Bayesian Parameter Estimation for Binary Neutron Star Inspiral

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“Gravitational Radiation?”

General Relativity Theory introduced the idea that space-time is curved by the presence of mass, energy and momentum within it. It also implies the existence of gravitational waves, which can be thought of as ‘ripples’ in space-time, caused by rapidly moving, heavy objects, and propagating at light speed.

By now this effect has only been observed indirectly, when the observed slight deceleration of the rotation of a binary pulsar matched exactly with the predicted deceleration due to the loss of energy through gravitational radiation [1].

The effect of gravitational radiation is very weak, so it takes very sensitive instruments to detect and measure it. The most prominent approach towards the measurement of gravitational waves at the moment is by means of Laser Interferometers: instruments that measure the effect of a passing gravitational wave by determining the phase shift of two laser beams that are sent through two orthogonal tubes of lengths of up to several kilometres.

Bayesian Analysis

The parameter estimation problem for binary inspirals has by now usually been addressed through the Maximum - Likelihood approach [3]; a Bayesian analysis however, might provide more sensible results [4]. Bayesian estimation has already been tried and proved promising in a simplified setting, i.e. using fewer parameters and data from a single interferometer [5]; our aim is to extend this approach to the complete parameter set and multiple interferometers.

The posterior distribution (that does not have a simple, closed form) is investigated using an MCMC algorithm.

Priors

The a priori information about the parameters can be formulated (as ‘proper’ priors) given both theoretical or observational results, assuming e.g. that different orientations are equally likely, potential locations are evenly spread across time and space, and neutron star masses comply with observed frequencies.

Model

Assumptions made for the analysis are:

- measurement is signal plus superimposed interferometer noise
- signal follows the ‘(2.0) post-Newtonian’ approximation [2]
- interferometer noise is Gaussian with a certain (interferometer specific) spectrum
- noise is independent between different interferometers

A binary system’s last 10th second before coalescence, measured by an interferometer as a ‘chirp’.

Likelihood

The likelihood for a single interferometer $f$ is computed based on the (complex-valued) Fourier-Transformation ($\tilde{f}$) of the data ($y$) and also depends on the interferometer’s noise spectrum:

$$p[f]\propto \exp \left( -\sum_{\text{freq}} |\tilde{f}(y \times \Delta f) - \tilde{f}(y \times \Delta f_0)|^2 \over 2\text{noise spectrum} \right)$$

The template $\tilde{\phi}$ denotes the Fourier-transformed (theoretical) response of detector $f$ depending on the parameter vector $\theta$ [6].

Assuming that the noise is independent across different interferometers, the detector network likelihood is the product of the individual likelihoods:

$$p[z|\theta] = \prod_f p[f|z]$$. 

Likelihood computation in the frequency domain allows to easily filter out the signal frequencies that fall into the interferometers’ operational ranges.

Results so far

The estimation procedure is still under development and has by now been tested by investigating simulated data sets. A first stage was to implement a simplified model including only 5 parameters and processing data from one interferometer. This model was used to figure out appropriate procedures for different stages especially of the likelihood computation, including downsampling, filtering, Fourier transformation and windowing, and eventually sensible tuning of the MCMC algorithm.

After this model worked satisfactorily, it is now being extended to the realistic case of 9 parameters and processing data from several detector sites. Some methods applied in the simpler model to improve the efficiency of the MCMC sampler evidently are not appropriate for the extended model, mostly due to the much larger parameter space.

The plots in the upper right illustrate marginal posterior distributions for some of the parameters for the extended model; the posteriors are still slightly off the true values indicated in red.

Outlook

Further work is required on the implementation’s accuracy and efficiency.

Convergence of the MCMC sampler to the actual stationary distribution still is very slow; the most promising approach to the solution of this problem at the moment is the use of parallel tempering [7].

By now the method has only been run on simulated data sets, ‘real’ measurements are not yet available but are expected in the near future.

References