475.101 / 102 / 108 Semester 1, 2000
Assignment 3 Solutions

Question 1.

(a) **(i)** $\text{pr}(X \le 19) = 0.0228$

(ii) $\text{pr}(X < 19) = \text{pr}(X \le 19) = 0.0228$

(iii) $\text{pr}(X > 21) = 1 - \text{pr}(X \le 21) = 1 - 0.1587 = 0.8413$

(iv) $\text{pr}(24 \le X \le 27) = \text{pr}(X \le 27) - \text{pr}(X \le 24) = 0.9772 - 0.6915 = 0.2857$

(b) **(i)** The least amount of soft serve ice cream that is needed so that the driver can satisfy demand on 90% of afternoons is 10.24 litres

(ii) The upper quartile for ice cream sales = 9.46 litres.
The lower quartile for ice cream sales = 7.74 litres.
The interquartile range for the ice cream sales = 9.46 – 7.74 = 1.72 litres.

(c) **(i)** $\text{pr}(X > 6) = 1 - \text{pr}(X \le 6) = 1 - 0.8495 = 0.1505$

(ii) $\text{pr}(3.7 \le X \le 5.6) = \text{pr}(X \le 5.6) - \text{pr}(X \le 3.7)$
$= 0.7173 - 0.0538 = 0.6635$

(iii) $\text{pr}(X \le x) = 0.6$ when $x = 5.32$

$X \sim \text{Normal}(\mu = 5.1, \sigma = 0.87)$

x	$\text{pr}(X \le x)$
6	0.8495
3.7	0.0538
5.6	0.7173

$\text{pr}(X \le x)$	x
0.6	5.3204

(d) **(i)** $Y = 3X - 3W = 3 \times X + (-3) \times W \Rightarrow$

$E[Y] = E[3X - 3W] = 3 \times E[X] + (-3) \times E[W] = 3 \times (-3) + (-3) \times 5 = -24$ and

$$\text{sd}[Y] = \text{sd}[3X - 3W] = \sqrt{3^2 \times sd[X]^2 + (-3)^2 \times sd[W]^2}$$
$$= \sqrt{3^2 \times 5^2 + (-3)^2 \times 3^2}$$
$$= \sqrt{225 + 81} = \sqrt{306} \approx 17.5$$

(ii) We cannot say anything about the shape of the distribution of Y as we don't know about the shape of the distributions of X or W.

Question 2.

(a) The z-score for the mothers mark is $\frac{63 - 50.4}{13.5} = 0.933$.

The z-score for the daughters mark is $\frac{72 - 58.0}{15.3} = 0.915$.

The mother and daughter had similar rankings for their years (with the mother being slightly higher). Their marks are reasonably similar.

(b) The competition in the mothers year was likely to have been stronger as the 41% of students who sat were probably the more able students. Taking this into account, the mothers mark appears to have been a better result.

Question 3.

(a) $T = 9 \times M$ is assuming that all nine males have <u>exactly</u> the same weight.

Let M_i be the weight of the i^{th} male. $M_i \sim \text{Normal}(76, 10)$ for each $i = 1,...,9$

$M_{Tot} = M_1 + ... + M_9$

$E[M_{Tot}] = E[M_1] + ... + E[M_9] = 9E[M_i] = 684$

$\text{sd}[M_{Tot}] = \sqrt{sd[M_1]^2 + ... + sd[M_9]^2} = \sqrt{9}\,\text{sd}[M_i] \approx 30$

As the M_i are all Normally distributed, so is M_{Tot}. So $M_{Tot} \sim \text{Normal}(684, 30)$.

(b)

Number of Males	mean for MTOT	std dev for MTOT	pr($X \le 800$)	pr($X > 800$)
9	684	30.00	0.99994	0.00006
10	760	31.62	0.89705	0.10295
11	836	33.17	0.13886	0.86114
12	912	34.64	0.00061	0.99939

(i) The probability of the weight of 9 males exceeding the 800 kg weight limit of the elevator is 0.00006.

(ii) For 10 males, the probability of the total weight exceeding the 800 kg is 0.103.
For 11 males, the probability of the total weight exceeding the 800 kg is 0.861.
For 12 males, the probability of the total weight exceeding the 800 kg is 0.999.

(iii) The maximum number of males in a group such that there is less than 5% chance of the of the elevator being overloaded would be 9.

(iv) The maximum number of males in a group such that there is less than 1% chance of the of the elevator being overloaded would be 9.

(c)

Number of Females	mean for FTOT	std dev for FTOT	pr($X \le 800$)	pr($X > 800$)
9	612	27.00	1.00000	0.00000
10	680	28.46	0.99999	0.00001
11	748	29.85	0.95925	0.04075
12	816	31.18	0.30391	0.69609

(iii) The maximum number of females in a group such that there is less than 5% chance of the of the elevator being overloaded would be 11.

(iv) The maximum number of females in a group such that there is less than 1% chance of the of the elevator being overloaded would be 10.

(d) Based on the above calculations, it would appear to be much safer to limit the number of people on the elevator to 9 people rather than 11.

Notes: - In reality, this is a much more complex issue which involves other factors.

- These calculations have been based on the assumptions that groups of males (or females) were random and thus their weights independent. Is this a reasonable assumption?

(e) Let M_i be the weight of the i^{th} male. M_i ~ Normal(76, 10) for each i = 1,...,6

$M_{Tot} = M_1 + ... + M_6$ $\quad M_{Tot}$ ~ Normal(6×76 = 456, $\sqrt{6}\times 10 \approx 24.49$).

Let F_i be the weight of the i^{th} male. F_i ~ Normal(68,9) for each i = 1,...,7

$F_{Tot} = F_1 + ... + F_7$ $\quad F_{Tot}$ ~ Normal(7×68 = 476, $\sqrt{7}\times 9 \approx 23.81$).

We want to know pr($M_{Tot} > F_{Tot}$) = pr($M_{Tot} - F_{Tot} > 0$) = pr($D > 0$)

where $D = M_{Tot} - F_{Tot}$ ~ Normal(456 – 476 = -20, $\sqrt{24.495^2 + 23.812^2} \approx 34.16$).

From computer, pr($D > 0$) = 1 – pr($D < 0$) = 1 – 0.7209 = 0.2791.

A randomly selected group of 6 males will weigh more than a randomly selected group of 7 females 28% of the time.

Question 4.

(a) Let X be the amount of coffee in a packet. X ~ Normal(μ = 305 gm, σ = 2.3 gm)

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Normal with mean = 305.000 and standard deviation = 2.30000

        x      P( X <= x)
  300.0000        0.0149
  310.0000        0.9851
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pr(300 ≤ X ≤ 310) = pr(X ≤ 310) – pr(X ≤ 300) = 0.9851 – 0.0149 = 0.9702.

The managers requirement will be exceeded if the machines specifications are correct.

(b) <u>**Stem-and-leaf plot of coffee weights**</u>

```
300 |  1
301 |  1  6  7
302 |  4  6  7  8
303 |  0  2  6  9
304 |  1  1  2  3  5
305 |  1  4  4  6
306 |  1  5  6  6  9
307 |  4  7
308 |  8
309 |  6
```

Units: 300 | 1 = 300.1 grams.

(c) **(i)** The sample of weights does not have an exactly Normal distribution. The plot is not perfectly symmetric and the tails seem to short.

(ii) There are no features of the stem-and-leaf plot that suggest major departures from the Normal distribution. The data is roughly symmetric with one central mode.

(iii) It is <u>plausible</u> that the sample data could have come from a Normal distribution.

(d) Let X_i = be the weight of the ith packet of coffee. X_i ~ Normal (μ = 305 gm, σ = 2.3 gm)

Let $\overline{X}$ be the mean weight from a sample of 30 packets of coffee.

As X_i is Normal, $\overline{X}$ is also Normal.

$$\overline{X} = \frac{X_1 + X_2 + ... + X_{30}}{30} = \frac{1}{30}\sum_{i=1}^{30} X_i$$

E[$\overline{X}$] = μ_X =305, and sd[$\overline{X}$] = $\frac{\sigma_X}{\sqrt{n}} = \frac{2.3}{\sqrt{30}}$ = 0.4199.

$\overline{X}$ ~ Normal (μ = 305 gm, σ = 0.4199 gm)

The Central Limit Theorem was not needed to be able to answer this since the distribution of the weight of a single packet of coffee was Normal, then the distribution of the mean weights of a sample of packets of coffee must also be Normal.

(e)

```
Normal with mean = 305.000 and standard deviation = 0.419900

        x      P( X <= x)
  304.6000        0.1704
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pr($\overline{X}$ < 304.6) = 0.1704. It would not be very unusual to get a sample of 30 packets with a mean of 304.6 grams or less. Mean weights lower than this can be expected in 17% of samples - roughly 1 time in 6 – so it is not unusual when it occurs.

(f) Assuming the specifications for the machine are correct, then p, the true proportion of packets of coffee outside the 300 – 310 gram weight range, is 1 – 0.9702 = 0.0298.

$\hat{P}$ is approximately Normal ($\mu_{\hat{P}} = p = 0.0298$, $\sigma_{\hat{P}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.0298\times 0.9702}{500}}$ = 0.0076)

(g)

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Normal with mean = 0.0298000 and standard deviation = 0.00760420

        x      P( X <= x)
    0.0220        0.1525
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pr($\hat{P} < \frac{11}{500} = 0.022$) = 0.1525 so pr($\hat{P} > \frac{11}{500} = 0.022$) = 1 – 0.1525 = 0.8475.

It is not unusual to get a sample of 500 packets of coffee with 11 falling outside the 300 – 310 gram weight range. Roughly 85% of samples will have 11 or more packets outside the desired weight range. (And roughly 15% of samples will have 11 or fewer packets outside the desired weight range.)

(h) From the stem-and-leaf plot we could see no problems with the data coming from a Normal distribution. The sample mean of 304.6 from the sample of 30 packets of coffee was not particularly unusual assuming the specifications were correct. Similarly, the proportion of packets with weights outside the 300–310 gm range from the sample of 500 packets was not particularly unusual assuming the specifications were correct. Overall, there is no reason to doubt the specifications were correct.