

Coherent Bayesian analysis of inspiral signals

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Overview:

1. The Bayesian approach
2. MCMC methods
3. The inspiral signal
4. Priors
5. Example application

The Bayesian approach

- idea: **assign probabilities to parameters** θ
- pre-experimental knowledge: **prior probabilities / -distribution** $p(\theta)$
- data model: **likelihood** $p(y|\theta)$

- application of **Bayes' theorem** yields the **posterior distribution**

$$p(\theta|y) \propto p(\theta) p(y|\theta)$$

conditional on the observed data y .

- posterior distribution combines the **information in the data** with the **prior information**

MCMC methods - what they do

- Problem -
given: posterior distribution $p(\theta|y)$ (density, function of θ)
wanted: mode(s), integrals,...
- what MCMC does:
simulate random draws from (any) distribution, allowing to approximate any integral by sample statistic (e.g. means by averages etc.)
- Monte Carlo integration

MCMC methods - how they work

- Markov Chain Monte Carlo
- random walk
- **Markov property**: each step in random walk only depends on previous
- **stationary distribution** is equal to the desired posterior $p(\theta|y)$
- most famous: **Metropolis- (Hastings-) sampler**
especially convenient: normalising constant factors to $p(\theta|y)$ don't need to be known.

MCMC methods

- Metropolis-algorithm may also be seen as **optimisation algorithm**: improving steps always accepted, worsening steps sometimes (\rightarrow *Simulated Annealing*)
- in fact: purpose often *both* **finding** mode(s) *and* **sampling** from them

The inspiral signal

- measurement: time series (signal + noise)
at, say, 3 separate interferometers
- **signal**: chirp waveform; 2.5PN amplitude, 3.5PN phase^{1,2}
- **9 parameters**: masses (m_1, m_2), coalescence time (t_c), coalescence phase (ϕ_0), luminosity distance (d_L), inclination angle (ι), sky location (δ, α) and polarisation (ψ)

¹K.G. Arun et al.: *The 2.5PN gravitational wave polarizations from inspiralling compact binaries in circular orbits*, Class. Quantum Grav. 21, 3771 (2004).

²L. Blanchet et al.: *Gravitational-wave inspiral of compact binary systems to 7/2 post-Newtonian order*. Phys. Rev. D 65, 061501 (2002).

The signal at different interferometers

- ‘local’ parameters at interferometer I :

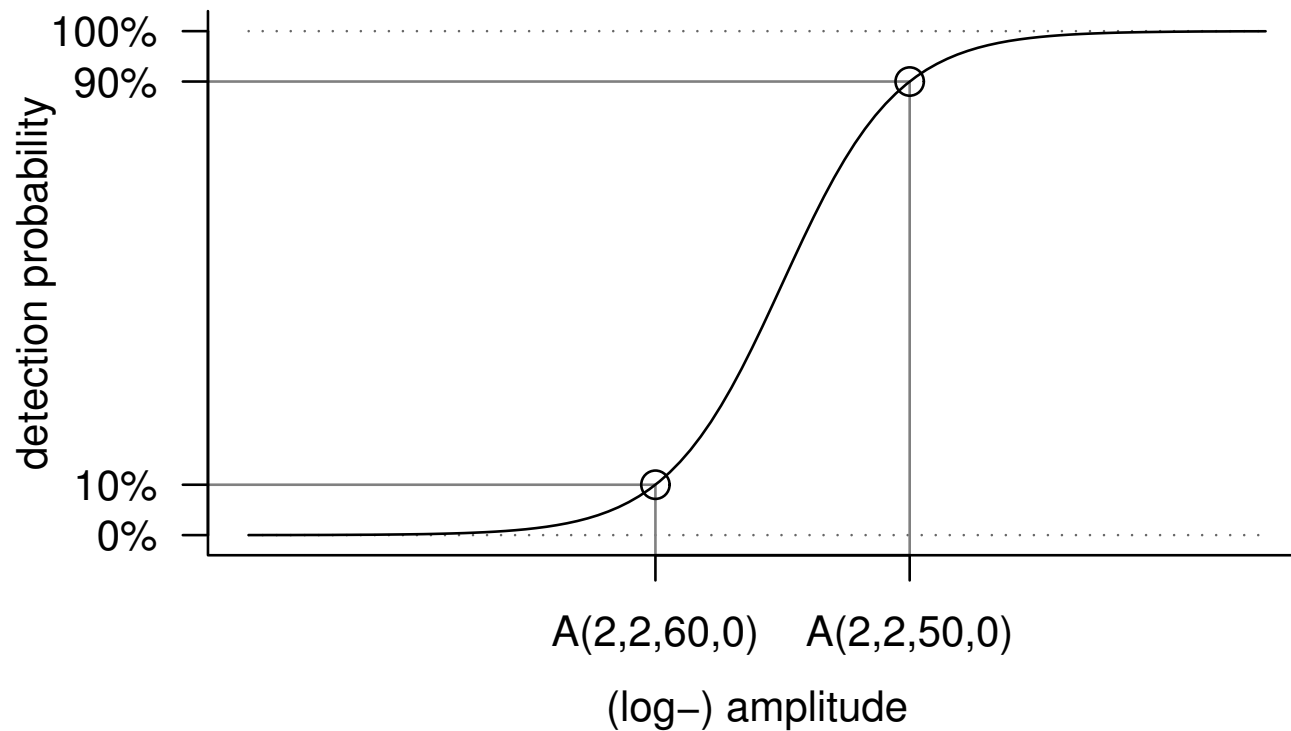
sky location (δ, α) → altitude $(\vartheta^{(I)})$ / azimuth $(\varphi^{(I)})$
coalescence time (t_c) → local coalescence time $(t_c^{(I)})$
polarisation (ψ) → local polarisation $(\psi^{(I)})$

- **noise** assumed **gaussian, coloured**; interferometer-specific spectrum
- **likelihood** computation based on Fourier transforms of data and signal
- noise **independent** between interferometers
⇒ coherent network likelihood is **product** of individual ones

Prior information about parameters

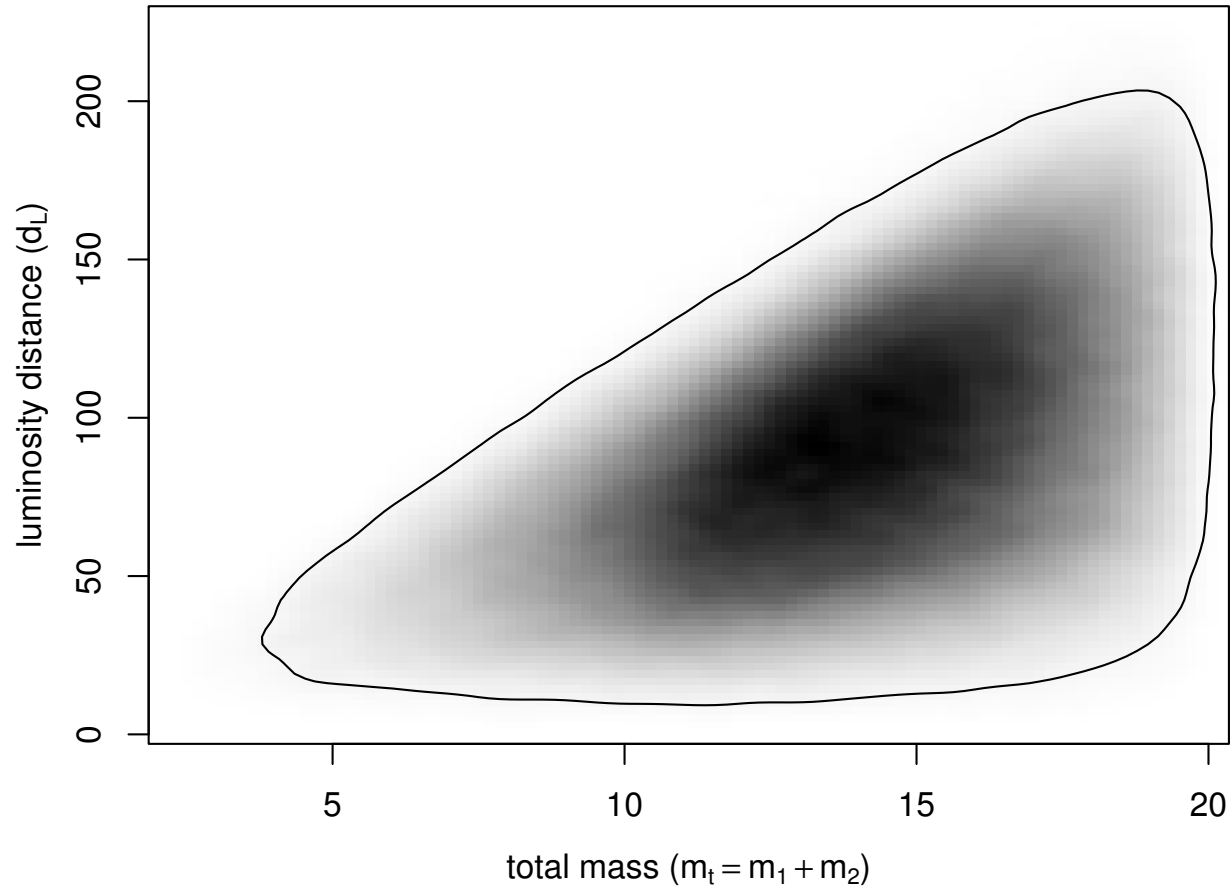
- different locations / orientations equally likely
- masses: uniform across $[1 M_{\odot}, 10 M_{\odot}]$
- events spread uniformly across space: $P(d_L \leq x) \propto x^3$
- but: **certain SNR required** for detection
- cheap **SNR substitute**: signal **amplitude** \mathcal{A}
- primarily dependent on **masses, distance, inclination**: $\mathcal{A}(m_1, m_2, d_L, \iota)$

- introduce sigmoid function linking **amplitude** to **detection probability**³

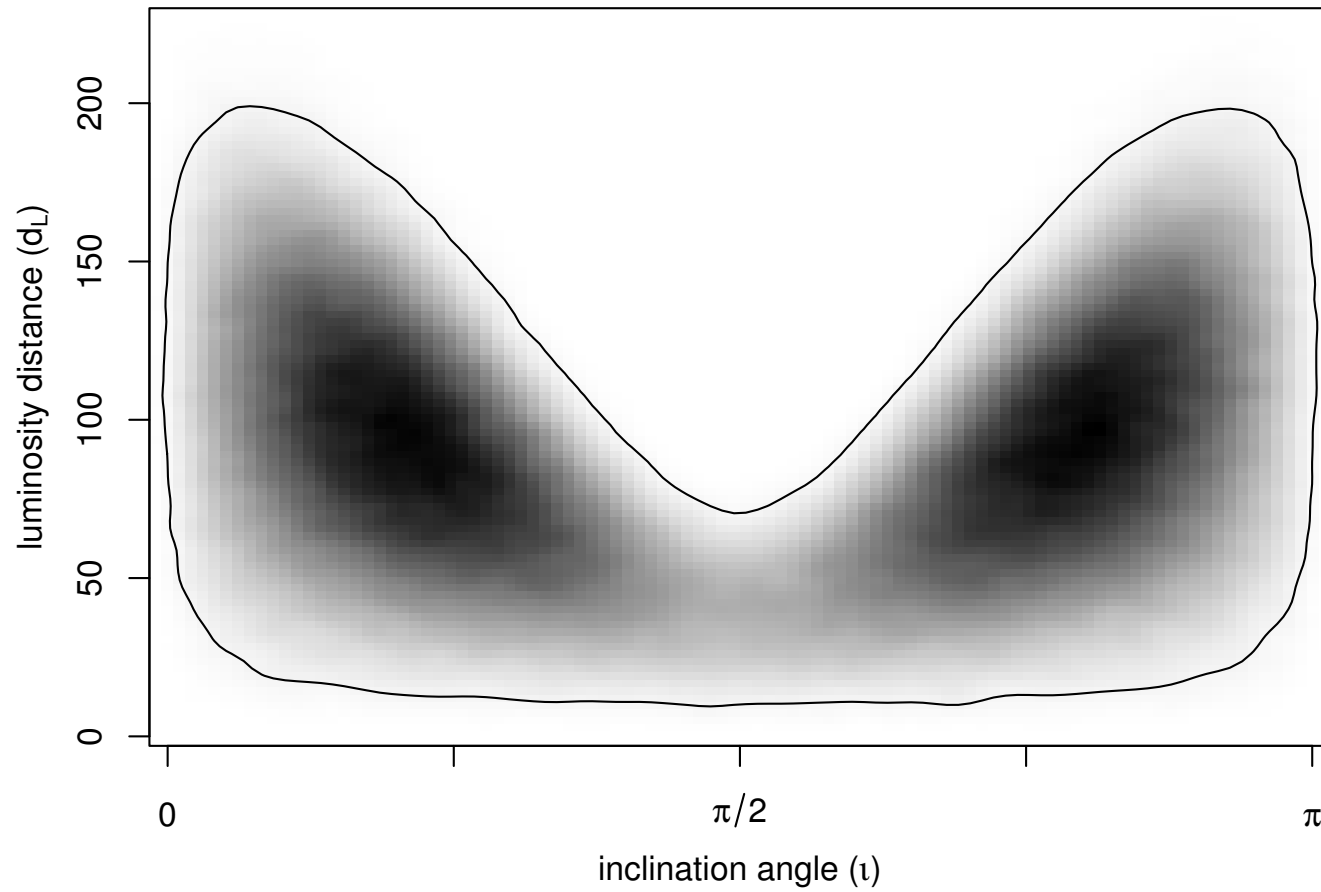


³R. Umstätter et al.: *Setting upper limits from LIGO on gravitational waves from SN1987a*. Poster presentation; also: paper in preparation.

Resulting (marginal) prior density

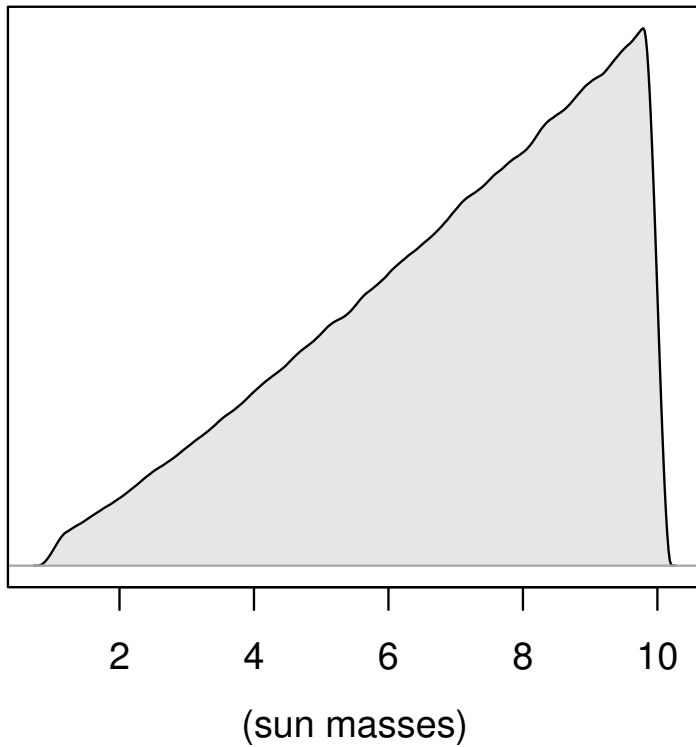


Marginal prior density

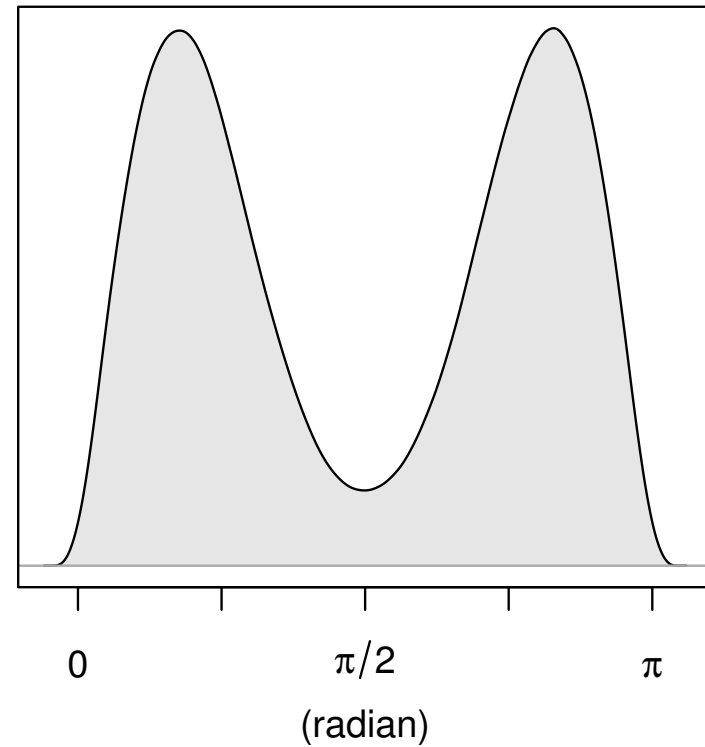


Marginal prior densities

individual masses (m_1, m_2)



inclination angle (i)



Prior

- prior 'considers' **Malmquist effect**
- more realistic settings once **detection pipeline** is set up

MCMC details

- **Reparametrisation**,
most importantly: **chirp mass** m_c , **mass ratio** η
- **Parallel Tempering**⁴
several *tempered* MCMC chains running in parallel
sampling from $p(\theta|y)^{\frac{1}{T_i}}$ for ‘temperatures’ $1 = T_1 \leq T_2 \leq \dots$
- **Evolutionary MCMC**⁵
‘recombination’ steps between chains—motivated by Genetic algorithms

⁴W.R. Gilks et al.: *Markov chain Monte Carlo in practice* (Chapman & Hall / CRC, 1996).

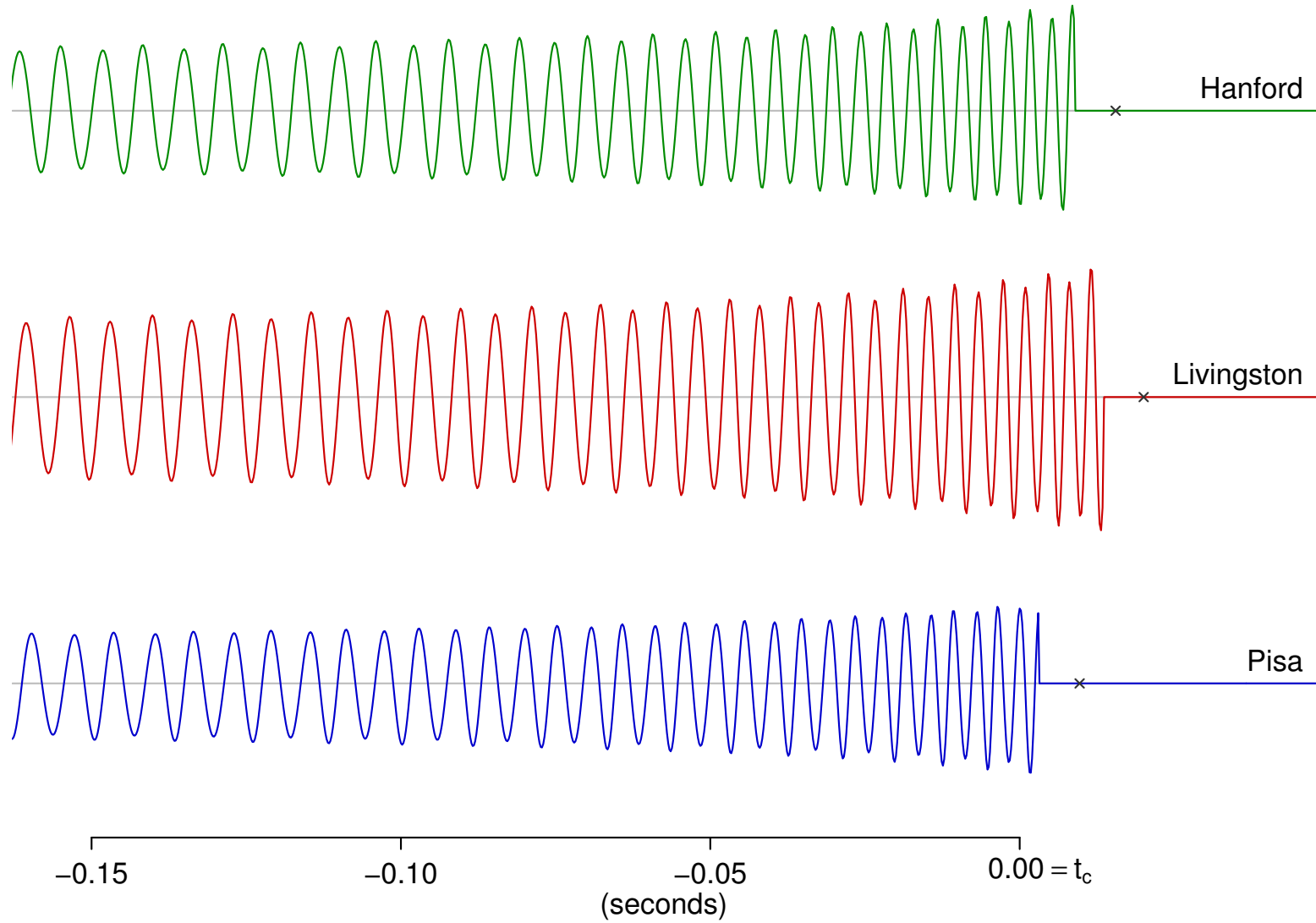
⁵F. Liang, H.W. Wong: *Real-parameter Evolutionary Monte Carlo with applications to Bayesian mixture models*. J. Am. Statist. Assoc. 96, 653 (2001)

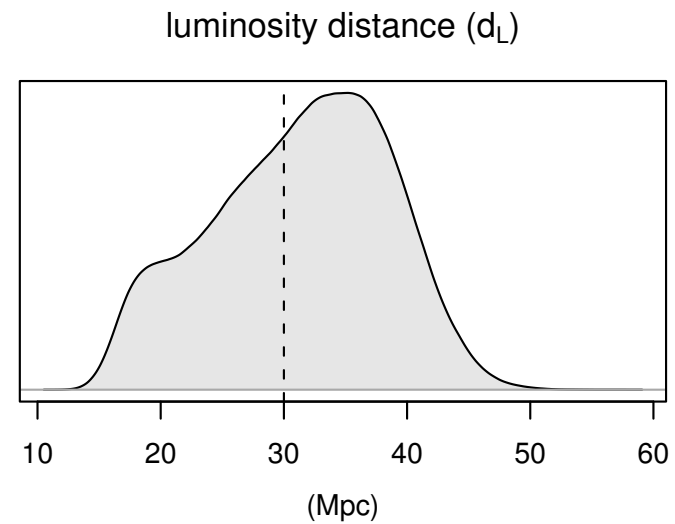
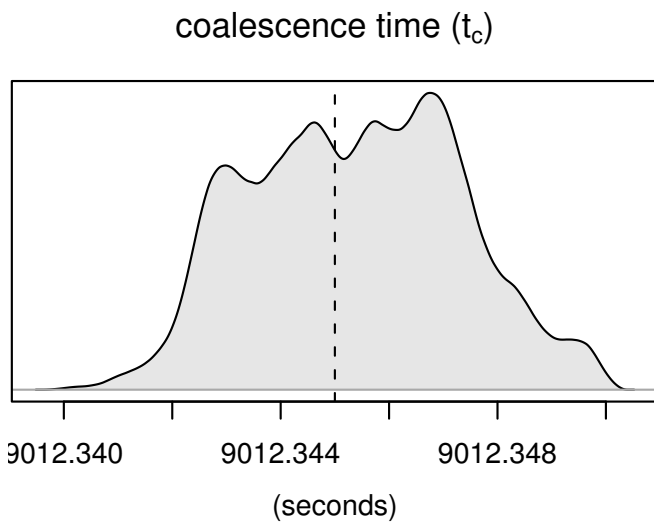
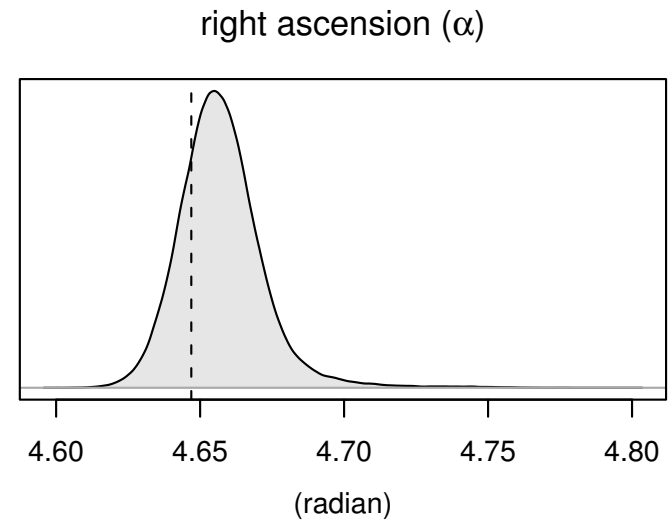
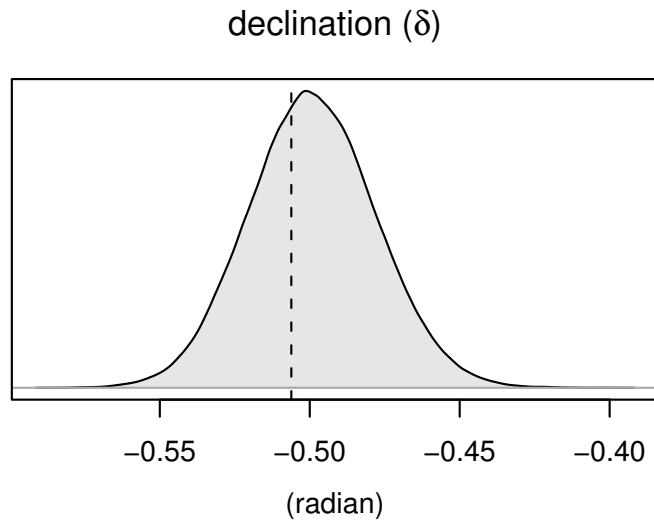
Example application

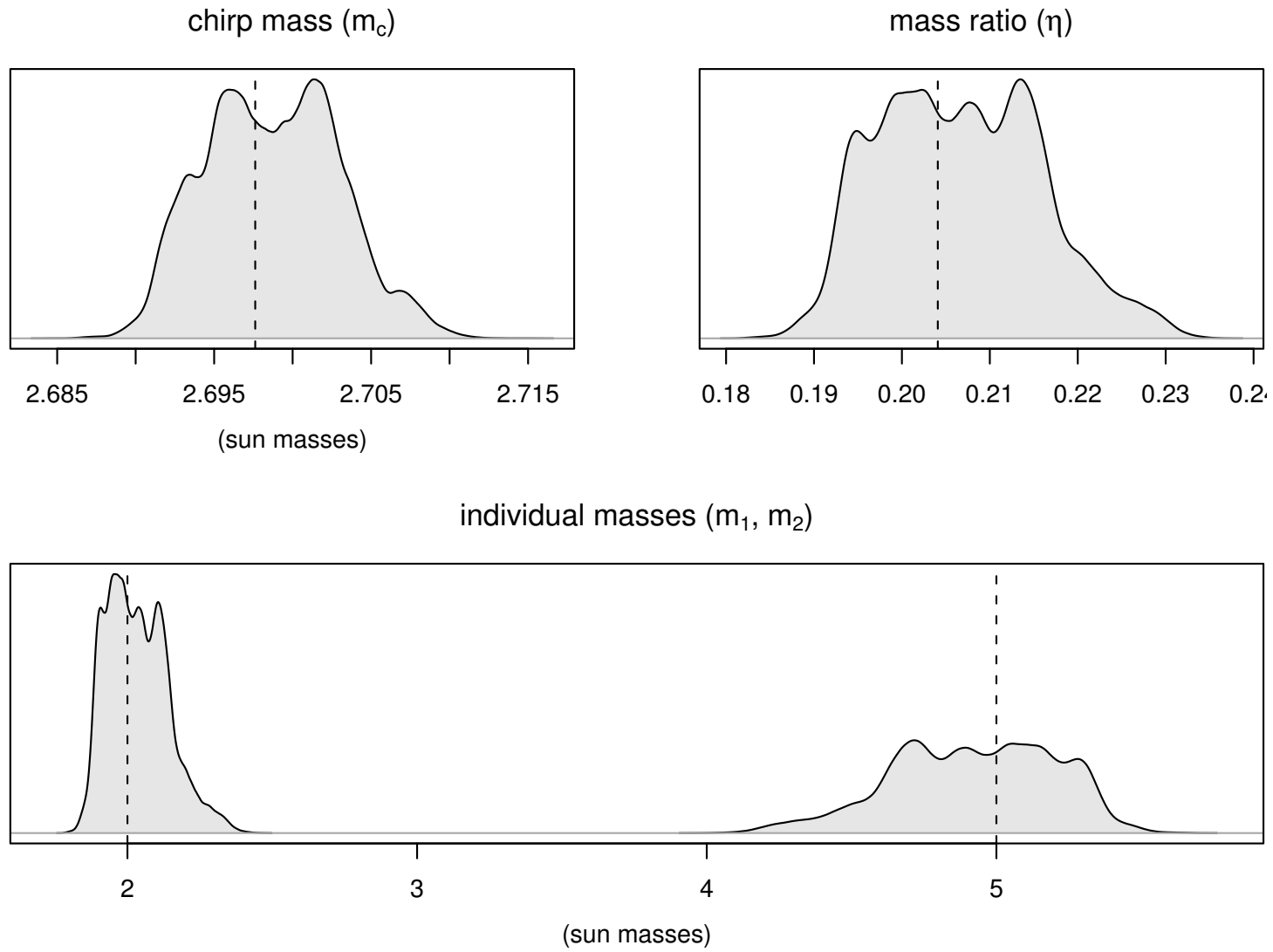
- simulated data:
2 M_{\odot} - 5 M_{\odot} inspiral at 30 Mpc distance
measurements from 3 interferometers:

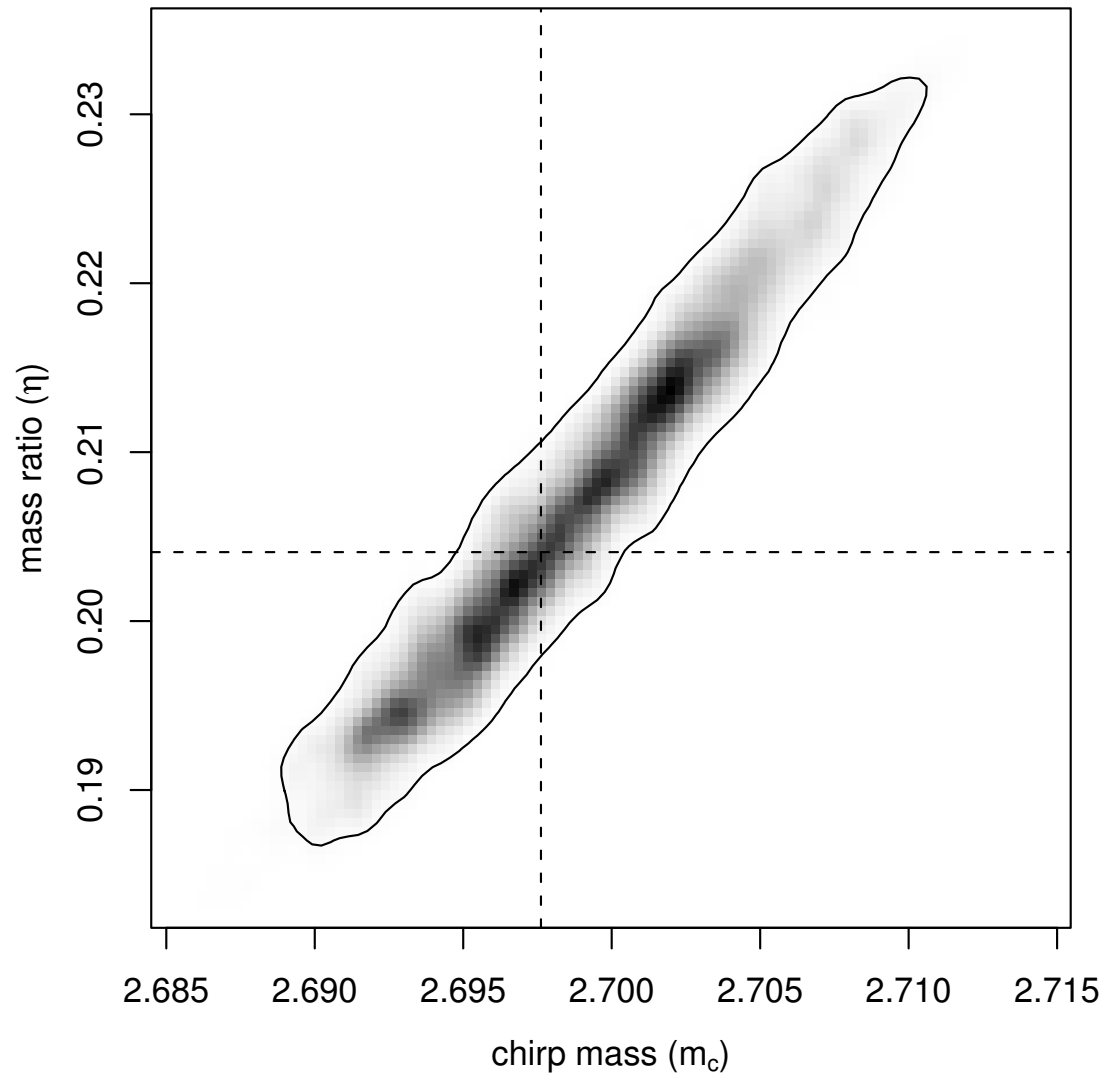
	SNR
LHO (Hanford)	8.4
LLO (Livingston)	10.9
Virgo (Pisa)	6.4
network	15.2

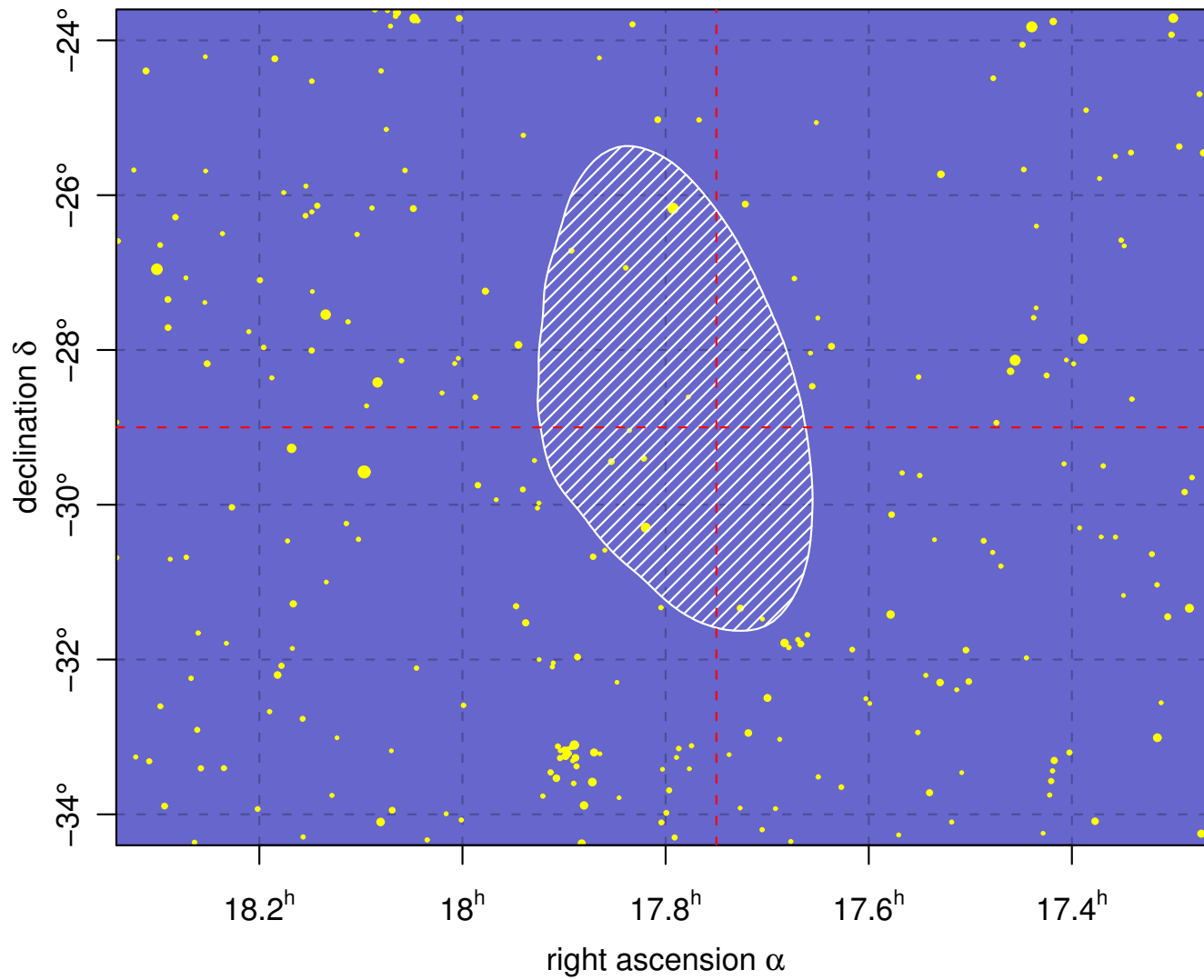
- **data:** 10 seconds (LHO/LLO), 20 seconds (Virgo) before coalescence,
noise as expected at design sensitivities
- computation **speed:** 1–2 likelihoods / second







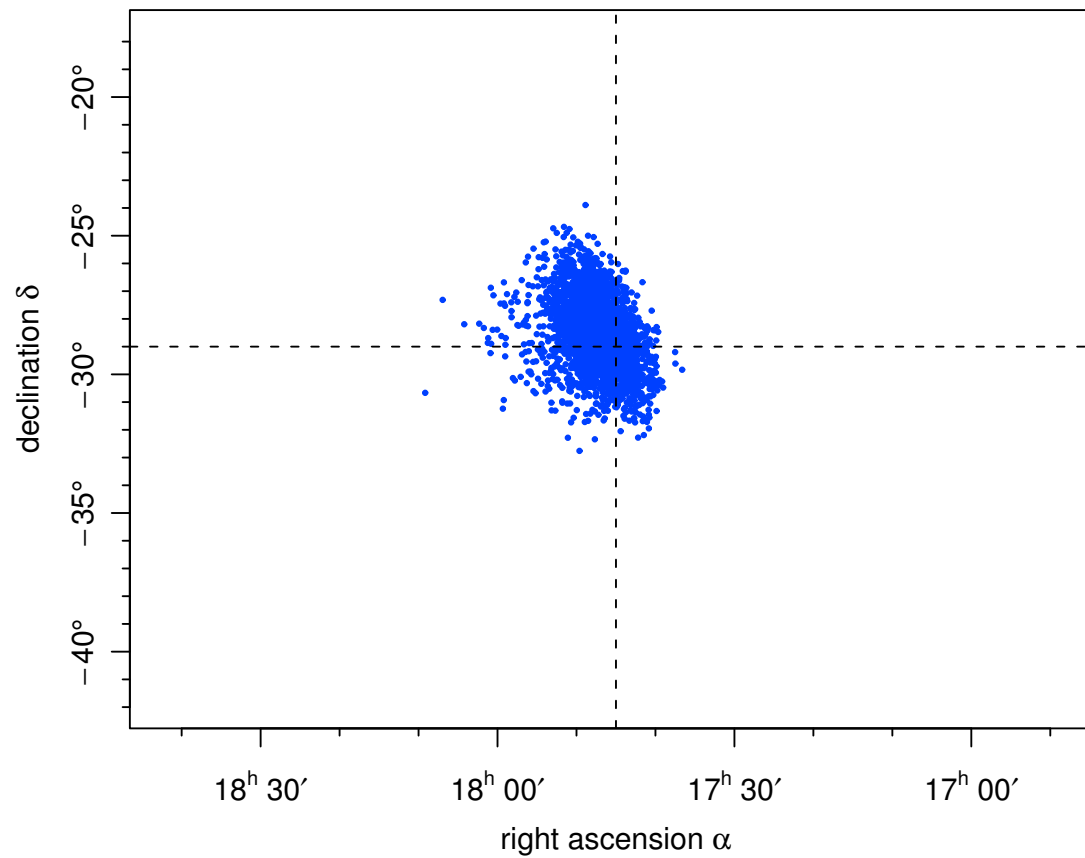




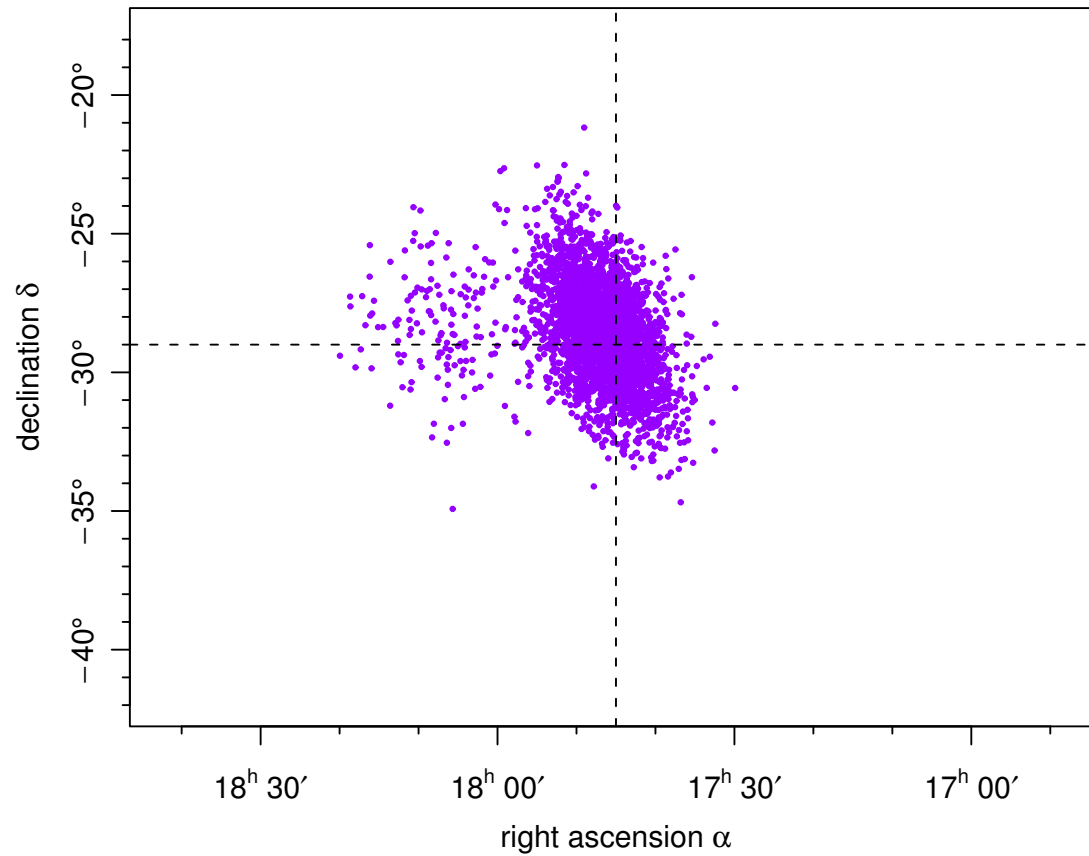
some posterior key figures

	mean	95% c.i.	true	unit
chirp mass (m_c)	2.699	(2.692, 2.707)	2.698	M_\odot
mass ratio (η)	0.207	(0.192, 0.225)	0.204	
coalescence time (t_c)	12.3455	(12.3421, 12.3490)	12.3450	s
luminosity distance (d_L)	31.4	(17.4, 43.5)	30.0	Mpc
inclination angle (ι)	0.726	(0.159, 1.456)	0.700	rad
declination (δ)	-0.498	(-0.539, -0.456)	-0.506	rad
right ascension (α)	4.657	(4.632, 4.688)	4.647	rad
coalescence phase (ϕ_0)			2.0	rad
polarisation (ψ)			1.0	rad

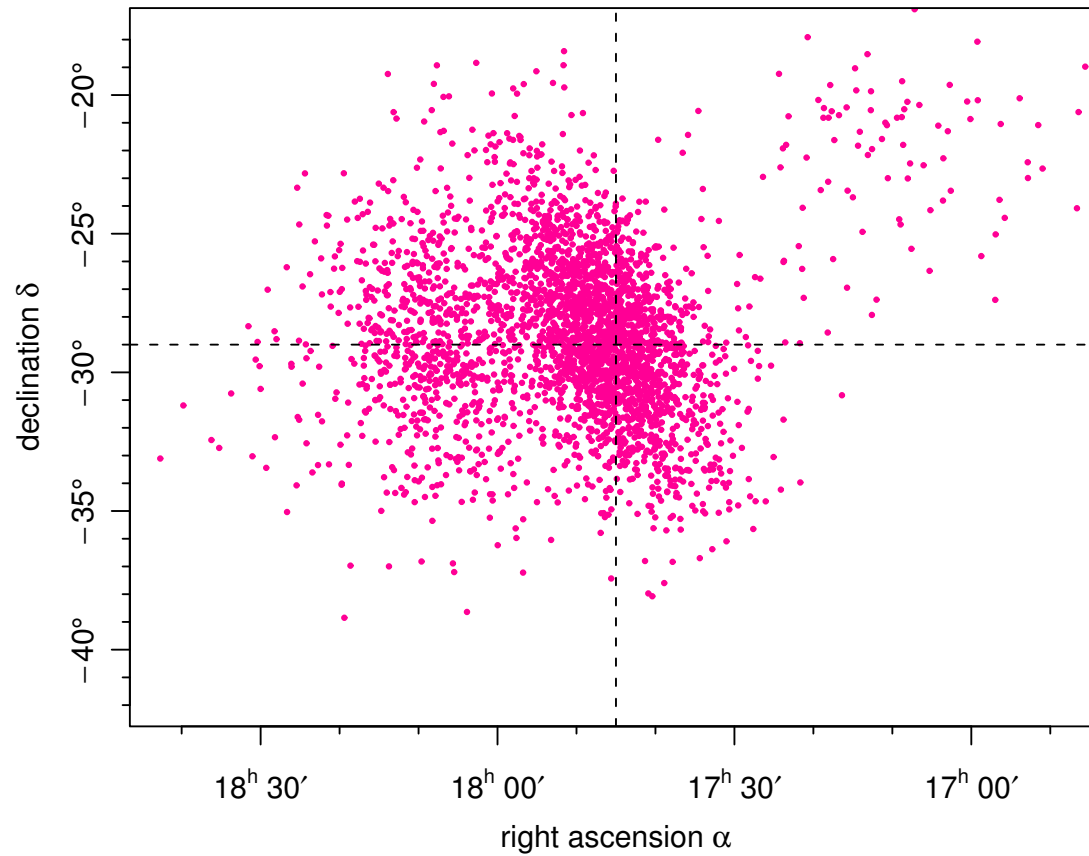
MCMC chain 1 — temperature = 1



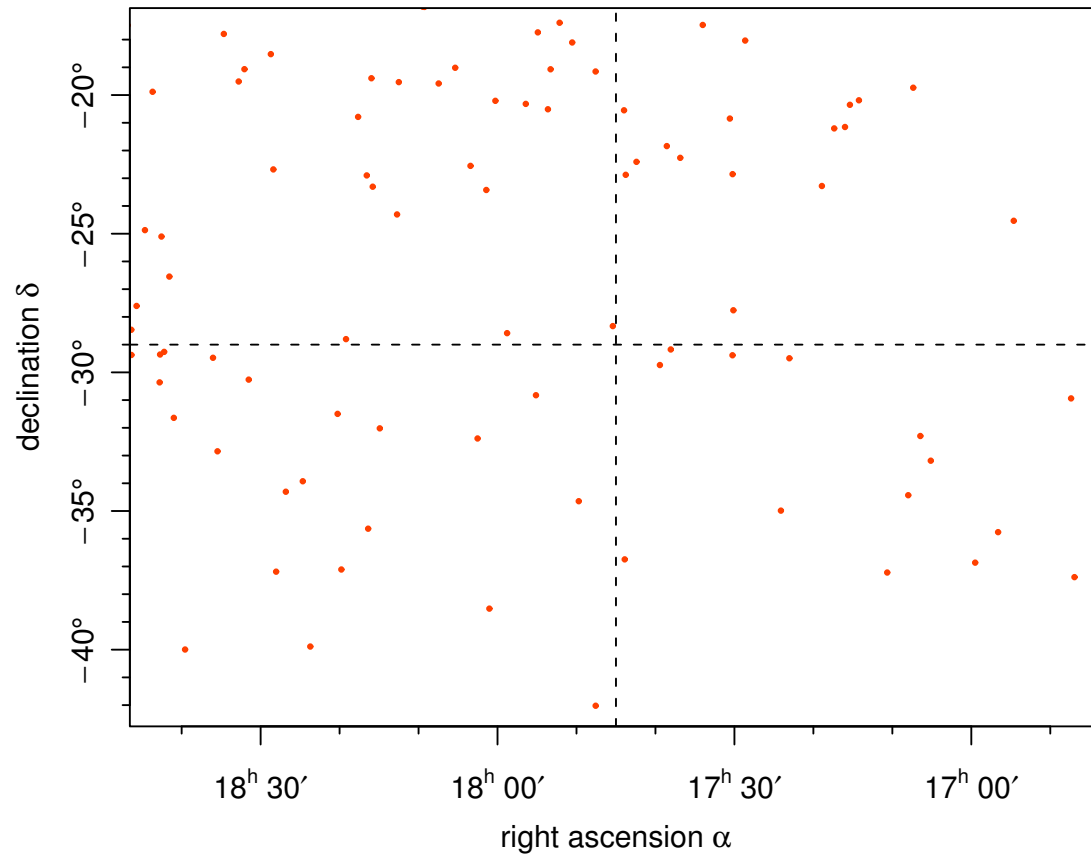
MCMC chain 2 — temperature = 2



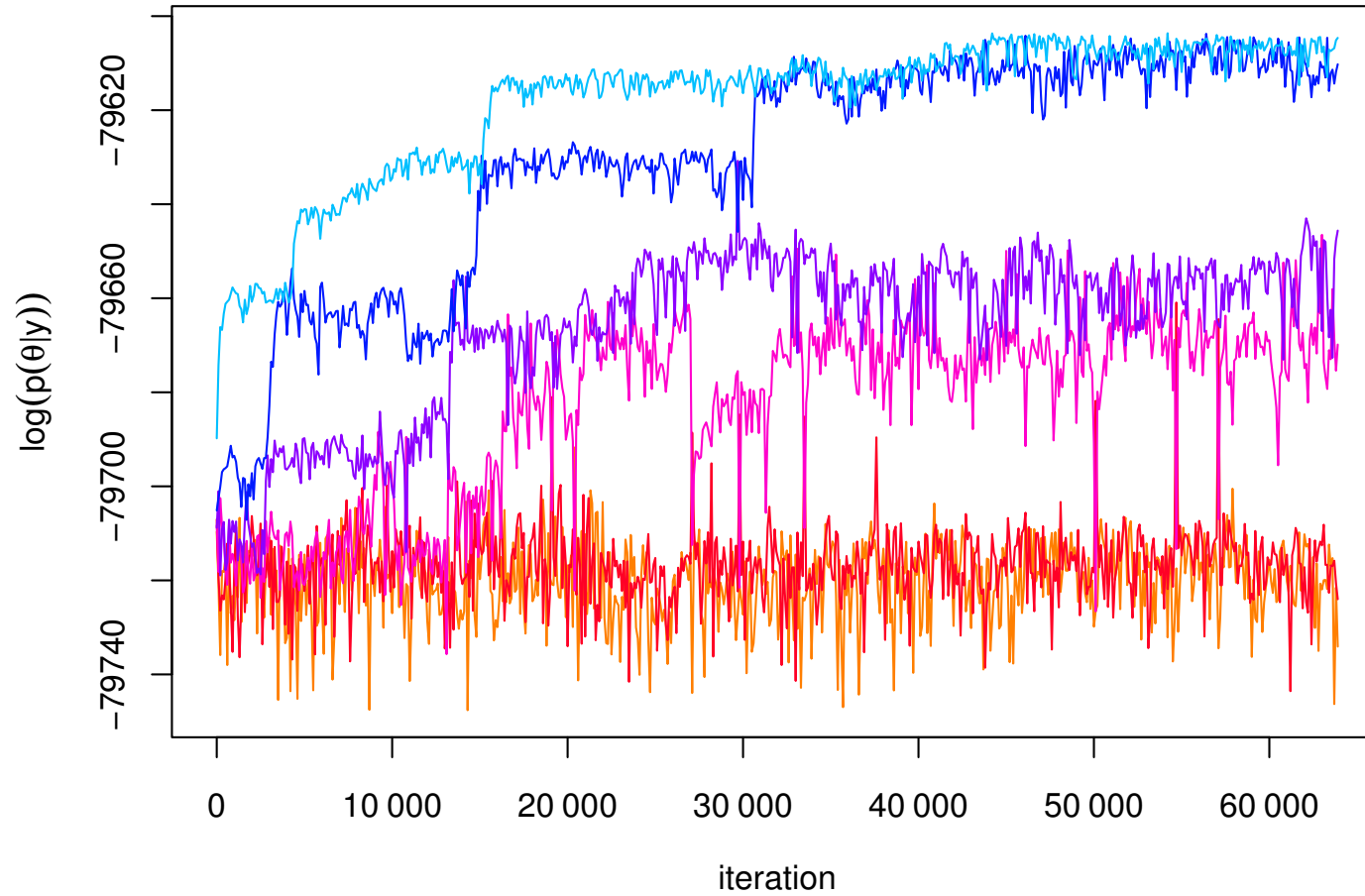
MCMC chain 3 — temperature = 4



MCMC chain 4 — temperature = 8



Six tempered chains 'in action'



Outlook

- incorporation into a 'loose net' detection pipeline for large mass ratio inspirals
- use information supplied by detection pipeline
- further parameters, e.g. spin effects