Bayesian Parameter Estimation for Binary Neutron Star Inspirals

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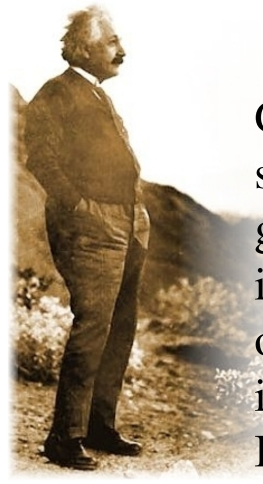
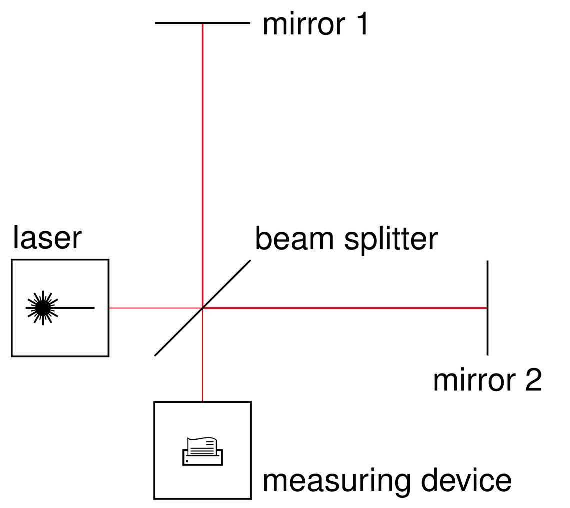
"Gravitational Radiation?"



General Relativity Theory introduced the idea that space-time is curved by the presence of mass, energy and momentum within it. It also implies the existence of gravitational waves, which can be thought of as 'ripples' in space-time, caused by rapidly moving heavy objects, and propagating at light speed.
By now this effect has only been observed *indirectly*, when the observed slight deceleration of the rotation of a binary pulsar matched exactly with the predicted deceleration due to the loss of energy through gravitational radiation [1].
The effect of gravitational radiation is very weak, so it takes very sensitive instruments to detect and measure it. The most prominent approach towards the measurement of gravitational waves at the moment is by means of *Laser Interferometers*: instruments that measure the effect of a passing gravitational wave by determining the phase shift of two laser beams that are sent through two orthogonal tubes of lengths of up to several kilometres.



Layout sketch and actual interferometer with 4-km arms.

Measurement of Gravitational Radiation would not only confirm the General Relativity Theory, but also complement to 'traditional' observations in the electromagnetic spectrum.

▶ Chirps in space-time

Gravitational radiation is emitted by all kinds of objects and processes, but one of the first events expected to be detected are *binary inspirals*, that is, a pair of heavy objects (neutron stars or black holes) that orbit each other rapidly at a decreasing distance and eventually collide. The non-constant orbit again is caused by the emission of energy through gravitational radiation, which causes the two objects to slightly slow down and approach each other until the system finally collapses.
The gravitational wave signal emitted by such an inspiral is a so-called '*chirp*', an oscillation with increasing frequency and amplitude; its exact shape is determined by nine parameters like

- masses of the involved objects (2 parameters),
- orientation of inspiral and interferometer (3 parameters),
- distance and direction to the inspiral (3 parameters), and
- coalescence time (1 parameter),

which in turn are to be estimated from the measured signal [2].

▶ Bayesian Analysis

The parameter estimation problem for binary inspirals has by now usually been addressed through the Maximum - Likelihood approach [3]; a Bayesian analysis however, might provide more sensible results [4].
Bayesian estimation has already been tried and proved promising in a simplified setting, i.e. using fewer parameters and data from a single interferometer [5]; our aim is to extend this approach to the complete parameter set and multiple interferometers.
The posterior distribution (that does not have a simple, closed form) is investigated using an MCMC algorithm.

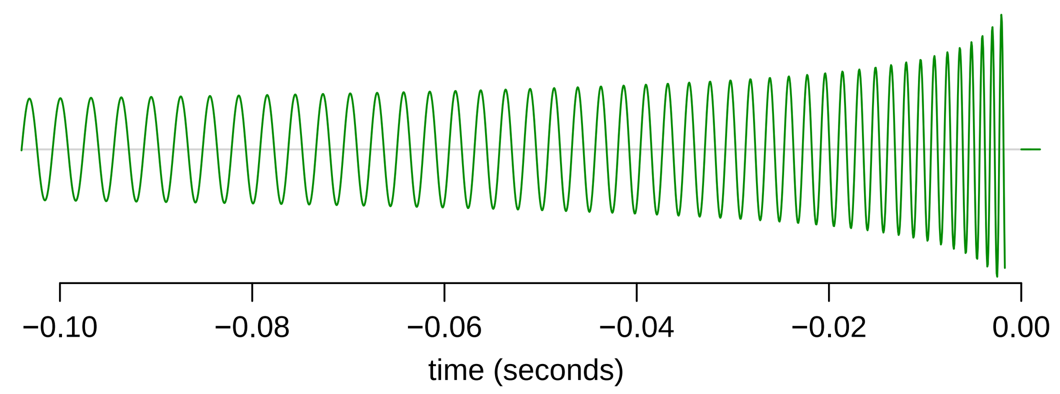
▶ Priors

The *a priori* information about the parameters can be formulated (as 'proper' priors) given both theoretical or observational results, assuming e.g. that different orientations are equally likely, potential locations are evenly spread across time and space, and neutron star masses comply with observed frequencies.

▶ Model

Assumptions made for the analysis are:

- measurement is signal plus superimposed interferometer noise
- signal follows the '*(2.0) post-Newtonian*' approximation [2]
- interferometer noise is Gaussian with a certain (interferometer specific) spectrum
- noise is independent between different interferometers



A binary system's last 10th second before coalescence, measured by an interferometer as a 'chirp'.

▶ Likelihood

The Likelihood for a single interferometer I is computed based on the (complex-valued) Fourier-Transformation ($\tilde{z}$) of the data (z) and also depends on the interferometer's noise spectrum:

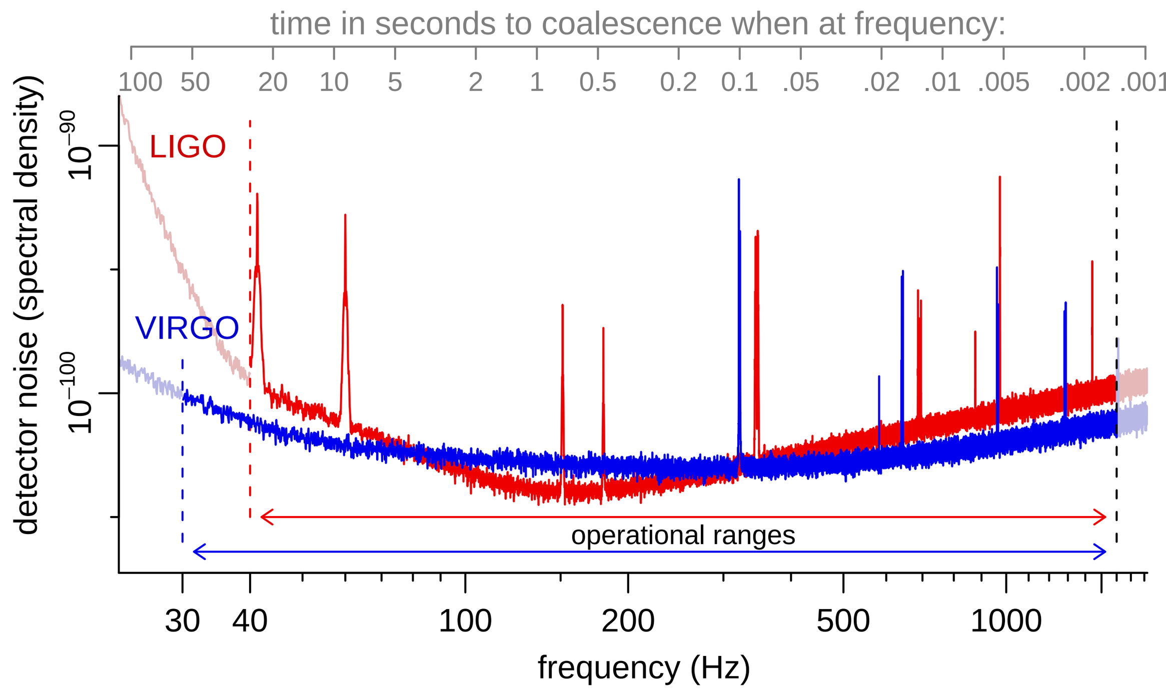
$$p^{(I)}(z|\vartheta) \propto \exp\left(-\frac{2}{\delta_t}\sum_{i=i_L}^{i_U} \frac{\big|\overbrace{\tilde{z}^{(I)}(i\times\Delta_f)}^{\text{data}} - \overbrace{\tilde{s}^{(I)}(i\times\Delta_f,\vartheta)}^{\text{template}}\big|^2}{\underbrace{S_n^{(I)}(i\times\Delta_f)}_{\text{noise spectrum}}}\right)$$

The 'template' $\tilde{s}^{(I)}$ denotes the Fourier-transformed (theoretical) response of detector I depending on the parameter vector ϑ [6].

Assuming that the noise is independent across different interferometers, the detector network likelihood is the product of the individual likelihoods:

$$p(z|\vartheta) = \prod_I p^{(I)}(z|\vartheta).$$

Likelihood computation in the frequency domain allows to easily filter out the signal frequencies that fall into the interferometers' operational ranges.

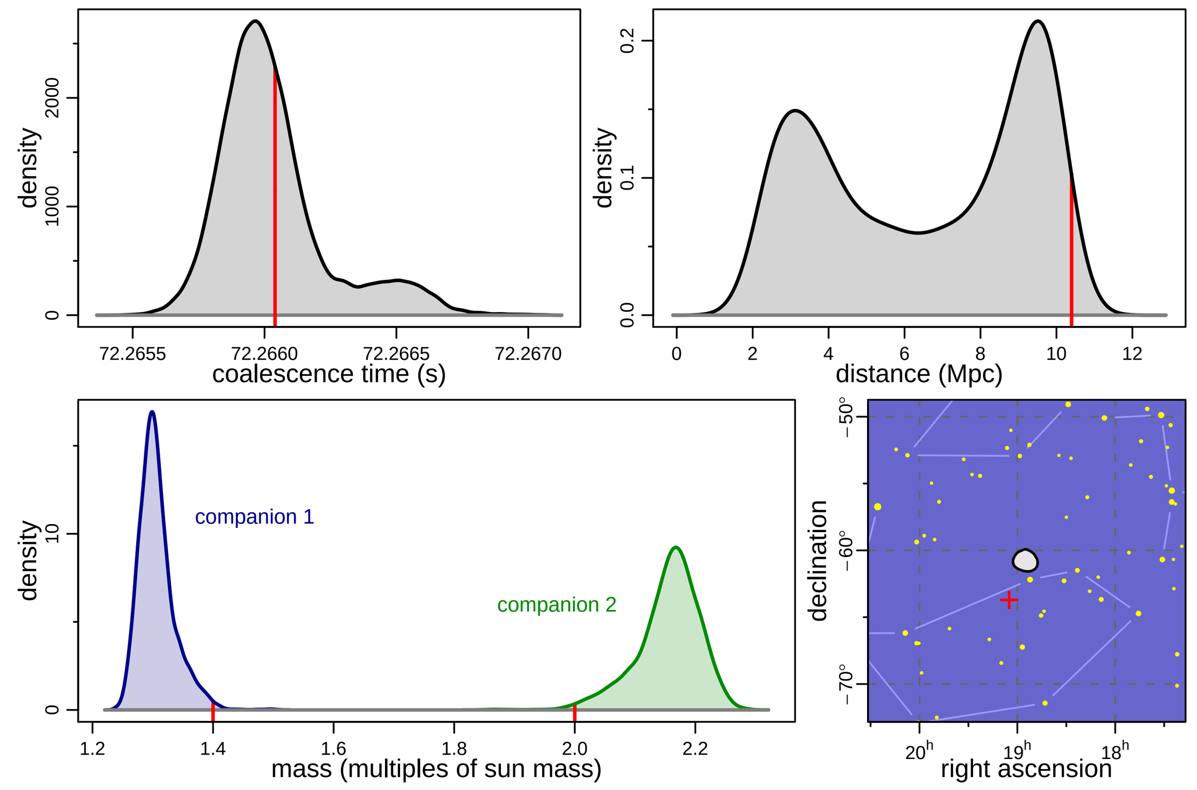


Two interferometer noise curves and the times a binary system spends orbiting at corresponding frequencies.

A binary inspiral will be detectable within today's interferometers' operational ranges for its last 20–100 seconds before coalescence.
Due to the signal's nature, some parameters (e.g. direction) require measurements from different detector sites in order to be estimable.

▶ Results so far

The estimation procedure is still under development and has by now been tested by investigating simulated data sets. A first stage was to implement a simplified model including only 5 parameters and processing data from one interferometer. This model was used to figure out appropriate procedures for different stages especially of the likelihood computation, including downsampling, filtering, Fourier transformation and windowing, and eventually sensible tuning of the MCMC algorithm.
After this model worked satisfactorily, it is now being extended to the realistic case of 9 parameters and processing data from several detector sites. Some methods applied in the simpler model to improve the efficiency of the MCMC sampler evidently are not appropriate for the extended model, mostly due to the much larger parameter space.
The plots in the upper right illustrate marginal posterior distributions for some of the parameters for the extended model; the posteriors are still slightly off the true values indicated in red.



Posterior densities for an inspiral's coalescence time, distance, its individual masses, and a 99% confidence region for its sky position (preliminary results).

▶ Outlook

Further work is required on the implementation's accuracy and efficiency.
Convergence of the MCMC sampler to the actual stationary distribution still is very slow; the most promising approach to the solution of this problem at the moment is the use of *parallel tempering* [7].
By now the method has only been run on simulated data sets, 'real' measurements are not yet available but are expected in the near future.

References

[1] J. H. Taylor and Weisberg J. M. Further experimental tests of relativistic gravity using the binary pulsar PSR1913+16. *The Astrophysical Journal*, 345:434–450, October 1989.

[2] L. Blanchet, B. R. Iyer, C. M. Will, and A. G. Wiseman. Gravitational waveforms from inspiralling compact binaries to second-post-Newtonian order. *Classical and Quantum Gravity*, 13(4):575–584, April 1996.

[3] A. Pai, S. Dhurandhar, and S. Bose. Data-analysis strategy for detecting gravitational-wave signals from inspiraling compact binaries with a network of laser-interferometric detectors. *Physical Review D*, 64(4), 2001.

[4] T. J. Loredo. The promise of Bayesian inference for astrophysics. In E. D. Feigelson and G. J. Babu, editors, *Statistical challenges in modern astronomy*, chapter 12, pages 275–297. Springer-Verlag, New York, 1992.

[5] N. Christensen, R. Meyer, and A. Libson. A Metropolis-Hastings routine for estimating parameters from compact binary inspiral events with laser interferometric gravitational radiation data. *Classical and Quantum Gravity*, 21(1):317–330, January 2004.

[6] L. S. Finn and D. F. Chernoff. Observing binary inspiral in gravitational radiation: One interferometer. *Physical Review D*, 47(6), March 1993.

[7] K. Hukushima and K. Nemoto. Exchange Monte Carlo method and application to Spin Glass simulations. *Journal of the Physical Society of Japan*, 65(6):1604–1608, June 1996.