Prove by induction: $\sum_{k=1}^{n} k(k+1) = \frac{1}{3} n(n+1)(n+2)$ $(*)$

for $n=1, 2, 3, \ldots$

Base case: prove $(*)$ for $n=1$.

$$\text{LHS} = \sum_{k=1}^{1} k(k+1) = 1 * 2 = 2.$$

$$\text{RHS} = \frac{1}{3} * 1 * 2 * 3 = 2 = \text{LHS}.$$

So $(*)$ is proved for base case $n=1$.

Prove by induction: $\sum_{k=1}^{n} k(k+1) = \frac{1}{3} n(n+1)(n+2)$ for $n = 1, 2, 3, \ldots$

General case: assume ⊛ is true for $n = x$ for some x.

So we may assume: $\sum_{k=1}^{x} k(k+1) = \frac{1}{3} x(x+1)(x+2)$. ⓐ

R.T.P. that ⊛ is also true for $n = x+1$.

So R.T.P. $\sum_{k=1}^{x+1} k(k+1) = \frac{1}{3}(x+1)(x+2)(x+3)$ (**)

$$
\begin{aligned}
\text{LHS of } (**) &= \sum_{k=1}^{x+1} k(k+1) \qquad\qquad k = x+1 \downarrow \\
&= \sum_{k=1}^{x} k(k+1) + (x+1)(x+2) \\
&= \frac{1}{3} x(x+1)(x+2) + (x+1)(x+2) \\
&= (x+1)(x+2)\left(\frac{1}{3}x + 1\right) \\
&= \frac{1}{3}(x+3)(x+1)(x+2) \\
&= \text{RHS of } (**).
\end{aligned}
$$

So if ⊛ is true for $n = x$, we have proved ⊛ true for $n = x+1$.

⊛ is true for base case $n = 1$, so we have proved ⊛ true for all $n = 1, 2, 3, \ldots\ldots$ □