Coherent Bayesian inference on compact binary inspirals using a network of interferometric gravitational wave detectors

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Overview:

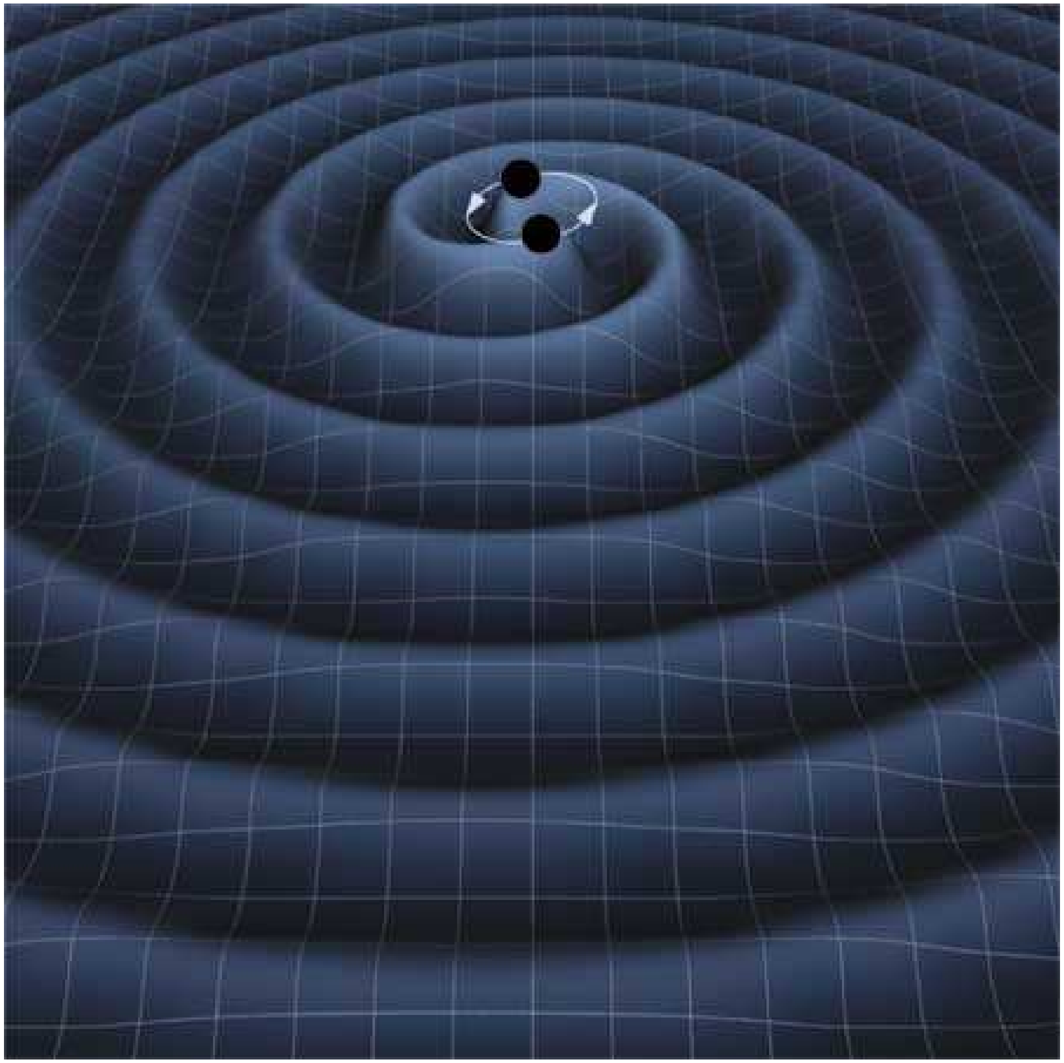
1. gravitational waves
2. measuring gravitational waves
3. the binary inspiral signal
4. prior & model
5. MCMC details
6. example application

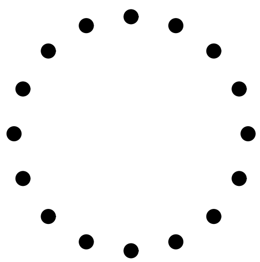
Gravitational waves

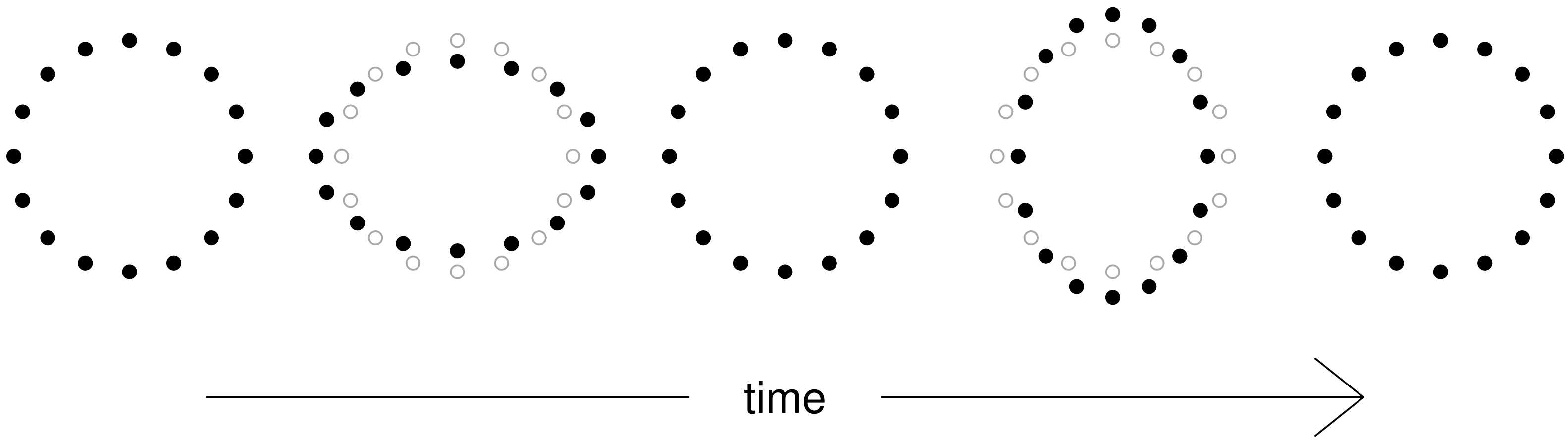
- general relativity: space-time curved by masses
- implication: existence of **gravitational waves**
  (pointed out in 1916)
- existence proven in 1979
- measurement attempted since 1960s
- no *direct* measurement yet

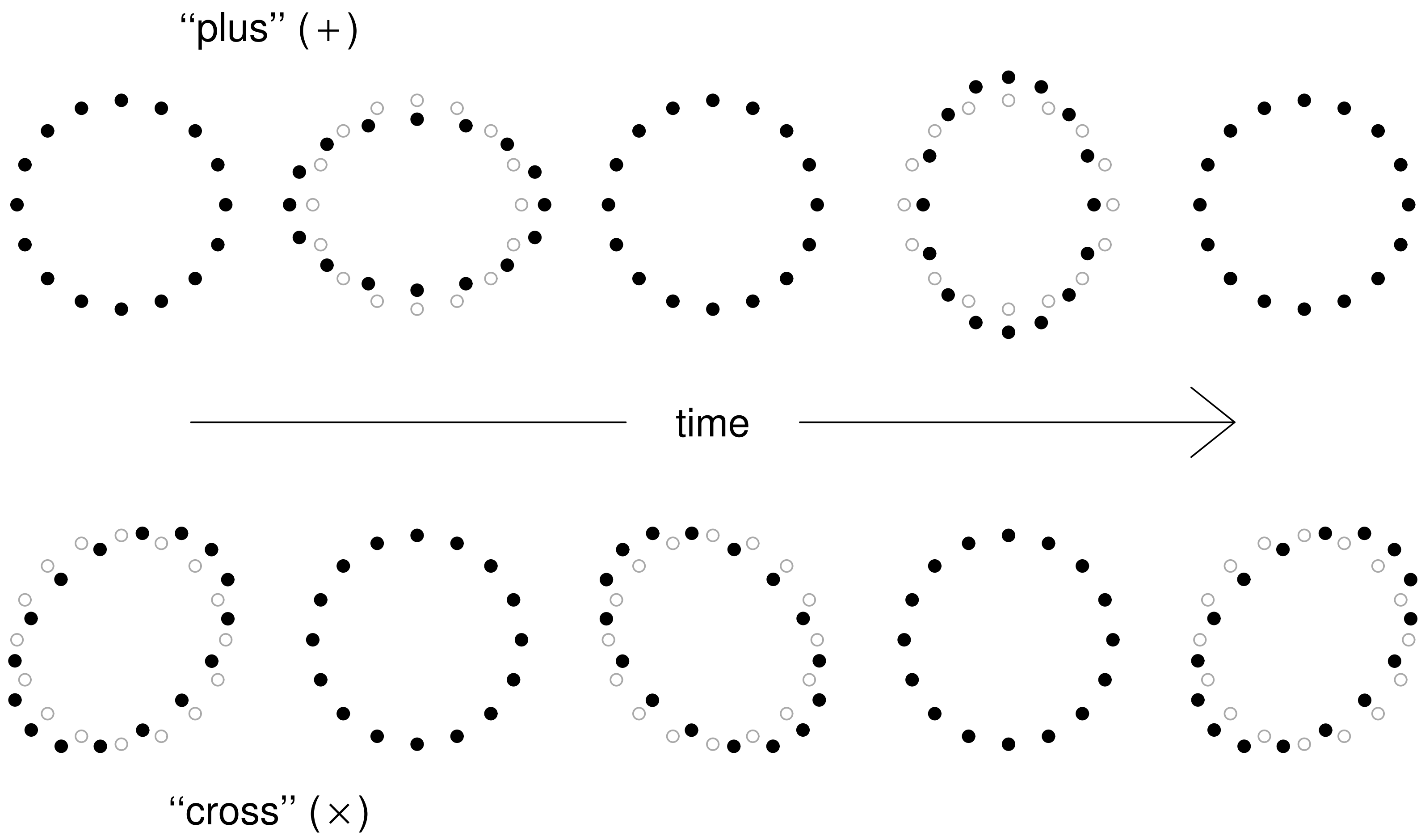
Gravitational waves

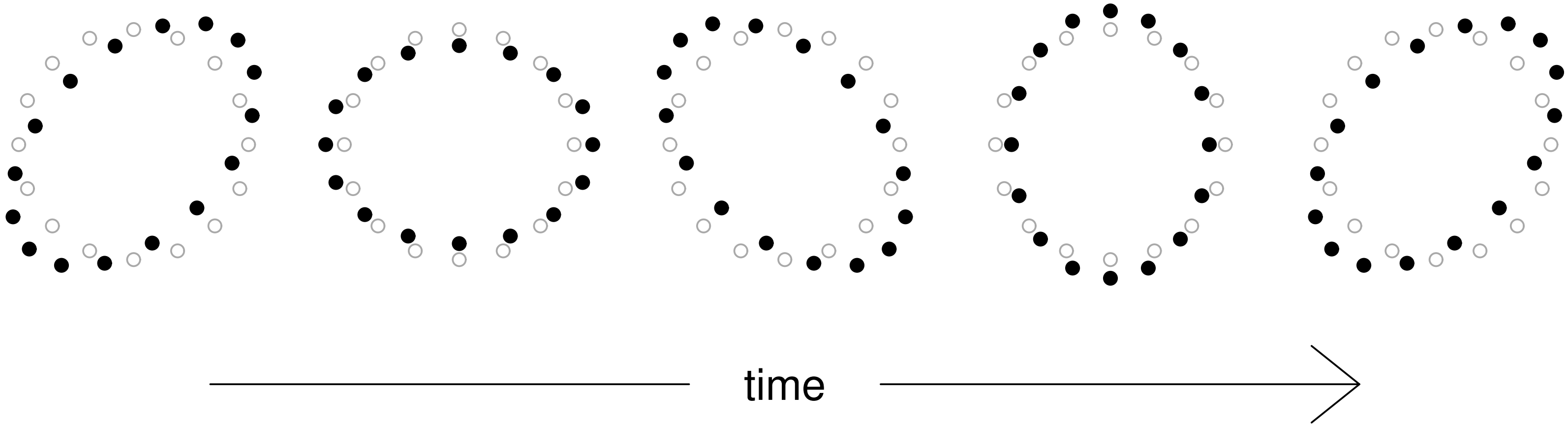
- very **weak effect**
- emitted by **rapid**ly moving, **heavy** objects
- event candidates:
  - supernovae
  - big bang
  - **binary star systems**
  - . . .

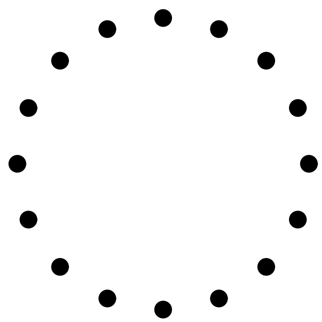


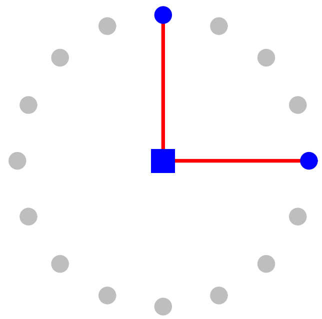


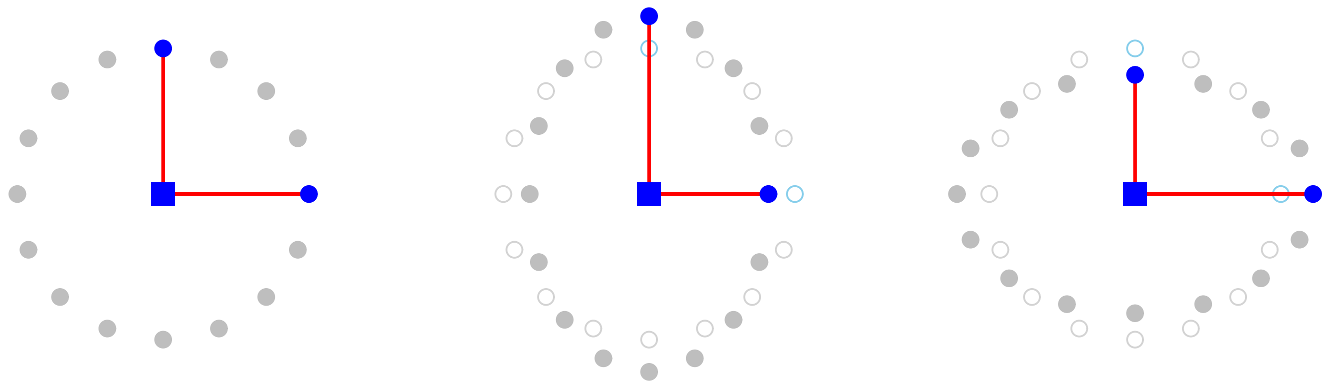

time


“plus” (+)
time
“cross” (×)


time







Hanford, WA

Pisa, Italy

Livingston, LA

Hannover, Germany

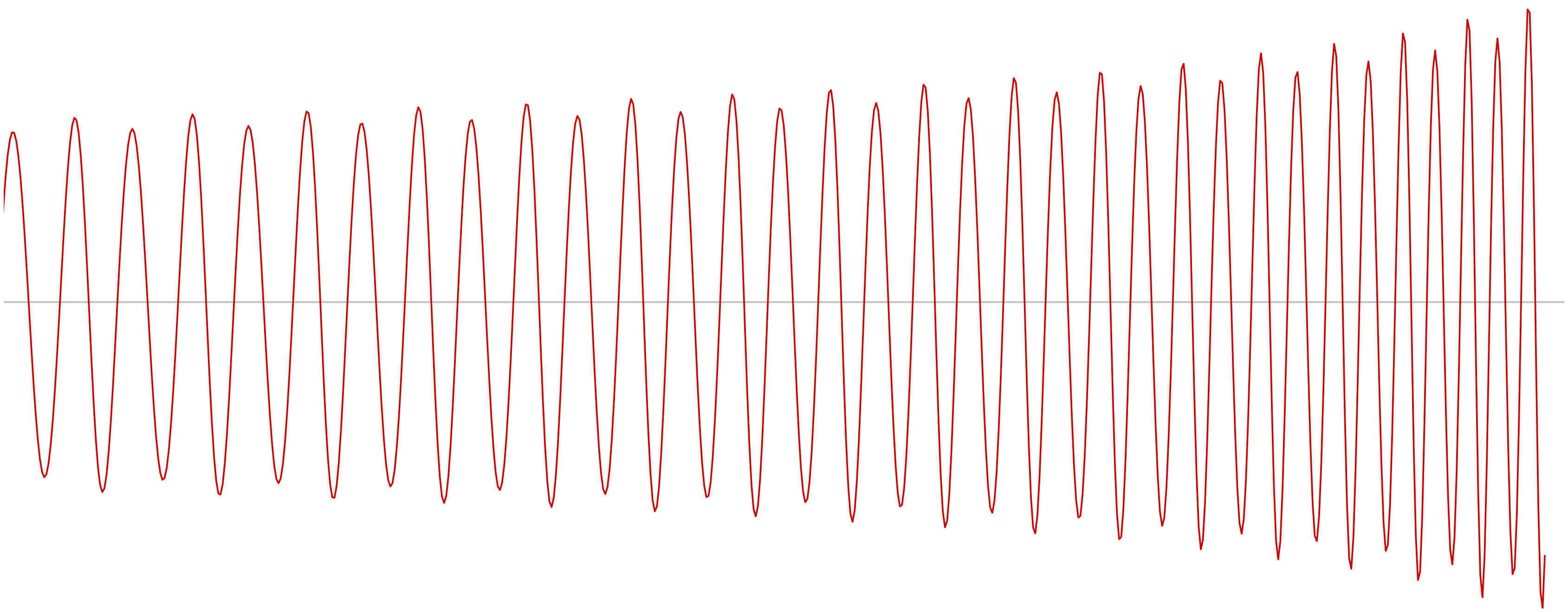
Measuring gravitational waves

- **laser interferometry**
- output: a **time series**
- problems: signal detection, **parameter estimation**, . . .

Binary inspiral events

- **binary star** system, orbiting around their barycentre
- energy is radiated in the form of gravitational waves
- orbits shrink, rotation accelerates
- $\rightarrow$ **“chirping” GW signal**
  (increasing frequency and amplitude)

The "chirp" signal



(3.5PN phase / 2.5PN amplitude approximation)

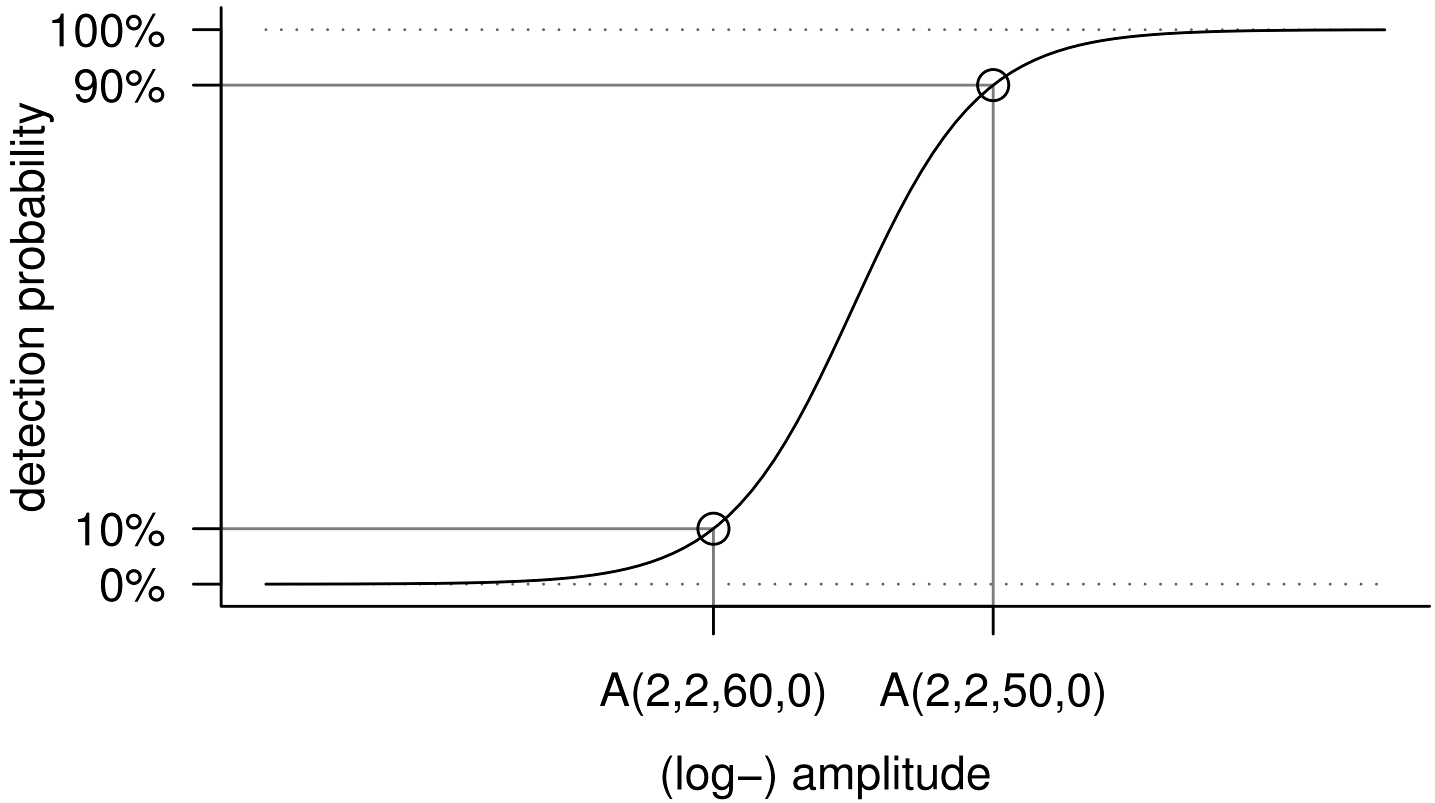
The 9 signal parameters

- **masses**: m_1, m_2
- luminosity **distance**: d_L
- **sky location**: declination δ, right ascension α
- **orientation**: inclination ι, polarisation ψ, coalescence phase ϕ_0
- coalescence **time**: t_c

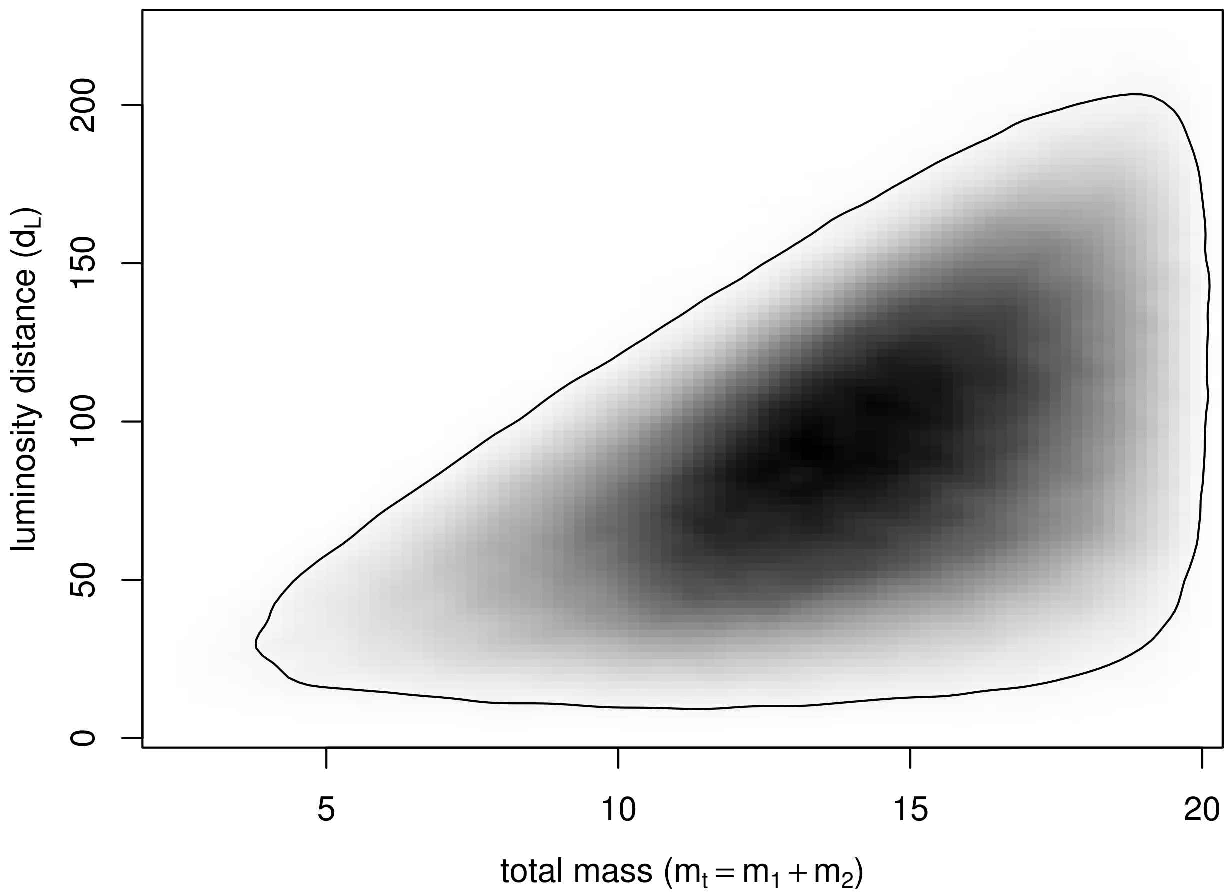
Prior information

- different locations / orientations equally likely
- masses: uniform across $[1\,\mathrm{M}_\odot,\ 10\,\mathrm{M}_\odot]$
- events spread uniformly across space: $\mathrm{P}(d_L \leq x) \propto x^3$
- but: **certain SNR required** for detection
- cheap **SNR substitute**: signal **amplitude** $\mathcal{A}$
- primarily dependent on **masses**, **distance**, **inclination**: $\mathcal{A}(m_1, m_2, d_L, \iota)$

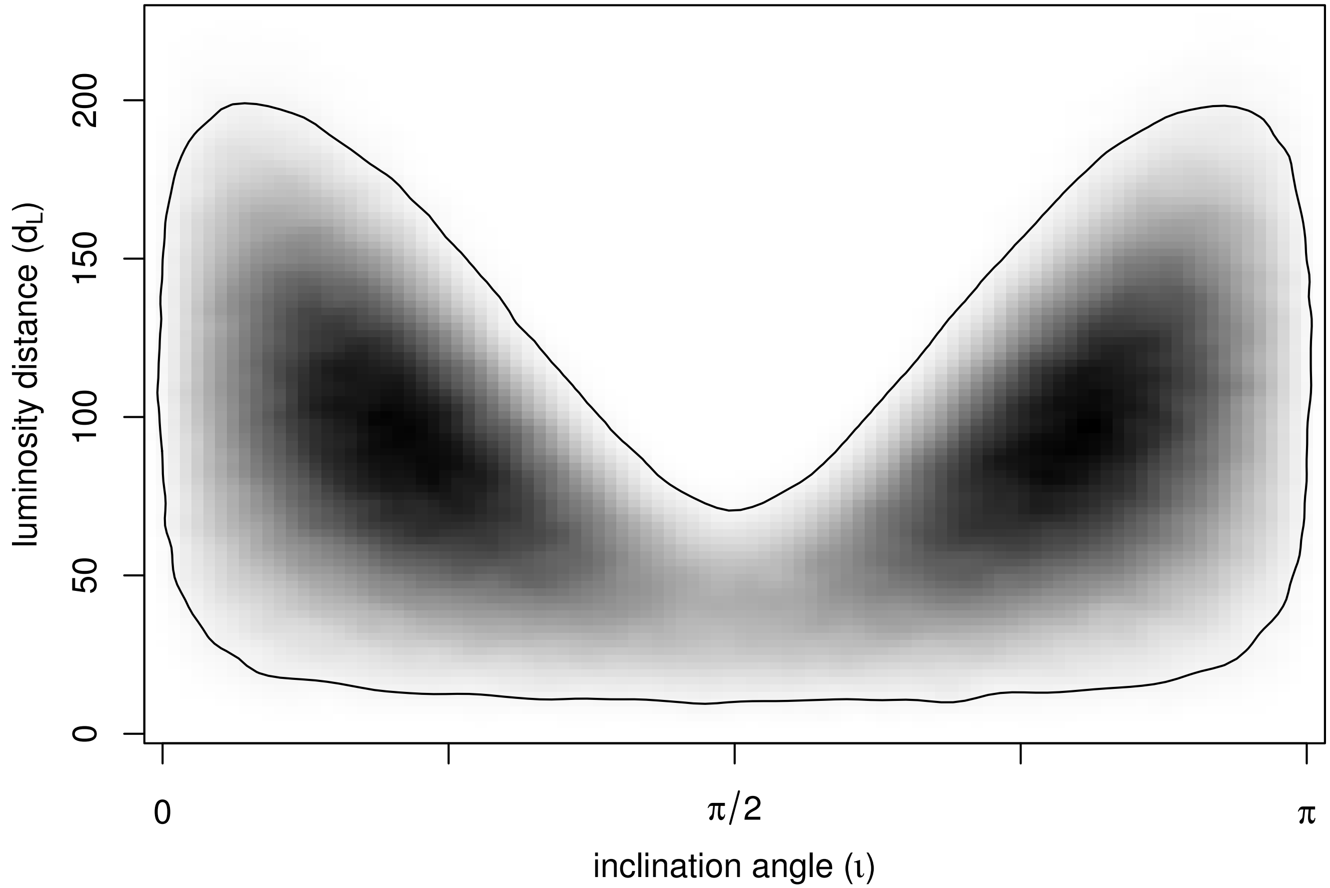
- introduce sigmoid function linking **amplitude** to **detection probability**:



Resulting (marginal) prior density

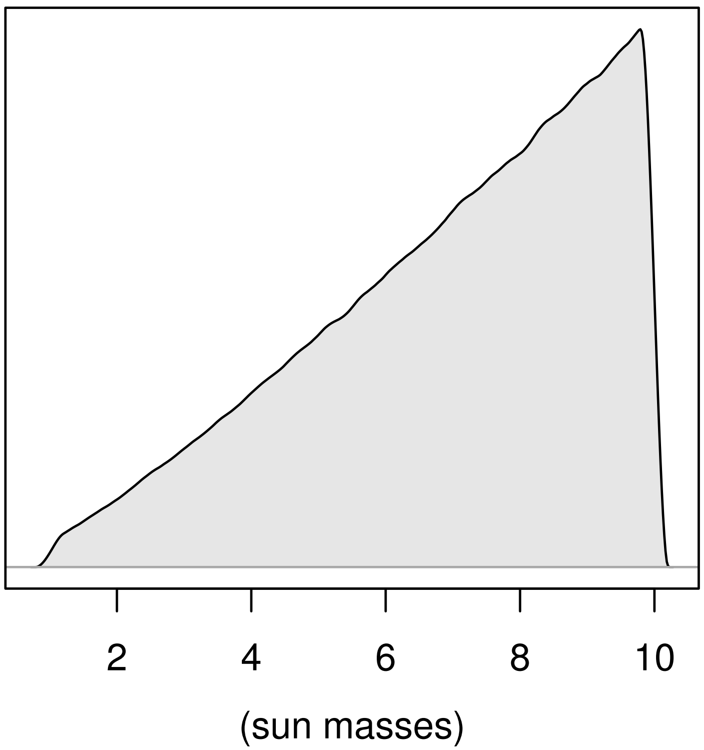


Marginal prior density

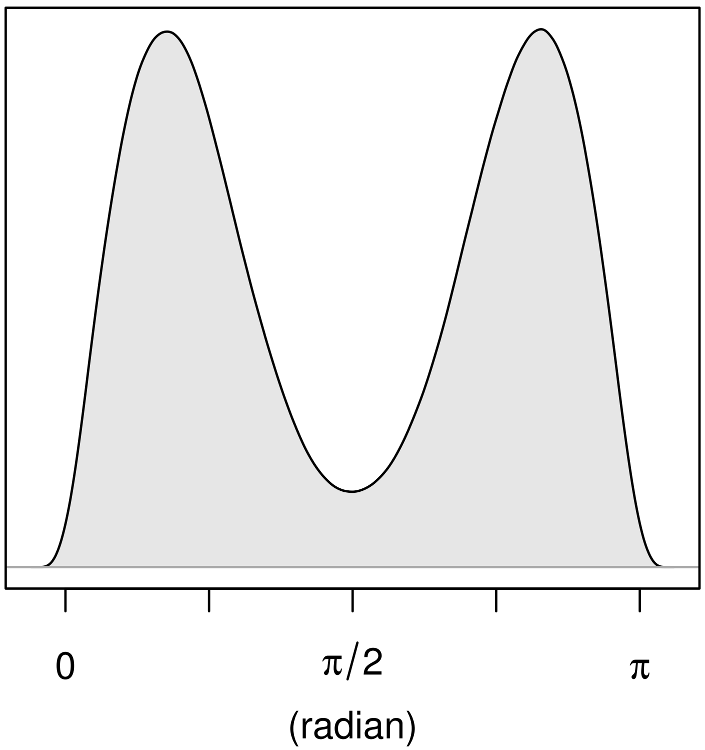


Marginal prior densities

individual masses (m_1, m_2)



inclination angle (ι)



Prior

- prior ‘considers’ **Malmquist effect**
  (selection effect)

- more realistic settings once **detection pipeline** is set up
  (“selection” of signals done by the signal *detection* algorithm)

Model

- data from **several interferometers**
- **noise** assumed **gaussian**, **coloured**; interferometer-specific spectrum
- noise **independent** between interferometers
  $\Rightarrow$ coherent network likelihood is **product** of individual ones
- **likelihood** computation based on Fourier transforms of data and signal

MCMC details

- **Metropolis**-algorithm
- **Reparametrisation**,
  most importantly: **chirp mass** m_c, **mass ratio** η
- **Parallel Tempering**[1]
  several *tempered* MCMC chains running in parallel
  sampling from $p(\theta)\, p(\theta|y)^{\frac{1}{T_i}}$ for 'temperatures' $1 = T_1 \leq T_2 \leq \ldots$

[1] W.R. Gilks et al.: *Markov chain Monte Carlo in practice* (Chapman & Hall / CRC, 1996).

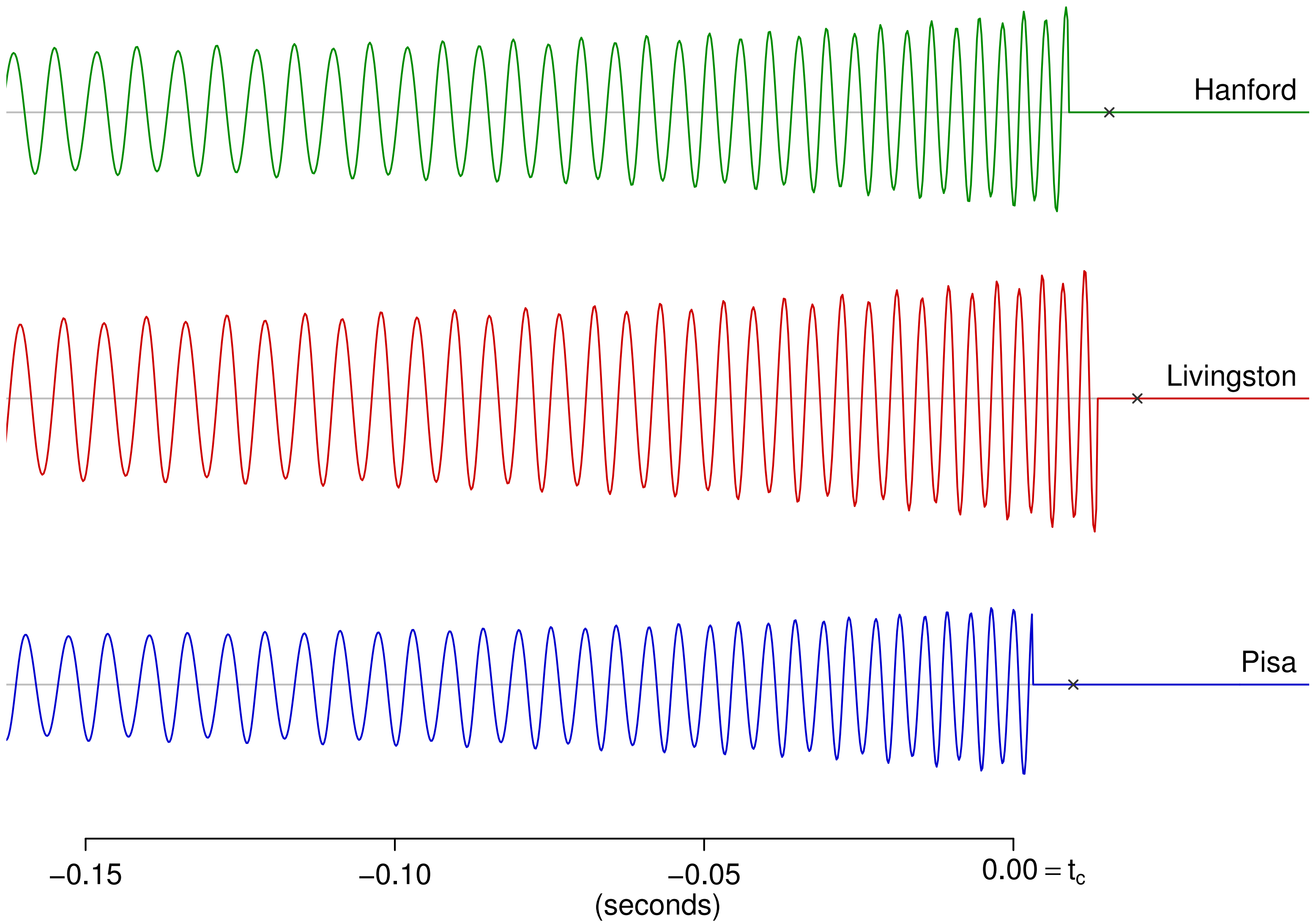
Example application

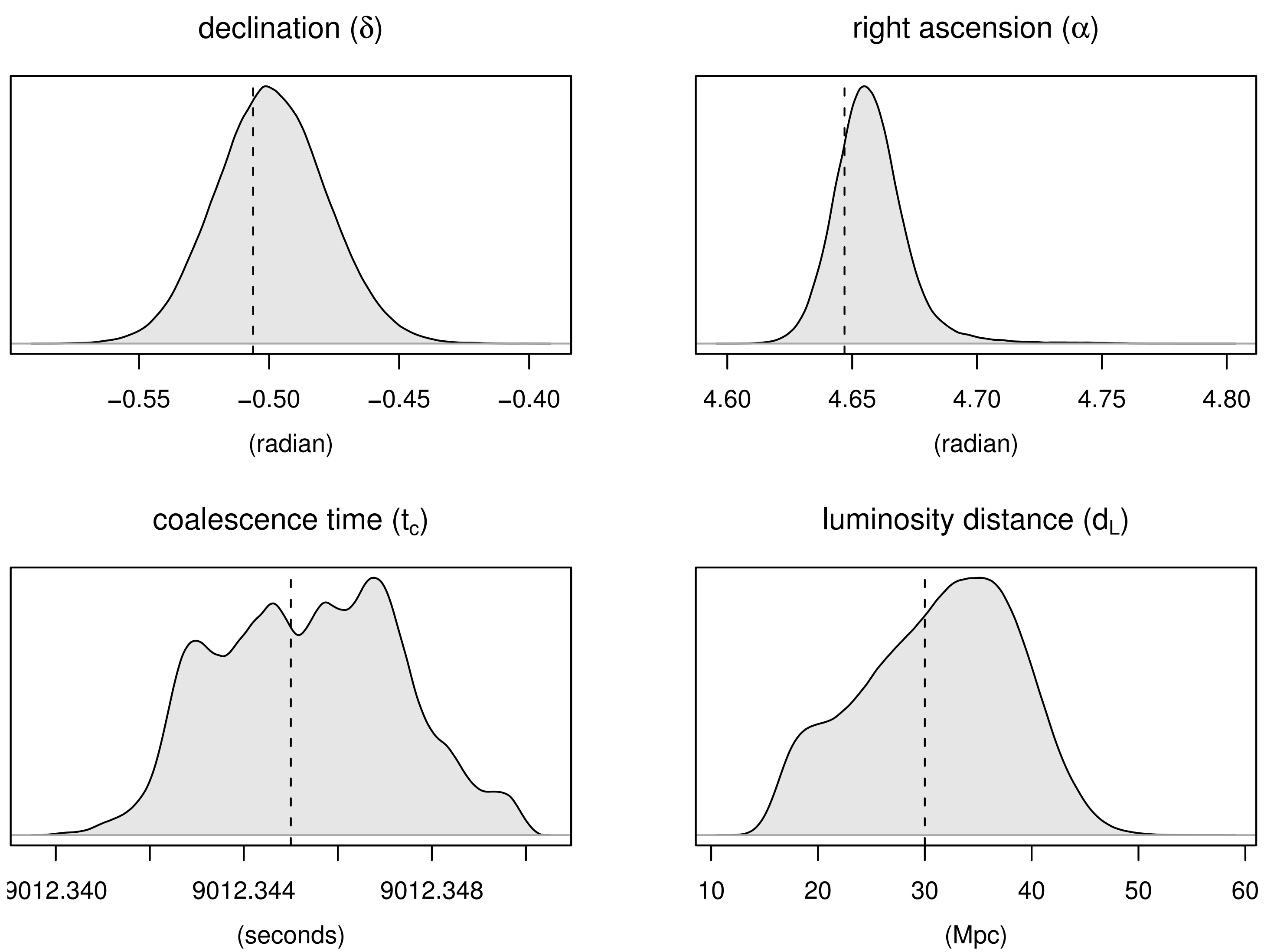
- **simulated data**:
  2 $M_\odot$ - 5 $M_\odot$ inspiral at 30 Mpc distance
  measurements from 3 interferometers:

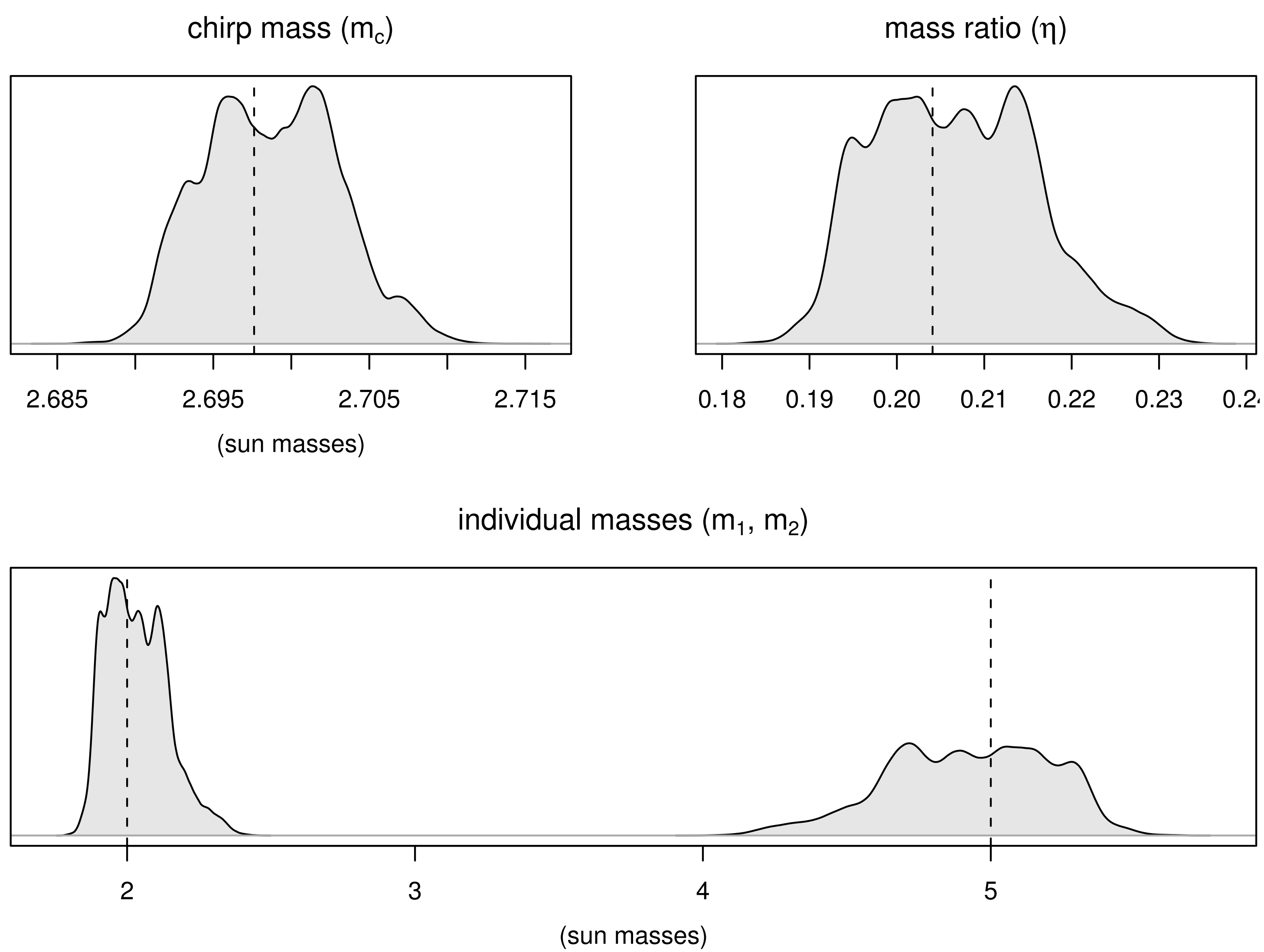
	SNR
LHO (Hanford)	8.4
LLO (Livingston)	10.9
Virgo (Pisa)	6.4
network	15.2

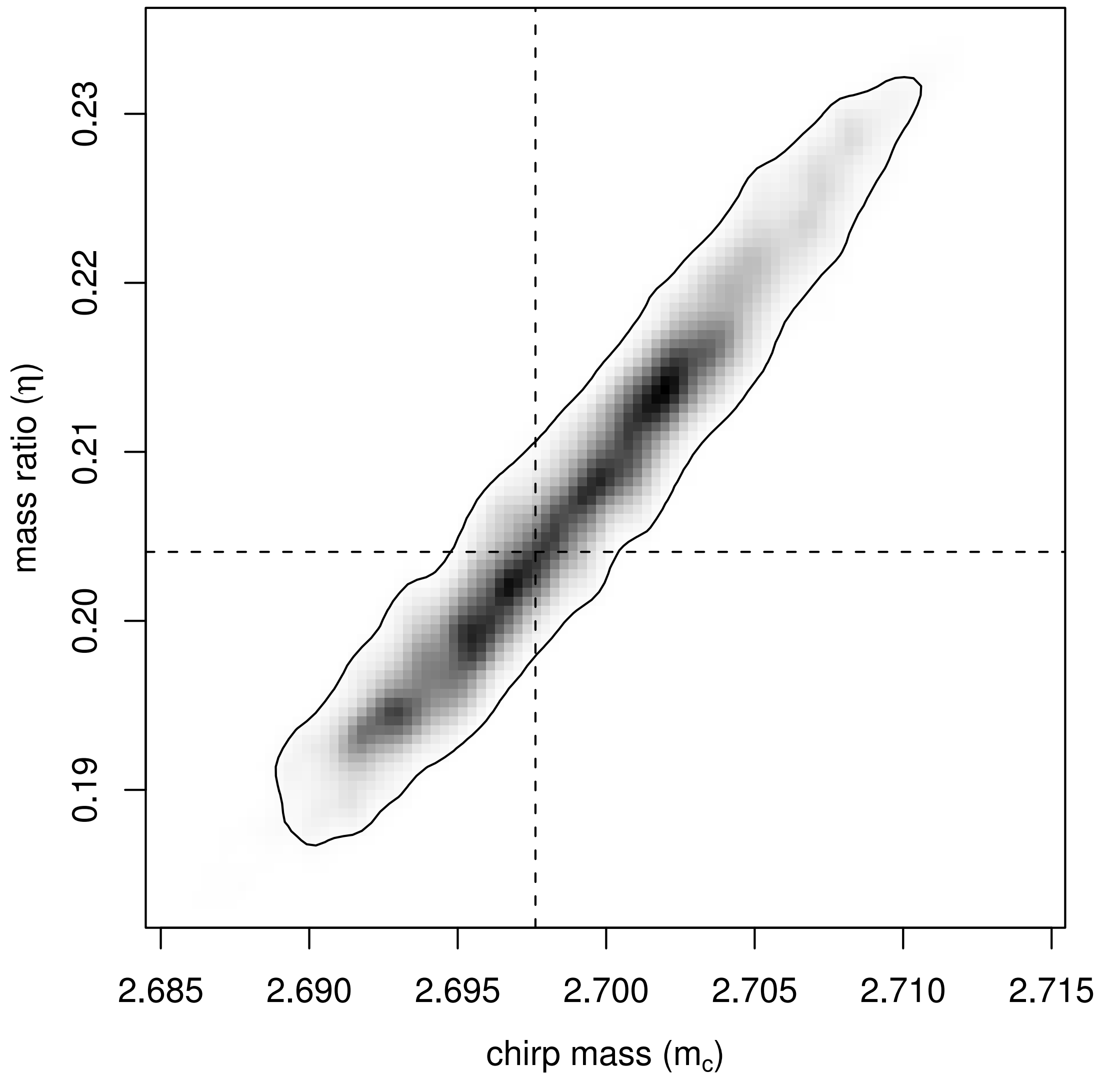
- **data**: 10 seconds (LHO/LLO), 20 seconds (Virgo) before coalescence, **noise** as expected at design sensitivities

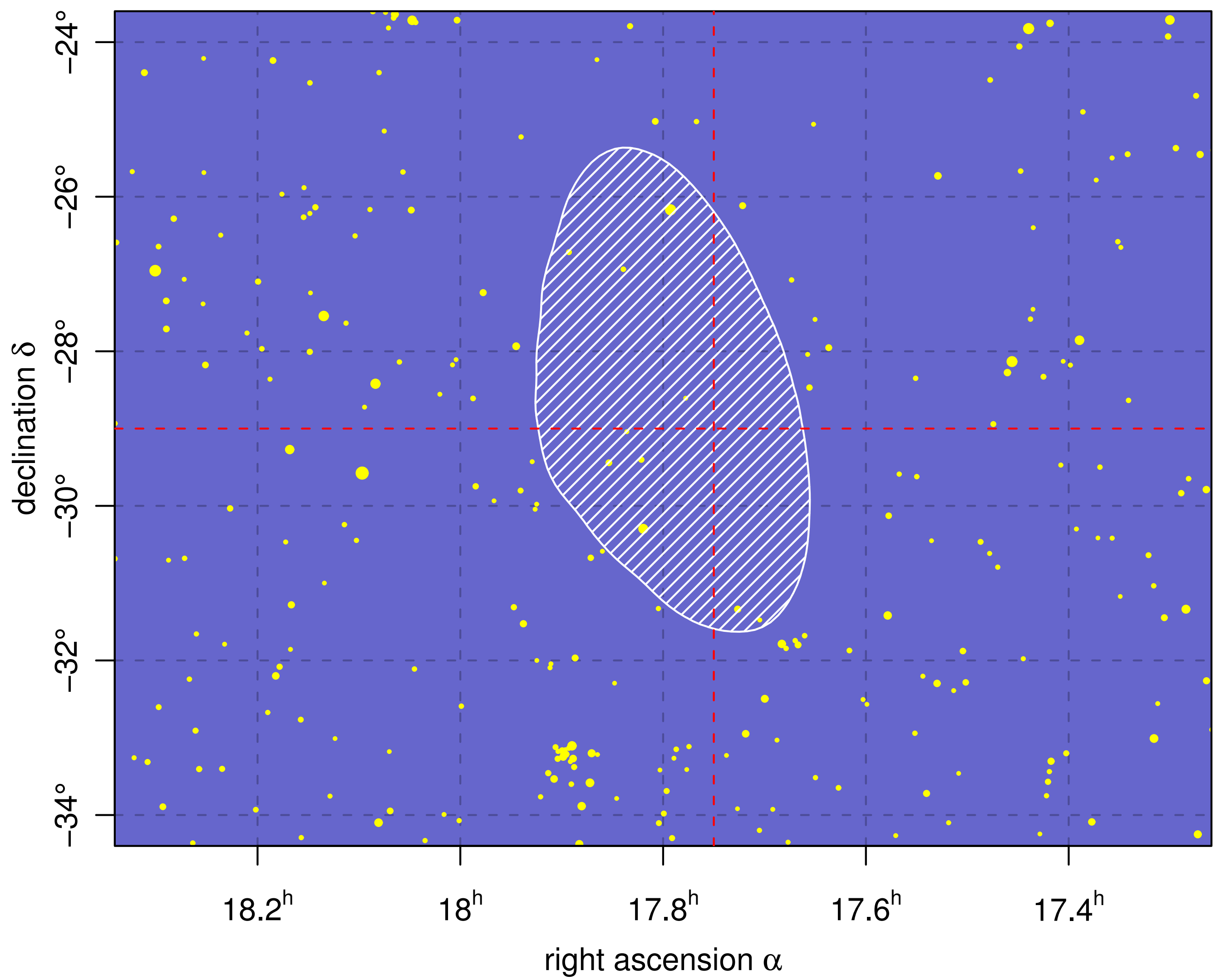
- computation **speed**: 1–2 likelihoods / second


Hanford
Livingston
Pisa
−0.15
−0.10
−0.05
0.00 = tc
(seconds)


declination (δ)
−0.55
−0.50
−0.45
−0.40
(radian)
right ascension (α)
4.60
4.65
4.70
4.75
4.80
(radian)
coalescence time (t_c)
9012.340
9012.344
9012.348
(seconds)
luminosity distance (d_L)
10
20
30
40
50
60
(Mpc)


chirp mass (m_c)
2.685
2.695
2.705
2.715
(sun masses)
mass ratio (η)
0.18
0.19
0.20
0.21
0.22
0.23
individual masses (m_1, m_2)
2
3
4
5
(sun masses)


0.19
0.20
0.21
0.22
0.23
mass ratio (η)
2.685
2.690
2.695
2.700
2.705
2.710
2.715
chirp mass (mc)



−24°
−26°
−28°
−30°
−32°
−34°
declination δ
18.2h
18h
17.8h
17.6h
17.4h
right ascension α

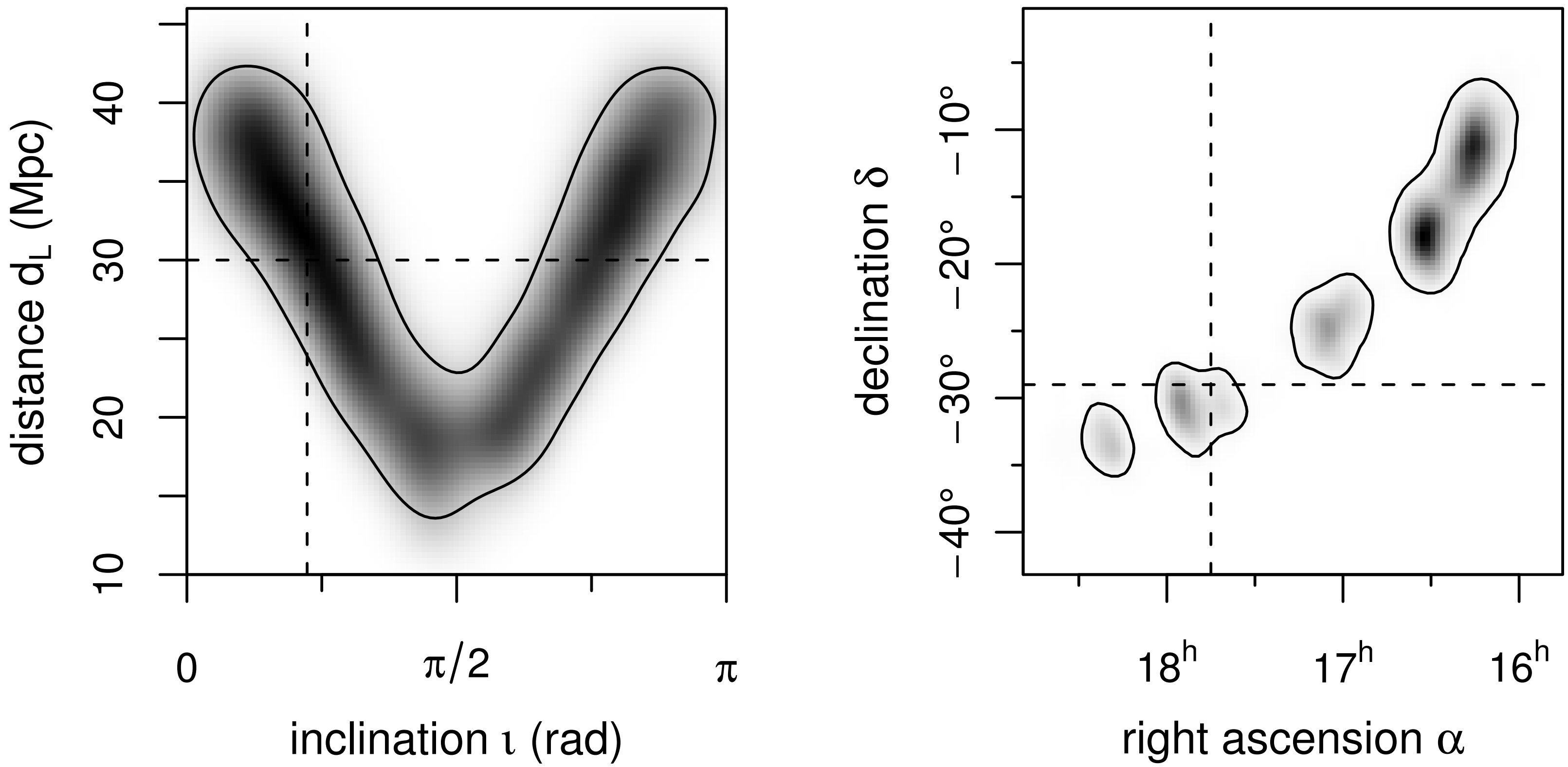
Additional examples

- lower (total) signal-to-noise ratio (SNR)

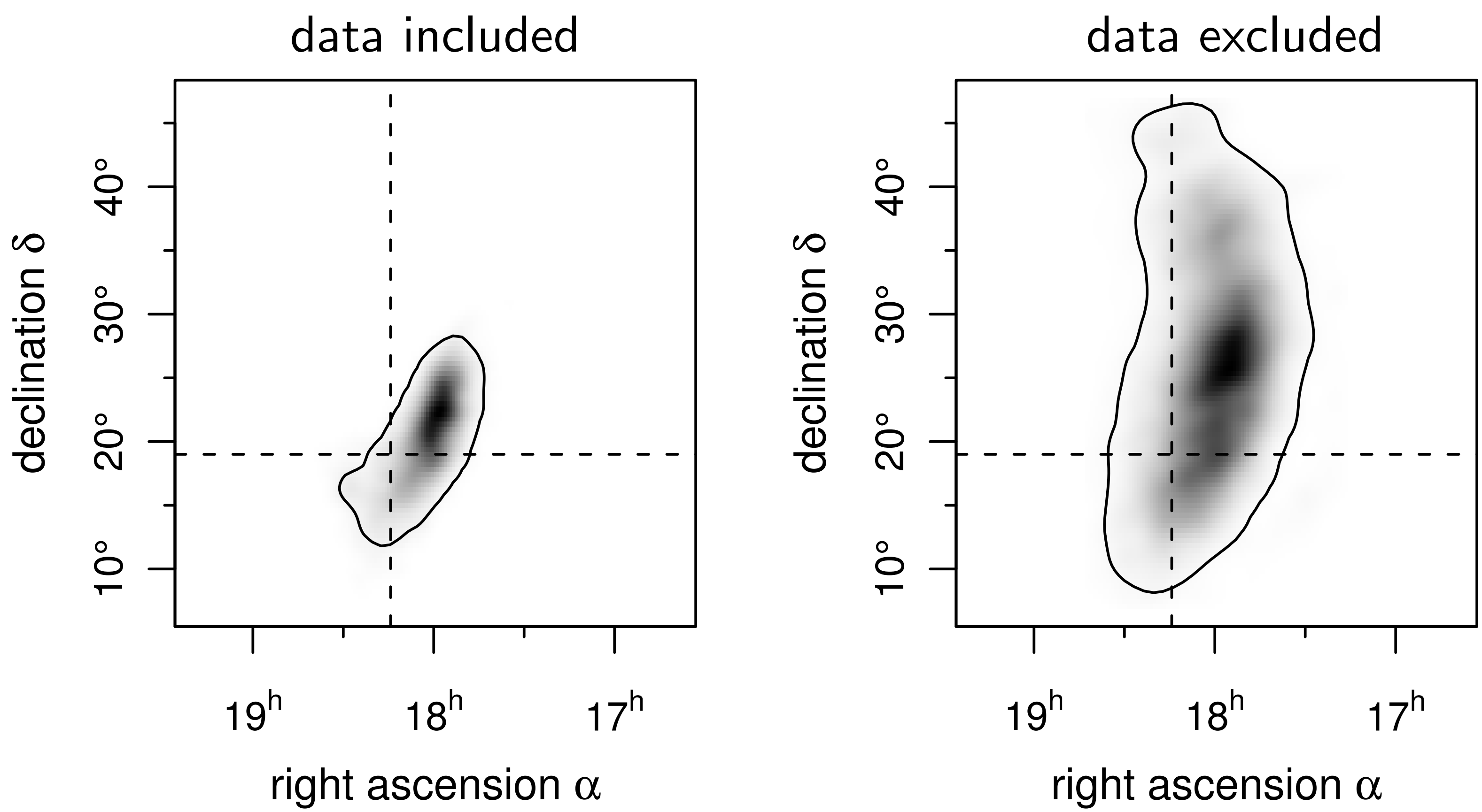
- 'unbalanced' SNR:

	SNR
LHO (Hanford)	9.6
LLO (Livingston)	13.9
Virgo (Pisa)	**0.2**
network	16.9

Low total SNR



Low SNR at one interferometer



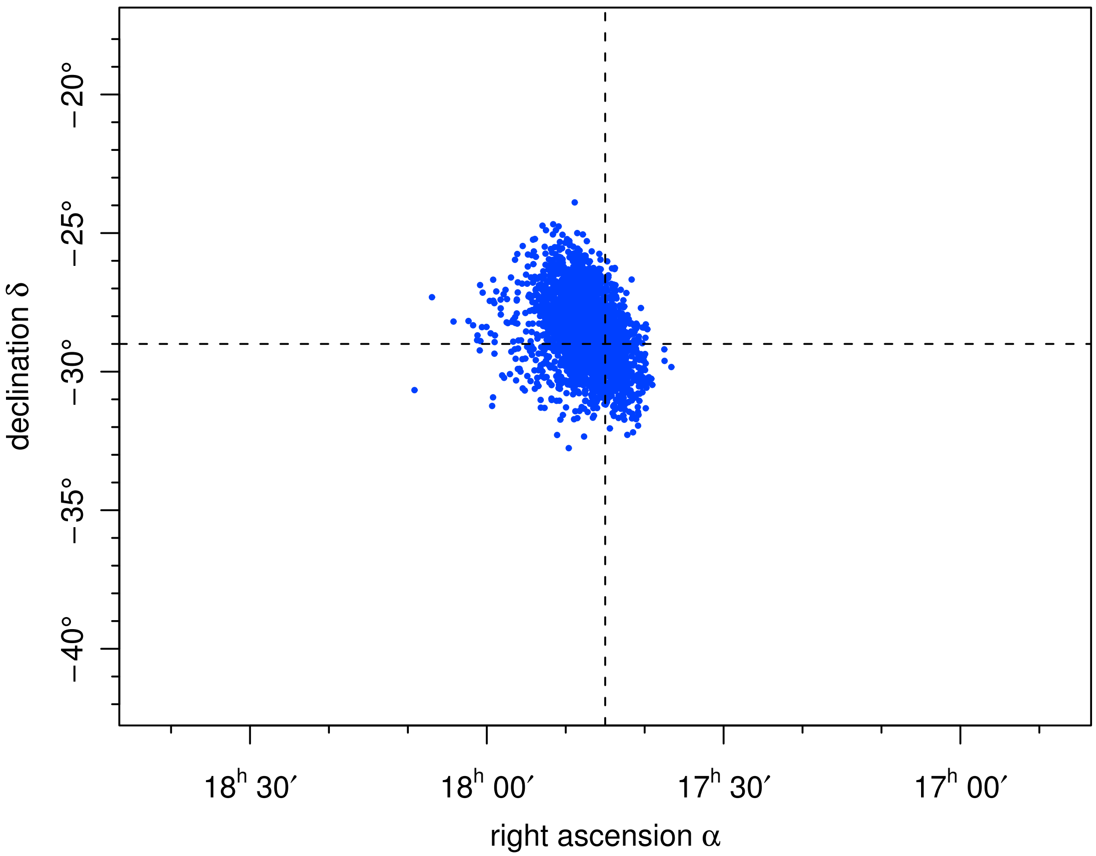
Parallel tempering MCMC

- several **parallel** MCMC chains
- **tempering**: sampling from *tempered* distributions

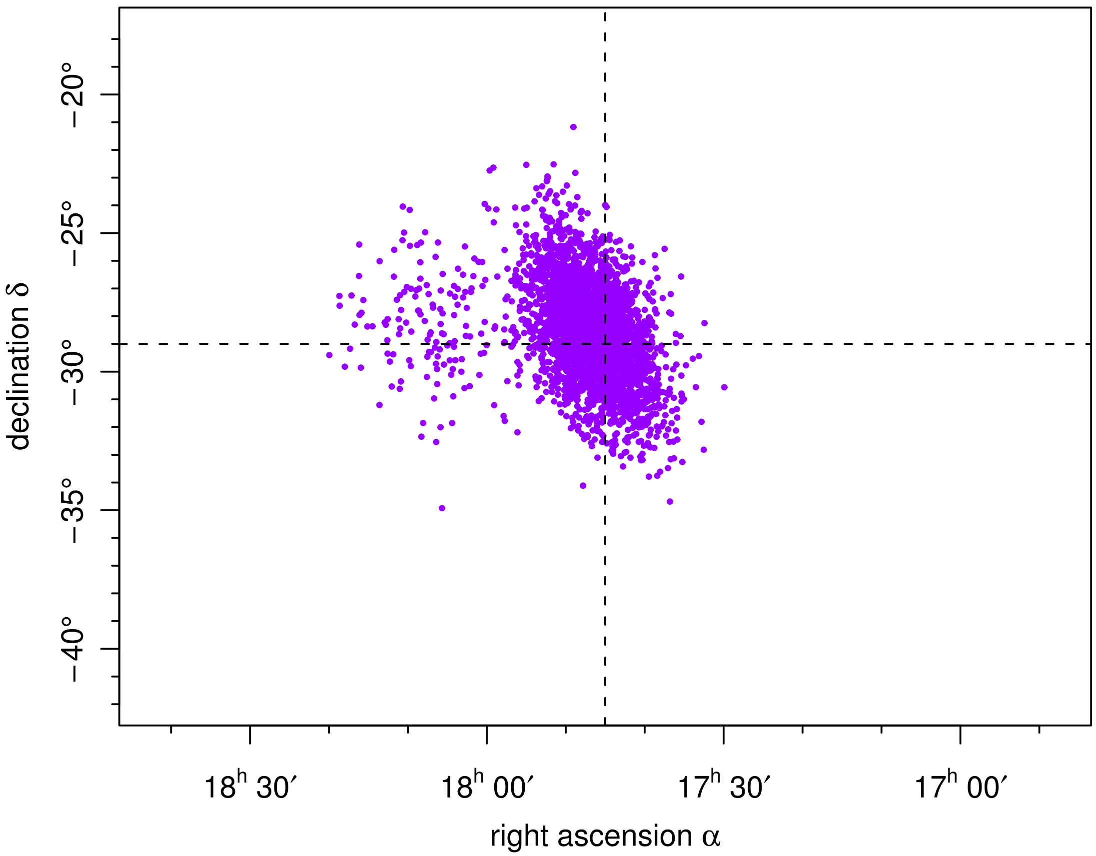
chain	temperature	sampling from
1	$T_1 = 1$	$p(\theta)\, p(y\|\theta)$
2	$T_2 = 2$	$p(\theta)\, p(y\|\theta)^{\frac{1}{2}}$
3	$T_3 = 4$	$p(\theta)\, p(y\|\theta)^{\frac{1}{4}}$
$\vdots$	$\vdots$	$\vdots$
		$p(\theta)$

- additional **swap** proposals between chains

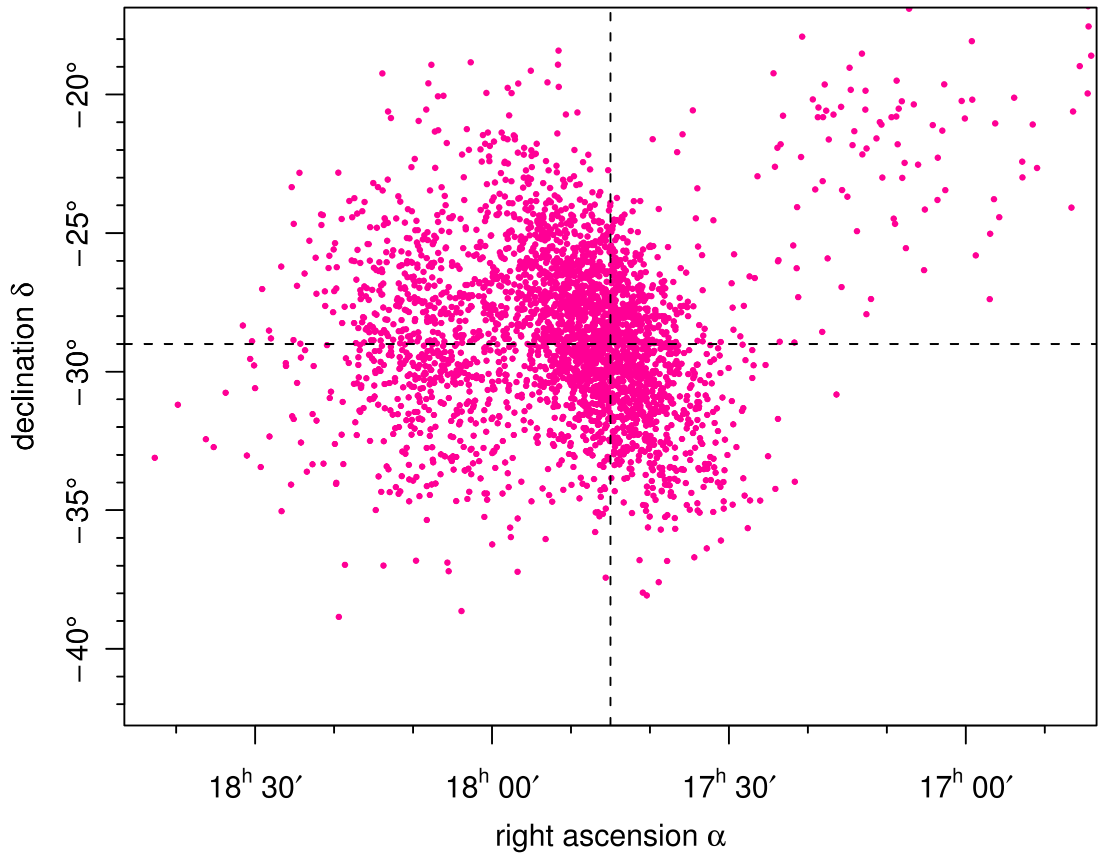
MCMC chain 1 — temperature = 1



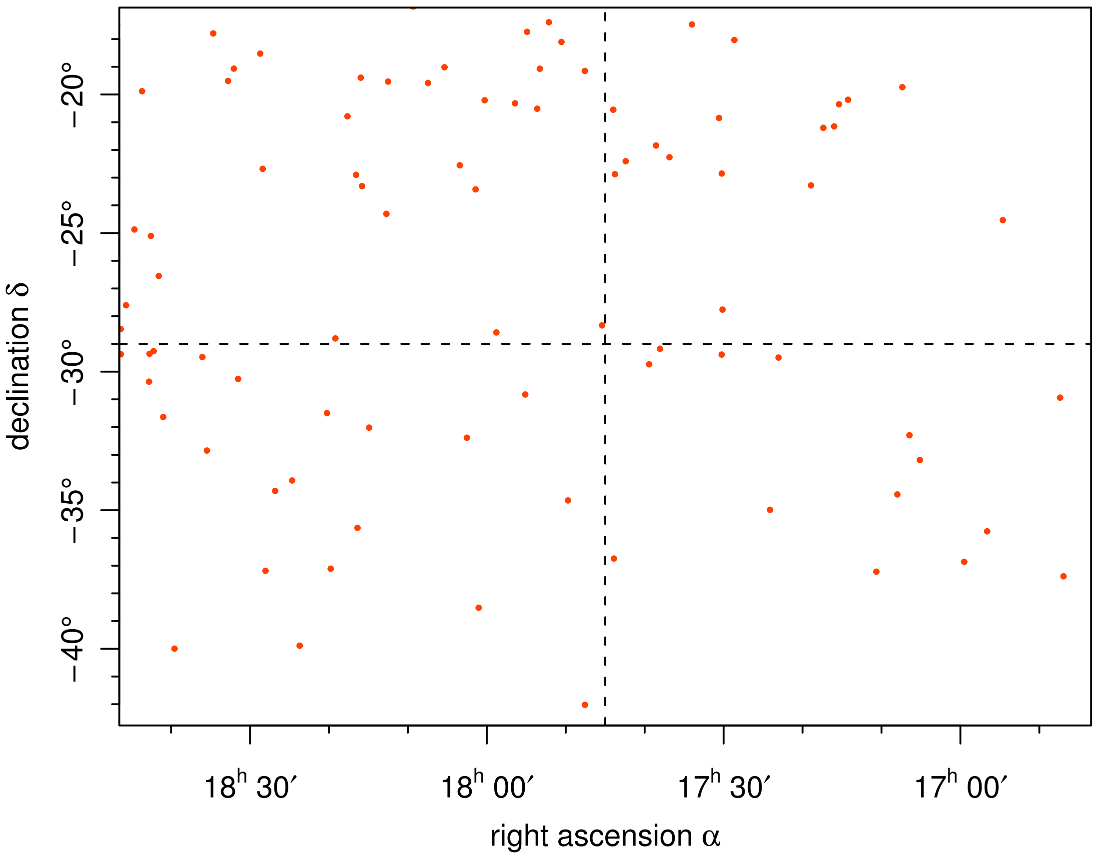
MCMC chain 2 — temperature = 2



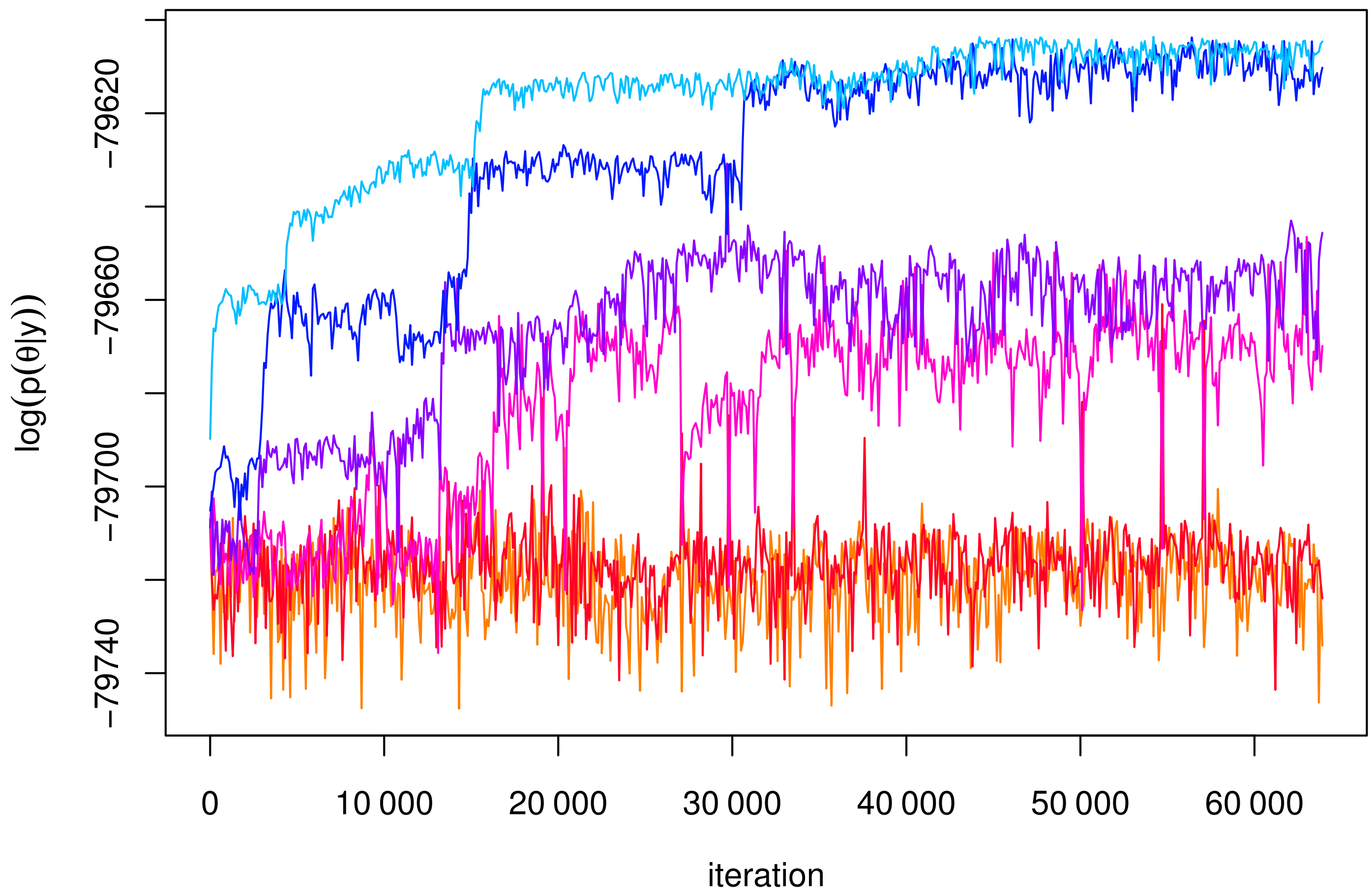
MCMC chain 3 — temperature = 4



MCMC chain 4 — temperature = 8



Six tempered chains over time



C. Röver, R. Meyer, N. Christensen: *Coherent Bayesian inference on compact binary inspirals using a network of interferometric gravitational wave detectors*. Physical Review D, 75(6):062004, March 2007.