we couldn't figure out what the question was. Yeah, the question had to do with the graphs.

(Year Five, Session Six, Mar 2005)

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their school district. The specifics of this phenomenon are beyond the focus of this analysis and will be analyzed in subsequent work.

Seven more critiques related to what was measured and how when the data were generated to address the question at hand. For instance, the teachers were concerned that students would not know what was a reasonable heart rate or that an appropriate allowance for a 6th grader cannot be decided by collecting data on allowances of students in one classroom. In order to justify all the critiques discussed to this point, the teachers drew on a perspective of a student in a classroom who is expected to engage in the statistical activity.

On the remaining 8 occasions, the teachers pointed out that the resources provided for them in the description of the task were insufficient. Six of these contributions mentioned lack of examples of ways in which students might approach the task, one lack of assessment tools in the unit, and one unrealistic time demands of the task as described. The teachers were very persistent in requesting that useful instructional materials should include examples of students' work to facilitate their planning.

Additional evidence that it was normative in the group to view significance of the problem as well as data generation discussion as key elements of a statistics lesson came from the second day of the summer workshop at the end of year 5, when the group reviewed the notes on launch generated in session 5 of year 4 (see Attachment AA). The group spent less than 8 minutes reading through these notes and no comments or questions were raised during that time. The teachers treated the notes as self-evident and moved on to the next activity.

Lastly, the teachers highlighted aspects of the tasks and proposed changes that indicated that their goal was to facilitate the emergence of specific forms of students' reasoning in the classroom. For instance, the teachers pointed out that it was important that in the AIDS scenario, the natural cut point split data in two groups in a way that if the students used the additive argument (i.e., the *number* of people whose T-cell count was above 550 was higher in the traditional treatment group), they would arrive at a different solution than students who reasoned about the data proportionally (i.e., the *proportion* of people whose T-cell count was above 550 was higher in the experimental group). On a different occasion, to help the students interpret 4-equal-groups representations, Erin and Muriel proposed having students put sticky dots on the poster with 4-equal-groups with data hidden. They proposed this would help to highlight in the classroom how the dots had to be "bunched up" in the narrow intervals in order to fit in (see Figure 25).

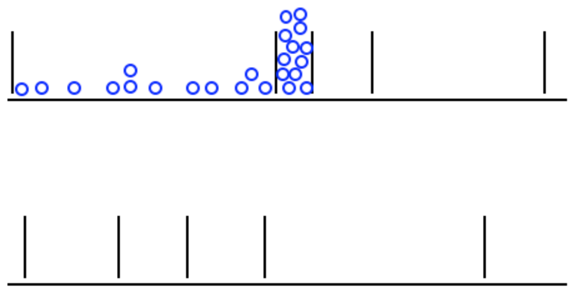


Figure 25. Four Equal Groups Representation. Data “bunched up” in the narrow intervals and “spread out” in the wide intervals.

During sessions 5, 6, and the summer workshop in year 5, the teachers proposed changes to datasets of different scenarios on 7 different occasions (e.g., using more skewed datasets or bimodal datasets to problematize solutions in which students averaged the high and low extreme data values to estimate where the “hill” was). Six of the 10 teachers present in the sessions contributed to proposing these changes. They justified each of their proposals in terms of the types of students’ reasoning, the emergence of which would be encouraged by the alternative proposals for the shape of the data.

Shifts in Students’ Reasoning as a Resource for Designing and Planning Classroom Instruction

In addition to adopting a student’s perspective when reviewing instructional units and planning for instruction, the teachers also came to reason about instruction in terms of major shifts in students’ reasoning. During the first day of the summer session at the end of year 5, we asked the teachers to outline the main instructional goals in different phases of teaching univariate data analysis. The purpose for this activity was to align the ideas within the group about what the major goals were, and then use these goals as the primary orientation in proposing adaptations to the instructional sequence. In this way, the resulting instructional sequence would address the teachers’ needs in their schools while simultaneously building on the learning of the professional teaching community.

In contrast to the summer workshop at the end of year 4, the teachers had a very strong grasp of the key issues that were critical within different phases of instruction during year 5. The following excerpt comes from the discussion of the goals of Minitool 1, and shows how all the teachers had a meaningful way to contribute to outlining the primary goals for instruction. This pattern of participation and the nature of the teachers' contributions are representative of the entire summer workshop in year 5. After spending 20 minutes compiling notes individually while reviewing printouts of different task scenarios, each teacher shared one item from their notes. As they shared, one of the researchers typed up the items while they were being projected on the screen in front of the room.

Naomi:	[One of the goals for the class is] actually seeing the bars as data.
Researcher 1:	And obviously, as we go around, people should feel free to comment, or elaborate, suggest rewording, we could do that as we go as well. So it sounds like seeing bars, actually seeing data as measures of something. Erin?
Erin:	To start comparing datasets, to get them into the game of analyzing data.
Researcher 1:	The game of analyzing the data. Lisa?
Lisa:	Um, getting them to discuss what they've thought about.
Researcher 1:	Norms of discussion?
Lisa:	[nodding] The discussion aspect of it.
Researcher 1:	Muriel?
Muriel:	... Starting to look at the shapes and where the "most" is, beginning to partition it.
Researcher 1:	Maybe [put down] beginning to organize data by partitioning. ...
Bruno:	Discovery of range by ordering the bars ... When you order the bars [by size and by color in Minitool 1], you could see the range from the shortest to the longest.
…	
Ben:	Seeing range beyond just subtract the smallest from the biggest.
Erin:	Right, what it *means*. What does it tell you when you get this number.
...	
Kate:	Connect data to the situation.
Researcher 1:	OK. Is there anything left Julie?

Julie: Well, I guess I have one thing that might be different. Giving multiple reasons. Like using multiple methods to analyze the data, not just one or two.

Researcher 2: That to me is part of norms of discussion, so maybe we can put it under there?

Researcher 1: Yeah. Is there anything that anybody else has that hasn't sort of come up in different words so far?

Ben: I don't know if it's same or not. Data and situation—like making decisions based on data.

(Summer 5, Day 1, Jun 2005)

Not only did the teachers re-create the main goals that were more elaborated than the notes of major shifts in students' reasoning from summer workshop in year 4, they also found it reasonable to use the statistics instructional sequence as the basis for their instructional materials. We did not expect that there would be a consensus in the group about this issue since the teachers also reviewed textbook units that were used in their schools and could propose to take those units as the primary starting point in developing the instructional materials for their classrooms. We therefore planned justifications for why approaching the design of the instructional unit by making adaptations to the statistics instructional sequence would be reasonable. Contrary to our expectations, no such justifications were required.

In addition to re-creating the major goals for different phases of the instructional sequence the teachers also actively drew on these goals when they made decisions about which activities to include and which not to include in their unit. In day 2 of the workshop, the teachers explained how different goals were most important to advance during different activities within the sequence. Ben's report is again representative of the teachers' participation.

Ben: We did it [looked at the main goals] by activity. Almost all of them [statistical activities] have that goal of learning in the situation. I don't know if I could call that the main goal... Kind of an overarching goal on all of them.

Kate: [Inaudible] piece of unifying goals.

Researcher: Would it make sense? So if kids aren't analyzing data … If it isn't about analyzing data in order to understand the situation, to get insight into the situation, then everything else is pretty hopeless. So if that doesn't happen then pretty much all of the other goals all the way through to Minitool 2 are pretty much [lost]. So that has to be the initial central focus. And obviously, that's gonna have implications for the data generation process which we move to in a while. ...

Ben: The next one is Driving to School. The main thing we saw there was context, or the data in the situation again. And this is before they've seen the Minitools. ... Because, a big part of the context in this one is—if you are in a hurry you want [to drive to school] the one way, [but] if it doesn't matter [to that you arrive before certain time] you might try the other [way]. So depending on the situation I might choose this one or the other one.

Researcher: There is that option. Something that is a real resource for you—when you want to bring out this importance of taking account of a situation or context.

Ben: Because you can really - either one is right, depending on the situation you are dealing with.

Researcher: And so that is a nice activity to have there. ...

Ben: And then that actually goes into Minitool 1 where you actually look at it [the same data in Minitool 1]. Which we thought the main goal then is seeing the bars as individual pieces of data. ... [I could tell my students] "Remember when we talked about it takes *this long* when you go this way? Well there it is, on Minitool." And, then just getting them familiar with what Minitools can do and some of that.

(Summer 5, Day 2, Jun 2005)

Performance Assessment Summary

The teachers' participation in the activities of year 5 indicated to us that anticipating the ways in which students might both become interested in analyzing data and reason during specific instructional activities was now relevant to teachers' planning, at least when they planned collectively in professional development sessions. In addition, the "shifts" in students' reasoning that the group derived in bottom up fashion by analyzing the observed activity of students in the design experiment classroom in summer of year 4 became resources with which the teachers reasoned as they planned for "main goals" to accomplish in different phases of the sequence.

CHAPTER VIII

CONCLUSIONS: BASIS FOR A DOMAIN-SPECIFIC PROFESSIONAL DEVELOPMENT THEORY

In this chapter I discus implications of the analyses I have presented for professional development design and research. In doing so, I formulate a revised trajectory for the learning of a professional teaching community. My overall goal is to contribute to the development of an empirically grounded professional development theory that is specific to the domain of middle-school statistics. This effort is consistent with the *design orientation* described in Chapter II. The reader will recall that in this design orientation, an important goal is to develop resources that guide the work of professional development facilitators and researchers in supporting the learning of professional teaching communities. The domain-specific professional development theory that I describe in this chapter is intended to inform decision-making so that the successive forms of reasoning and the means of supporting their emergence can be reproduced in other professional teaching communities. Because of its empirical grounding, this theory can be tested, revised, and adapted to contingencies of the new sites (P. Cobb, Confrey et al., 2003).

In formulating the professional development theory, I first clarify the end-points and the starting points of the revised conjectured learning trajectory. Second, I address two types of professional development situations in which the norms of general participation in the group evolved and summarize our learning about the means that supported renegotiation of these norms. Third, I specify two major developments in pedagogical reasoning that occurred in the teacher group during the years 3-5 of the design experiment, and the means by which these shifts were supported. In doing so, I bring to the fore the shifts in mathematical reasoning for teaching that were initiated by the first shift in pedagogical reasoning and facilitated the second shift in pedagogical reasoning. Against this background, I discuss the main modification to the original conjectured learning trajectory that proposes a way of building on teachers' current instructional practices rather than attempting to support teachers' focus on students' reasoning directly from the outset.

Professional Development Endpoints

The analysis of the actual learning of the professional teaching community indicates that teachers' development of instructional practices that place students' reasoning in the center of instructional planning and decision-making is a viable goal for professional development that occurs outside the teachers' classrooms. The teachers who participated in the study reported in this dissertation developed professional practices in which instructional planning that occurred in the professional development setting involved anticipating the forms of statistical reasoning that students might produce on specific instructional tasks. Our documentation of the teachers' classroom practices was not sufficient to establish whether or to what extent the teachers took their students' reasoning as a basis for instructional decisions in their classrooms and how they planned for instruction in their schools. We also did not study what support the teachers would need in order to start taking their students' reasoning as a basis for instructional planning in content areas other than statistics. Further design studies are needed that investigate both the relevant institutional and professional development support structures that would make professional practices of this kind a reality in the teachers' everyday instruction.

I have documented that a conception-based perspective did not become normative in the group when examining and supporting students' learning. However, the analysis indicates that several teachers adopted this perspective and found it relevant as they planned instruction. I therefore conjecture that with additional support, that would in particular involve teachers' experimentation in their classrooms followed by structured reflections in professional development sessions, the goal of supporting teachers to develop a conception-based perspective of learning might also be a viable goal.

Professional Development Starting Points

It is important to stress that the actual learning trajectory that I documented in this dissertation occurred after the teacher group had already satisfied the criteria proposed by Dean (2005) for a professional teaching community. These criteria included that the group had already developed a shared purpose, a shared technical repertoire, and productive norms of mutual engagement. As Dean clarifies, these criteria take the deprivatization of teachers' instructional practice "as necessary for the emergence of a professional teaching community and also acknowledge the situatedness of the professional teaching community within the institutional setting of the school and district" (p. 167). In other words, the teachers' practices were no longer treated as private and they constituted a major resource for our work as professional development facilitators throughout the last three years of the design experiment.

In addition, the teachers' knowledge of statistical ideas that are central in middle-school statistics was substantial at the beginning of year 3 and the teachers took over the primary responsibility for the further development of their statistical understandings. Although we continued to provide support for their statistical learning, it was in response to the teachers' questions and initiatives. We could focus on a pedagogical agenda for the teachers' learning because a number of crucial developments had already occurred, as documented by Dean.

On the Robustness of Norms of General Participation

I documented two types of situations in which the norms of general participation were disrupted. The first type of disruptions occurred temporarily whenever a group of newcomers joined the community. We anticipated these disruptions and supported the group in re-negotiating norms of general participation. Two points seem particularly relevant to share in this regard. First, it proved beneficial to initially engage the group in activities that did not draw heavily on the old-timers' history of participation in the community (e.g., the *Traffic School and Hobbies* activity) and provided the newcomers with opportunities to make substantial contributions to group discussions. Second, even when the newcomers' practices were deprivatized and they participated substantially in different types of professional development activities, it proved challenging for them to make significant contributions to the initial discussion of their colleagues' classroom videos. This indicates that the activities that involve the analysis of classroom videos required additional support for the newcomers. I conjecture that organizing the initial newcomers' video analyses as an activity in small, mixed groups of the newcomers and the old-timers (rather than a whole group discussion) will better support the newcomers' initial participation in these discussions.

The second disruption of the norms of general participation occurred when 5 principals and vice-principals from the teachers' schools joined the group for several hours during session 6 in year 3. The reader will recall that pairs formed by a teacher and a school leader interviewed a student, trying to understand what the students knew about fractions. The major goal that we and the teachers had established for the session was to help the school leaders problematize their views of mathematics teaching as a straightforward and routine endeavor. The school leaders' participation in the session was viewed as an important step in introducing changes in the institutional context of the teachers' schools and gaining access to needed resources. After the interviews, the group discussed the students' understandings of fractions. During this discussion, several teachers' contributions suggested that they constructed the activity as an evaluative situation in which the school leaders would interpret students' lack of understanding as a sign of poor instruction. The fact that we did not anticipate that the school leaders' presence would profoundly alter the ways in which the teachers participated in professional development activities limited the opportunities for the school leaders' learning. I suggest that when other researchers use similar activities, they should organize them in ways that would not jeopardize how outsiders to the group perceive the teachers' skills and professionalism. For instance, inviting students of teachers who were not members of the group could have minimized some of the teachers' concerns and bring the intended goals of the activity to the fore.

Major Shifts in Normative Reasoning

I described how during professional development activities in year 3, most of the teachers focused on what they needed to do during instruction (e.g., what questions to ask). In other words, the teachers focused on a *teaching trajectory* rather than on a learning trajectory in statistics. The first shift in the norms of pedagogical reasoning occurred at the beginning of year 4, when the teachers began to adopt a student's perspective during discussions of the initial launch phase of lessons. This shift was supported primarily by summer workshop activities at the end of year 3, during which we (a) capitalized on the teachers' concerns about how to motivate students, and (b) helped the teachers come to see students' motivation as situated, rather than as an inherent characteristic of students that was beyond their control. As a result, the teachers began adopt to a student's view as they attempted to envision which types of problem scenarios would likely capture students' interests. As they reflected on their own and other teachers' classroom experiences, their views of which aspects of instructional activities were critical in promoting students' interests in analyzing data evolved.

The teachers' initially viewed the launch as a non-statistical part of a lesson in which they attempted to entice students' engagement. The launch was then followed by a statistical activity in which the students produced graphs and calculations. Adopting a student's perspective made it possible for the teachers to investigate launch more closely and resulted in shifts in their views of a statistical activity. The teachers came to view discussions of both the significance of the problem at hand and of the process of generating data that would address the problem as important in developing students' interests in analyzing data. Launch discussions began to include issues that were both inherently statistical and facilitated development of students' interests in analyzing data. This constituted the major development in the normative ways of reasoning about statistics for teaching (i.e., part of mathematical reasoning), which was crucial to the group's subsequent learning.

While adopting a student's perspective to envision whether students would be interested in statistical activities, the teachers came to expect that not only launch, but also the activity of analyzing data and discussing the analyses should contribute to students' developing interests. The teachers' reflections on their own participation in statistical activities as students contributed to development of this insight. In particular, difficulties in understanding others' explanations became constituted in the group as an impediment to the listening student's development of an interest in analyzing data as well as to their learning from others' analyses. Several teachers subsequently reported that they started to mediate their students' contributions in ways that they hoped would make these contributions easier to understand for the listening students. These developments supported the second shift in the norms of pedagogical reasoning, as the teachers subsequently found it reasonable to analyze the diverse ways in which students analyzed data in the design experiment classroom and in this way reconstruct the trajectory of these students' statistical learning.

It was only when the teachers reviewed and critiqued instructional units in statistics and designed their own instructional unit in year 5 that they began to focus explicitly on the forms of statistical reasoning that they intended to support. This illustrates that even though the teachers initially struggled when they attempted to outline the major shifts in students' reasoning during summer workshop at the end of year 4, they subsequently drew on the resulting learning trajectory to guide to their planning decisions during year 5.

Revisions of the Conjectured Learning Trajectory

As I have documented, we initially attempted to support the teachers' focus on students' reasoning directly by using student work and classroom videos. However, these efforts were not successful even though we repeatedly modified and adjusted the professional development activities. This finding and the subsequent learning of the professional teaching community provide the grounds for proposing a modification to the initial conjectured learning trajectory. This modification seems especially relevant when working with groups of teachers whose primary instructional goal is to get through lessons smoothly while addressing specified objectives. Instead of focusing on students' reasoning directly, we found it more effective to first address the issues of students' motivation and help teachers' develop insights into how they can cultivate students' mathematical interests. The focus on students' motivation allowed us to build on teachers' current concerns and practices, while treating their disinterest in issues of students' reasoning as reasonable (Simon et al., 2000).

Even though we attempted to build on the teachers' current practices and strove to view their participation as reasonable from the outset of the professional development experiment, it was not initially obvious how we might capitalize on the teachers' current instructional practices to pursue our overriding goal of making students' reasoning central to instructional decision making. The modified learning trajectory therefore captures significant learning on part of the research team and can provide initial guidance to other researchers and professional development facilitators. Indeed, the comments that we have received on the preliminary reports of this work from other researchers working with teachers indicate that concerns about their students' lack of motivation are not unique to group of teachers with whom we collaborated.

This analysis is not intended to be a prescription for how to support the learning of other professional teaching communities. Nonetheless, the broad process of supporting teachers' learning that I have delineated is generalizable to other cases. The developed conceptual resources—specifically the revised trajectory for supporting the learning of professional teaching communities substantiated by both the actual learning trajectory of the teacher group and the associated explanations of why specific means of support promoted learning of this group—can inform adjustments in the professional development activities and tools to the needs of other sites in a conjecture-driven manner. In this way, the reported findings transcend the specific context in which they were developed, and generalize to new sites and situations (Steffe & Thompson, 2000). The figure 26 summarizes the key developments in the forms of reasoning that were constituted as normative in the teacher group.

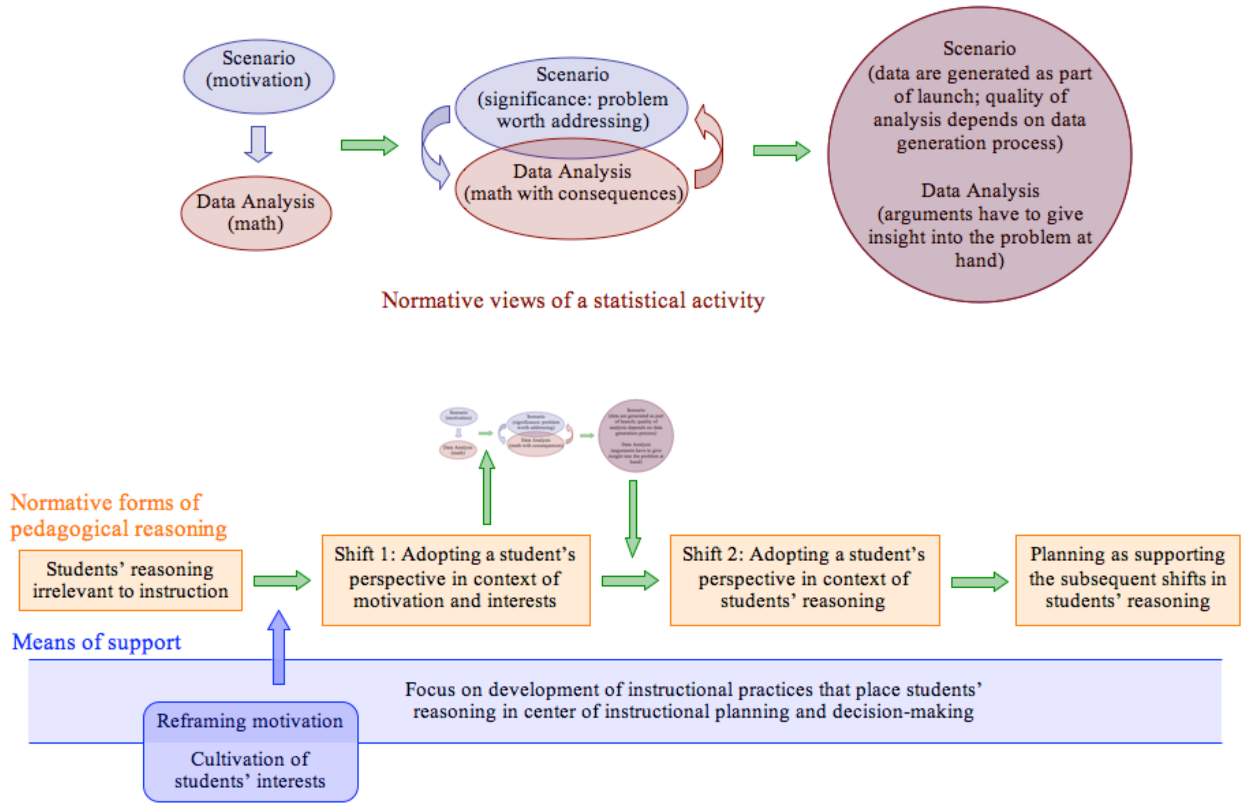


Figure 26. Realized Learning Trajectory

It might be argued that the ideas I have discussed while outlining the revised learning trajectory for a teacher group are pertinent only to the content domain of middle-school statistics. By being inherently about context and numbers (G. W. Cobb & Moore, 1997), statistics is well suited for a learning trajectory that leverages aspects of an effective launch in supporting the teachers' development of a more productive view of a statistical activity. I contend that other content domains, at least in middle-school mathematics, share this characteristic. In making this claim, I draw on Freudenthal's (1971; 1973) notion of *mathematizing*.

> In Freudenthal's view, students should be given the opportunity to reinvent mathematics by organizing or mathematizing either real world situations or mathematical relationships and processes that have substance for them. In developing this position, Freudenthal emphasized that the material students are to mathematize should be real for them. (P. Cobb et al., 2008)

In case of middle-school students, especially those who appear unmotivated during mathematics classes, it is reasonable to anticipate that mathematical relationships and processes might not initially be realistic in these students' view. In order to cultivate these students' mathematical interests, the process of mathematizing can provide a way of approaching everyday problems that the students view as significant from a mathematical perspective. For this reason, I would argue, a launch of any middle-school mathematical activity that is set in an everyday context not only can but should include discussions that are inherently mathematical. I therefore conjecture that the discussed contributions to the development of an empirically-grounded professional development theory that is specific to the domain of middle-school statistics can provide guidance for researchers working with middle-school mathematics teachers in other content domains.

Summary

In documenting the actual learning trajectory of a group of mathematics teachers I have illustrated how in order to support the learning of a professional teaching community effectively, we found it essential to both (a) capitalize on the teachers' current concerns and practices, and (b) support the subsequent sifts in the ways that the group reasoned about instruction towards the overall goal of using students' reasoning as an instructional resource (cf. Ball, 1993). We learned that focusing directly on students' reasoning from the outset in professional development might often be ineffective, especially when students' reasoning is not a part of the vision for high quality mathematics instruction promoted in the teachers' schools. In case of our study, students' reasoning initially appeared irrelevant to the issues for which school leaders held the teachers accountable.

In the process of repeatedly revising our initial approach that proved unviable, we learned to capitalize on the difficulties that teachers encounter as they attempt to entice students they perceive as unmotivated to engage in mathematical activity. The revised learning trajectory that I have outlined for a teacher group indicates how we used the teachers' concerns to support the learning of the group without abandoning the professional development agenda. Indeed, the initial focus on issues of students' motivation enabled us to support the teachers in reshaping their views of a high quality mathematics instruction as well as of the means by which it can be developed. As a result, the teachers developed professional practices in which students' reasoning was central to instructional planning and decision-making, and enacted these practices, at least in the resource-rich professional development setting. Subsequent design studies are needed that investigate both relevant institutional and professional development support structures that would make professional practices of this kind a reality in the teachers' everyday instruction.

APPENDIX A

PROFESSIONAL DEVELOPMENT PARTICIPATION

Teachers Participating in the Professional Development Collaboration by Year. The names in the table are pseudonyms. Parentheses signify a teacher who participated for a part of the given year:

	Year 1	*Year 2*	*Year 3*	*Year 4*	*Year 5*
Started in Year 1	Naomi Muriel Lisa Wesley Amy Marci Dot Rachel Jeremy	Naomi Muriel Lisa Wesley Amy Marci Dot Rachel Jeremy	Naomi Muriel Lisa Wesley Amy Marci	Naomi Muriel Lisa Wesley (Amy)	Naomi Muriel Lisa
Joined in Year 3			Brian Erin Kate Jane Hazel Helen	Brian Erin Kate	(Brian) Erin Kate
Joined in Year 4				Ben Bruno Dorothy Josh	Ben Bruno Dorothy (Josh)
Joined in Year 5					Julie (Ashley)

Numbers of Participating Teachers Working at Different Schools in Jackson Heights District. School names are pseudonyms:

	Year 1	*Year 2*	*Year 3*	*Year 4*	*Year 5*
Darlington	2	2	2	2	1
James Wood	2	2	3	3	2
Sinclair	2	2	2	2	2
Grindley	1	1			1
Hawthorn			2		2
Dogwood					2
Magnolia				2	1
Creston	2	2	3	3	1

Teachers' Rationales for Leaving the Group:

Year when left	***Teachers***	***Reason for leaving***	***When***
Year 2	Dot	Family time constraints	End of year
	Rachel	Teaching contract not renewed	End of year
	Jeremy	Graduate school/administrative position	End of year
Year 3	Marci	Graduate school/administrative position	End of year
	Jane	Moved out of the district	End of year
	Hazel	Childbirth	End of year
	Helen	Moved out of the district	End of year
Year 4	Amy	Retired	January
	Wesley	Moved out of the district	End of year
Year 5	Brian	Time conflict/department chair position	December
	Josh	Family health issues	November
	Ashley	Left teaching	January

APPENDIX B

"BENCHMARKS" FOR THE STATISTICS SEQUENCE

A list of "benchmarks" for the statistics instructional sequence co-created by the researchers and the teachers during Year Two, Session One (Dean 2005, p. 123)

Applet One

- Actually analyze data
- Developing data based arguments
- Justification with respect to question
- Using alternative methods to analyze data
- Partitioning data sets as a way to organize data in order to address question

Applet Two

- Focus on shape of data (trends and patterns in the data)
- Comparing data sets (unequal numbers)
- Majority (relative frequency)

APPENDIX C

FRACTION TASKS USED FOR STUDENT INTERVIEWS

Following pages were used in student interviews conducted during Year Two, Session Four, and during the session with principals in Year Three, Session Six.

Lauren, Michael, and Lawrence are training for a Walk-a-Thon from Durham to Raleigh to raise money for cancer research. On the first day of training Lauren walked ⅓ of the distance, Michael walked ½ of the distance, and Lawrence walked ¼ of the distance. Volunteers are being placed on training teams according to their first day results. Who walked the farthest distance? Who walked the shortest distance?

Lauren ⅓ of distance

Michael ½ of distance

Lawrence ¼ of distance

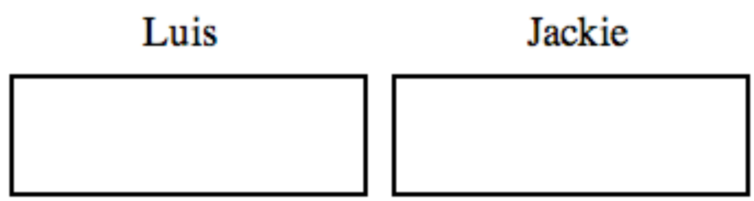
Dana, Marcus, and Juanita have joined a beginning gymnastics class. Today they are learning how to walk on the balance beam. Dana walked across ½ of the beam before losing her balancing, Marcus walked ⅝ of the beam, and Juanita walked ¾ of the beam. The gymnastics teacher wants to give certificates for 1st, 2nd, and 3rd place according to who walked the longest across the beam. Which place goes to each student?

Dana 1/2 of beam

Marcus 5/8 of beam

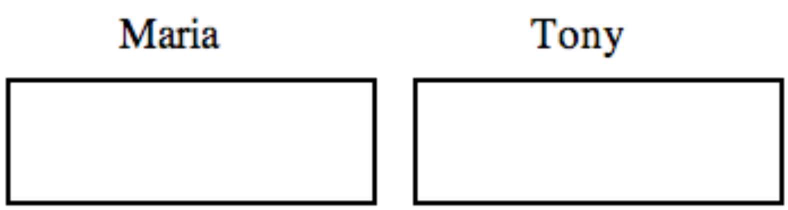
Juanita 3/4 of beam

Luis gets 1/2 of a candy bar and Jackie gets 2/3 of a bar. Who gets more? How much more?



Maria gets 2/3 of a candy bar and Tony gets 5/6 of a candy bar. Who gets more?

How much do they get altogether?



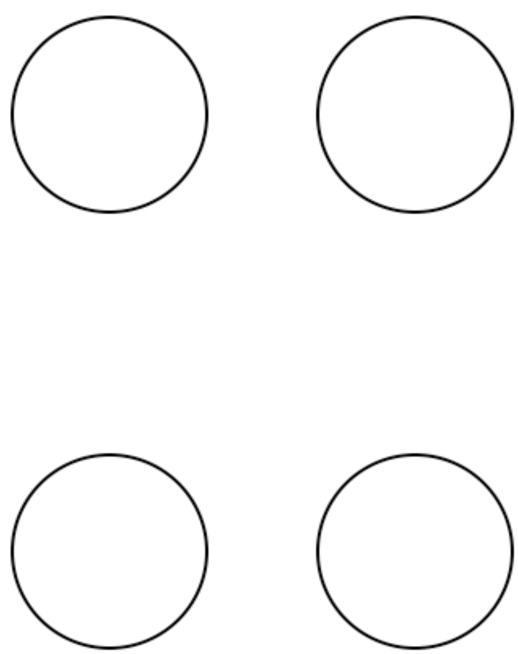
Anne gets 3/4 of a candy bar and Jeff gets 5/6 of a bar. Who gets more? How much more?

How much do they get altogether?

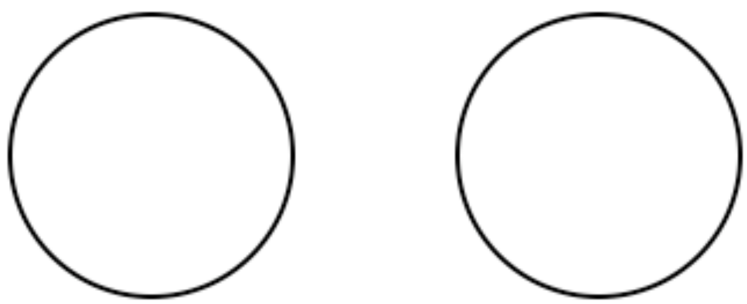
Anne   Jeff

One pizza is divided into 1/4s and another is divided into 1/5s. If I take a piece from each pizza, how much pizza do I eat?

One pizza is divided into 1/3s and another is divided into 1/5s. If I take two pieces from each pizza, how much do I eat?

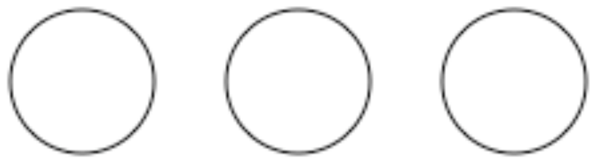


One pizza is divided into 1/5s and another is divided into 1/7s. Josh takes two pieces from the first pizza and Karen takes two pieces from the second pizza. Who eats more? How much more?

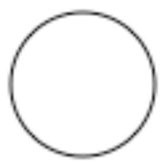


At the pizza house, several groups of people come for dinner. Show how the pizzas are distributed at each table.

1) 8 friends sit at the same table and order 3 pizzas to share equally. How much does each person get?

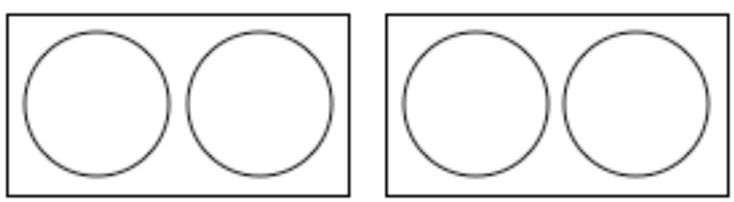


2) After they finish the first 3 pizzas, they order on more pizza to be shared equally. How much pizza does each person get to eat at this time?



3) How much did each person get to eat for dinner altogether?

4) Another group of 8 people sit at two separate tables. Four people sit at each table. Each group orders 2 pizzas. How much pizza does each person get to eat?



Place the following fractions in order from smallest to largest:

1/3 1/2 1/4

Place the following fractions in order from smallest to largest:

1/2 5/8 3/4

Which is larger? 2/3 or 3/5

3/4 or 5/6

Solve the following problems:

1/4 + 1/5=

2/3 + 2/5=

2/5 - 2/7=

APPENDIX D

A TRAJECTORY FOR SUPPORTING THE SCHOOL LEADERS

A conjectured trajectory for supporting the school leaders' development of deeper understandings of mathematics teaching (Dean, 2005, p. 161). Collective notes captured on a chart paper in professional development session, Year Two, Summer Session.

<u>Where are the school leaders right now?</u>
* Teaching is a routine, predictable process
* Focus on classroom management and covering content
* Describe current reform efforts in terms of generalities (e.g. using small group work, manipulatives, and real world problems)

<u>Where do we want school leaders to be?</u>
* Appreciate teachers' expertise
* Knowledge or importance of understanding students' mathematical reasoning
* Importance of collaboration to support focusing on student reasoning
* See value in doing math

<u>Intermediate steps with the Principals</u>
* Sensitize to what math teaching and learning is or should be
* Come to understand and value focus on issues of student reasoning
* Principals come to see value in making students' reasoning evident
* Communicate mathematical goals of the curriculum

APPENDIX E

"STRENGTHS AND WEAKNESSES" CHART PAPER

Strengths and Weaknesses Chart Paper Based on Results of "Dots Activity" (Dean, 2005, pp. 157-158). During the Dots Activity (Year Two, Summer Session), the teachers first identified and then ranked the major supports and the major hindrances to teaching mathematics in their schools and the district. The chart paper that captured results of the Dots Activity was used as a prompt for discussions in professional development Year Three, Session Three. Starred items were identified as the most important.

Strengths:

- District Math Leadership
- Statistics Project
- District Math Grant*
- Working with Colleagues*
- Curriculum
- District Math Specialists*
- Parent Organizations
- Teachers
- Administrators*

Weaknesses:

- State Accountability (Tests)
- Lack of Time*
- Lack of Instructional Time*
- Lack of Money*
- District Focus on Literacy
- Parent Attitude and Support
- Lack of Qualified Math Teachers
- Curriculum – Pacing, Flow
- Lack of Communication
    - Across Grade Levels
    - From State
    - Between Schools
- Students
- Teachers
- Administration

APPENDIX F

LIST OF SESSION ACTIVITIES

Year Three, Session Zero—August 8, 2002
(@ Ruth's house – lunch + afternoon)
Meeting new teachers
Old teachers explain objectives for the coming year

Year Three, Session One—September 18, 2002
Minitool 1: Braking distance (trucks vs. cars)
Teachers' perspective on teaching Braking Distance
Goals of the Project – Intro for new Teachers
Institutional Context: update on principals
Session Reflections

Year Three, Session Two—October 24, 2002
Video Analysis: Watermelon (Minitool 1, Marci, Naomi); launch
Minitool 1: Watermelon; data analysis discussion
Video Analysis: Watermelon (Minitool 1, Marci, Naomi); data analysis discussion
Minitool 1: Batteries; Wesley, Muriel, & Marci lead the activity
Session Reflections
Institutional Context: Brief update on meeting principals in January

Year Three, Session Three—November 21, 2002
Institutional Context: Dot Activity & update on principals in schools
Institutional Context: Joint planning
Institutional Context: Plans for meeting with principals
Student Work Analysis: Blood Drive (Minitool 1)

Year Three, Session Four—January 16, 2003
Video Analysis: Blood drive (Minitool 1, Muriel, Helen)
Institutional Context: Washington Hill visit summary
Meta: Process Model (launch, analysis, data analysis discussion)
Minitool 2: Speed Trap

Year Three, Session Five—February 19, 2003
Institutional Context: Teacher visit @ principal office discussion
Video Analysis: Speed Trap (Minitool 2, Brian & Wesley)
Student Work Analysis: Speed Trap (Minitool 2)
Meta: Process Model (launch, data analysis discussion)
Institutional Context: principals in session next time
Minitool 2u: Migraine (launch, Lisa leads)

Year Three, Session Six—March 17, 2003
Institutional Context: Preparing for session w/ principals (fraction tasks)

Minitool 2u: Anorexia (launch)
Institutional Context: Session w/ principals (fraction task)
Video Analysis: Migraine (Minitool 2u, Lisa & Kate)
Student Work Analysis: Migraine
Institutional Context: brief reflection on session w/ principals

Year Three, Summer Session—June 2003—Day 1
Intro: Developing a sustainable PTC (recruitment & time as resource: principals)
Motivation & Engagement Module 1

1. Why some students are motivated to learn math while others are not
2. Instructional techniques you would use to motivate students
3. Why students might become interested in analyzing data (Minitools)
4. Student Interview Analysis – obligations and valuations & interests

Motivation & Engagement Module 2

1. Possible supports for development of students' interests in DA
2. Video Analysis – focus of discussion, why of interest to students?
   - Braking Distance video

Year Three, Summer Session—June 2003—Day 2
Intro: Shift from Motivation language to looking at student interests (viewing math as worthy of students' engagement; situational and structural rationale)

3. Video Analysis – Batteries, Speed Trap, & AIDS
      - A: Activities and Tools
      - B: Classroom Norms and Discourse
      - C: The Teacher's Role
   - Investigate contribution of (A, B, C) to development of students' mathematical interests

Logistics: Inducting new people (which schools, shared responsibility within PTC)
Esther & Ruth – agenda for coming year (PTC seen as potential in developing local instructional leadership)

Year Three, Summer Session—June 2003—Day 3
Intro: Recap from yesterday, attention control and zoning out
Video Analysis Summary: Is this relevant to your classroom instruction
Student Statistical Reasoning Module

1. What students might learn as they analyze data using Minitools 1 & 2 (math & graphs as tools talk)
2. Video Analysis – Investigate nature of students' learning and means by which it was supported
      - A: Instructional Activities
      - B: Classroom Norms and Discourse
      - C: Students' Thinking
      - D: Role of the Teacher

Reflections

Year Four, Session Zero—July, 2003
(@ Ruth's house – lunch + afternoon; Data: Lori's notes)

Meeting new teachers
Old teachers explain objectives for the coming year

Year Four, Session One—October 28, 2003
Goals of the Project – Intro for new Teachers
Summer session retrospect
Motivation: Driver school/Hobbies activity
Minitool: How much TV; 2 contrasting data analysis discussions

Year Four, Session Two—November 13, 2003
Big Goals of the sequence: Situations of uncertainty, distribution, graphs as tools
Minitool 1: Driving to School
Minitool 1: Truck Stop
Reflections
Which activity to use in classrooms: Driving to School, Truck Stop, Speed Trap
Reflections

Year Four, Session Three—December 11, 2003
Minitools sequence long conversation (M1, M2, M3, tools)
Minitool 1u: Coats for Kids
Student Work Analysis: How much TV
Minitool 2: Fruits and Vegetables
Which activity to use in classrooms – discussion (Batteries, Coats for Kids)
Reflections

Year Four, Session Four—February 24, 2004
Minitools sequence long conversation (teachers raise topics)
Minitool 2: Sleeping Problems (measure not clarified in launch)
Reflection – what was missing in launch
Student Work Analysis: Batteries
Video Analysis: Batteries (Minitool 1, Wesley & Brian)
Minitool 1: Braking Distance, launch
Short debrief on video analysis

Year Four, Session Five—March 19, 2004
Minitool 2u: Reading Scores: 2 contrasting launches
Meta: Reflection on 2 contrasting launches experience and Process Model
Video Analysis: Braking Distance (Minitool 1, Bruno & Josh; listening students)
Minitool 2: Cholesterol
Reflections

Year Four, Session Six—May 14, 2004
Sequence overview – development from M1 & M2 to M3
Minitool 3: Bone Mineral Density: stacked data,
Institutional Context: New textbook intro next year – Esther, teacher leadership, stats unit development
Video Analysis: Cholesterol (Minitool 2, Kate & Lisa)
Minitool 2u: AIDS

Year Four, Summer Session—June, 2004—Day 1
Minitool 3: Salary & Gender
Reflection
Intro: Overview of learning trajectory vs. teaching trajectory
"Reconstruction of the Sequence" Module

1. Initial Thoughts: Overall goals of the sequence
2. Instructional endpoints
3. Instructional starting points
4. Video Analysis—Braking Distance & Batteries
    - First shifts – Minitool 1

Day 1 reflections

Year Four, Summer Session—June, 2004—Day 2
Reflection on the restructured notes from Day 1
Mean sequence—Jose Luis

5. Video Analysis—Speed Trap, Cholesterol, AIDS
    - Shifts – Minitool 2
6. Video Analysis—CO2 and speed
    - Shifts – pre-Minitool 3
7. Video Analysis—Salary and Gender
    - Final shifts – Minitool 3

Year Four, Summer Session—June, 2004—Day 3
Reflection on collective notes

8. Video Analysis—Cholesterol, AIDS
    - Differences in students' reasoning: Why some students have difficulties to understand others?

Logistics (new textbook implementation and PTC)

9. The proactive role of the teacher: [Misunderstanding]

Planning Year 5
Reflections

Year Five, Session One—September 2, 2004
Goals of the Project – Intro for new Teachers
Summer session reconstruction
Supporting the newcomers
Overview of available resources from past sessions [plans for website notes]
Video Analysis—Role of the teacher (#9 from summer 4)

Year Five, Session Two—November 2, 2004
Textbook unit 6 review—Activities 1-3 (Researchers-led)
Textbook unit 6—Activity 3 in Minitools

Year Five, Session Three—November 30, 2004
Textbook unit 6—Activities 4-5 (Researchers-led)
Textbook unit 7— Activities 1-2, some in Minitools (Researchers-led)

Year Five, Session Four—January 6, 2005
Textbook unit 7— Activities 3-4 (Researchers-led)
Textbook unit 8— Activities 1-3 (Teacher-led: Erin & Kate; Julie & Muriel; Bruno & Ben)

Year Five, Session Five—February 17, 2005
Textbook unit 8— Activity 4 (Teacher-led: Naomi & Dorothy)
Binder 6—Bare number problems (Researchers-led)
Binder 6—Minitool 1 problems (Teacher-led)
Scenario 4: Kate, Erin, Bruno
Scenario 5: Lisa, Julie, Naomi
Scenario 6: Ben, Pat, Dorothy
Binder 6—Minitool 2 problems, equal N's (Teacher-led)
Scenario 7: Julie, Dorothy, Lisa
Scenario 8: Ben, Erin, Kate
Scenario 9: Bruno, Naomi

Year Five, Session Six—March 24, 2005
Julie & Erin report on teaching Batteries task in Julie's class
Binder 7 (Teacher-led)
Bare number problems: Kate, Lisa
Minitool 1 problems: Researchers
Minitool 2 problems, unequal N's: Erin, Julie
Minitool 2 problems, equal N's: Ben, Bruno

Year Five, Summer Session—June, 2005—Day 1
Main goals of the statistics sequence
Binder 8 (Teacher-led)
Minitool 1: Kate, Muriel, Ben
Minitool 2 equal N's: Naomi, Erin, Lisa
Minitool 2 unequal N's: Julie, Bruno, Pat
Selecting tasks for new unit/adapted sequence

Year Five, Summer Session—June, 2005—Day 2
Relating main goals and activities (which goals are primary in which activities)
Clarification of launch/data generation process
Mean subsequence (Bakker (2004) dissertation chapter)
Identifying needed adaptations (State Mathematics Standards and tests)

Year Five, Summer Session—June, 2005—Day 3
Mean subsequence (choice and sequencing of activities)
Designing needed adaptations
New structure of Binder (resources for teachers in the unit)

APPENDIX G

EXCERPTS FROM STUDENTS' INTERVIEWS

Students' understanding of their obligations

Algebra class

All the students described algebra class as involving obligations in doing individual work and following certain procedures to solve problems.

I (interviewer): What about algebra class?
S: You need to be quiet and do your homework. (Sally 10-12)

M: You do your work and you show everything.
I: What do you have to do?
M: You have to write it all out. Like she does. Even the taking the number away, like if its two. It's like we know it already, but we still have to show the right steps. (Mark 12-16)

S: She says this is how you do it and you do it. (Brian and Sean 10-1)

Discussions in the algebra class involved the teacher demonstrating how to solve particular problems:

S: She [algebra teacher] says it is a discussion, but it's really her talking. We listen to her and copy the stuff down. Sometimes I don't really know what I'm copying down. I just copy it down to have it. (Stacy 12-17)

I: Tell me about the discussions in there [algebra class].
M: We don't really discuss anything because she tells us how to do it. If I missed something I ask somebody. (Mike 12-16)

V: You don't talk about it [algebra] with other people. You just ask her [algebra teacher] how to do it when she is back at her desk. Someone else can show you, like if you are absent, but you don't know if that's really the way she talked about. Sometimes people have told me the wrong things. (Valerie and Kate 10-23)

The following comments, in response to the question of describing the algebra class, are representative:

K: It is just a class [algebra class]. Most classes teach then they give you class work then homework. She [algebra teacher] goes over the homework. Then she goes over new stuff. Then we start on homework. And then it is time to go. (Kate 8-26)

B: We check our homework from last night from the board. She [algebra teacher] talks about the new problems then we try to do what she did with the problems. We can ask her questions if we don't remember how to do it. (Brad 9-9)

Two of the students expressed some frustration that the algebra teacher did not present explanations for the procedures that she presented in class. These students' comments are shown below:

J: When you ask her to work it out, she works it out, but she doesn't tell you how. (Janet 12-16)

K: She points to it on the board and tells you the steps to do, but she doesn't explain you why she's doing those things. (Valerie and Kate 10-14)

All eleven students indicated that the emphasis in the algebra class was on producing correct answers. The following responses are representative in this regard:

I: What have you learned in the algebra class?
B: It's a math class that's all we do, math, just straight problems. I guess I learned about … like when you have x's on both sides you want to have them on the same side to do the equation. (Brad 9-1)

I: What is something that you learned in Mrs. W's [algebra] class?
K: I guess you can just subtract a variable from another variable if it is on a different side of the equal sign. (Kate 8-26)

Statistics class

Students described obligations such as adequately explaining their own analyses or asking questions in order to understand other students' analyses.

S: You have to do a good job explaining how you looked at the problem. That's important since you didn't talk with everybody else when you were looking at the graph. (Stacy 9-23)

M: You talk about your way, or you add something to someone else's way. You can't just say that you agree or you disagree. Ms. M [the statistics teacher] makes you explain it. You have to ask questions about things that you don't understand.
I: What do you mean?
M: If you, um, don't understand why someone did something you have to ask them about it. You can't just say, oh yeah, that's okay, what you did. (Megan 9-1)

Five of the students noted whole-class discussions also involved explaining one's reasoning in how they analyzed the data. In these cases, students suggested that these explanations were important to other students in the class.

V: I knew what they [other students in statistics] did [in their analyses of the data] so I didn't want them to tell me what they were doing, but what were they thinking, yeah, what was your intention. (Valerie 10-6)

I: It sounds like what you're saying is when you talk about what and why you did it. It is important for the adults to know, but is it important for the other people in the class to know?
B: Yeah, it might change their opinions.
S: If you're just saying what you did then they are just comparing the different ways but not how you came about it. (Bryan and Sean 9-30).

I: What do you mean, what you ended up with?
S: You can't just talk about your conclusion because that doesn't let anybody know why you did things.
I: Is that important?
S: If you don't talk about what you were thinking about then we don't know if it all is okay…we can't figure out if it is a good point. (Sally 8-27)

Concerning the statistics class, students described obligations that involved analyzing and thinking. The students' comments point out obligations that relate to making decisions about what is relevant in the data. As Sean put it, "You have to analyze it and there is not just one right way of doing it."

S: It [statistics class] is not like science or something. You are not trying to find the right answer. You are trying to find the best answer. (Sean 12-15)

B: I like how we talk about all the different ways. There is usually one way that I like the best. Sometimes it is something that someone else found.
I: Is there one right answer:
B: No, but I think there is an answer that makes the most sense when someone explains it. (Bryan 9-8)

B: You have to work on the computers to find something in the data.
I: What do you mean?
B: You have to use the tool to look for stuff in the graphs.
I: That's what you have to do to do well in there?
B: Yeah, you have to look for what's consistent and then talk about it. You can use the tool to explain it. That helps me a lot. (Brad 9-24)

I: What is some advice you would give to a new student?
S: You should study the graphs so you have something to say about them.
I: What do you mean?
S: You look for like what you think stands out in the graphs. Then you sort of use that. It's kinda easier to use the tool to explain what you saw, you know, when you tell everyone your conclusion. (Sally 10-24)

Students' responses to the question of what they had learned in the statistics class indicated that in analyzing data, they gained insights into the phenomenon under investigation

M: You don't really learn things like you do in other classes. Like in our algebra class, we learn things like the process or how to do things step by step. In the Vandy class [statistics class] we learn about the world. Or what's happening. (Mike 12-15)

I: What have you learned so far?
V: I've learned a lot about the different subjects that we talk about. You all find out about it and give us data. Like the salary stuff for men and women. I knew it was going to be different, but it's like I can prove something now and that's different than just thinking it's different.
I: Is that important, to prove it?
V: I think it just gives more to what you say. Like, plus I didn't know about how they were different so that makes me want to learn why it's so different. (Valerie 10-23)

I: What did you learn [in the statistics class]?
M: I learned how to work the graphs. I really did enjoy that. It is different from what we normally do in math class.
I: What do you mean work the graphs?
M: I mean analyze the graphs, compare the data, look at the medians to compare.
I: Let's say that I don't know anything about analyzing data. What do you think is important to tell me, here is what you need to know. What would you say?
M: You should look at the median and that you should compare the two.
I: Why should I look at the median?
M: Because that is the highest concentration of data.
I: If I am looking at the median, what does that tell me?

M: If you look at the median it is easier to make judgments about the graph.
I: Is the median good to use on all graphs?
M: No, but on some it is…like the speed reading graph. That wouldn't really make sense … the scores were scattered all over. (Mike 12-16)

M: Now we know the terms like mean, median, range, and when you would want to use those terms.
I: You said you know what average is. How is it different from before? Average is something you talked about before the [statistics] class in other classes?
M: I learned when to use it to describe the data.
I: When should you use it?
M: Most of the time I don't use the average. I like using the range. I use the range when the points are spread out. If the points are around in a really small area you probably want to use the median since that would be a better way to let someone know about the [data] points. (Megan 12-16)

Students' Assessment of Their Own and Other Students' Competence

Algebra class

All four of the students who were succeeding in the algebra class had indicated that they did not see the value of the activities in which they were engaging in algebra, which seems to be in contrast with their view of the statistics class.

K: I'm not doing so good [algebra]. It's like my grades have dropped it's like she takes every little detail of every little problem. You are like, you leave out the x by accident and it's like wrong, wrong, wrong. (Kate, Valerie, and Sally 10-28)

K: I am pretty good in there [statistics] since I understand what we're doing. Sometimes it's hard for me to think about the problem at the end of the day, but I usually get into it when we start talking about it. (Kate, Valerie, and Sally 10-28)

M: I don't think she [algebra teacher] thinks I'm doing the best I can [in algebra]. I make too many mistakes on the quizzes. They're not that hard, I just forget about stuff. (Mike 12-15)

M: I think I'm doing okay [in statistics]. Brad and I talked about our way yesterday and a lot of people were talking about it. It was a good way. (Mike 12-15)

J: It [algebra class] drives me crazy. I don't understand anything. I feel okay about talking, but it's just that everybody around me knows how to work a problem. It's frustrating. I don't understand. I don't have any problems asking questions, but after a while you're just going how many questions have I asked. (Janet 12-16)

J: Yes, because we're special [statistics]. Because I know what I'm doing. We had these discussions. (Janet 12-16)

M: I understand it, but I am making some careless mistakes [algebra]. I have a C or something. I need to do better. Mrs. W already talked to me about how I should be doing better. (Megan 12-16)

M: Yeah, I think I'm a good student [in statistics]. I come up with some good ideas. (Megan 12-16)

When asked to indicate who was succeeding in the algebra class, students indicated specific students and most identified one student.

I: Who is a good student in algebra class?
K: Tyler, he is the only one who answers the questions right in our class. I think only one time he got a question wrong. He put a negative sign when it should have been positive. (Kate and Valerie 10-23)

When asked to summarize how to be a good student in the algebra class, the following response is representative:

I: What do you have to do to be a good student in algebra?
M: Get the right answers. (Mike 12-16)

Statistics class

All but three of the students said that they viewed all their peers to be succeeding in the design experiment class.

I: Do you feel like a good student in the [statistics] class?
K: No, I just feel regular. Equal.
I: You feel equal, the same? Doesn't anybody stand out?
K: Yeah, I think we are all equal. (Kate and Valerie 10-23)

I: Did you feel smart in the statistics class?
V: Yeah, everybody felt smart. You're like, "hey, look what we did." We talked about global warming, we analyzed these graphs. (Kate and Valerie 10-23)

I: Who is smart in the statistics class?
S: I think we're all smart. We talk about things so everyone knows what we're talking about. We kind of go at the same speed.
I: What about the algebra class?
S: Emily. (Sean 12-15)

I: Who is smart in the statistics class?
J: I can't think of one person. We all do a good job because she explains what we're supposed to do. We can talk to our partner about it. (Janet 12-16)

When asked how to be a good student in the statistics class, the students gave the responses as the following:

I: How can you tell a good student in the Vandy class.
M: Someone who does their work, has good ideas.
I: What do you mean, does their work?
M: You look at the graphs and decide what they tell you. You don't just come up with the answer. You work with the graphs and look at them. That's why you ask questions when most people are explaining. Some people can really tell you…so you don't have to ask too many questions, but most of the time you do. (Sally and Mike 10-24)

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