Coherent Analysis of Gravitational Wave Chirps and Bursts

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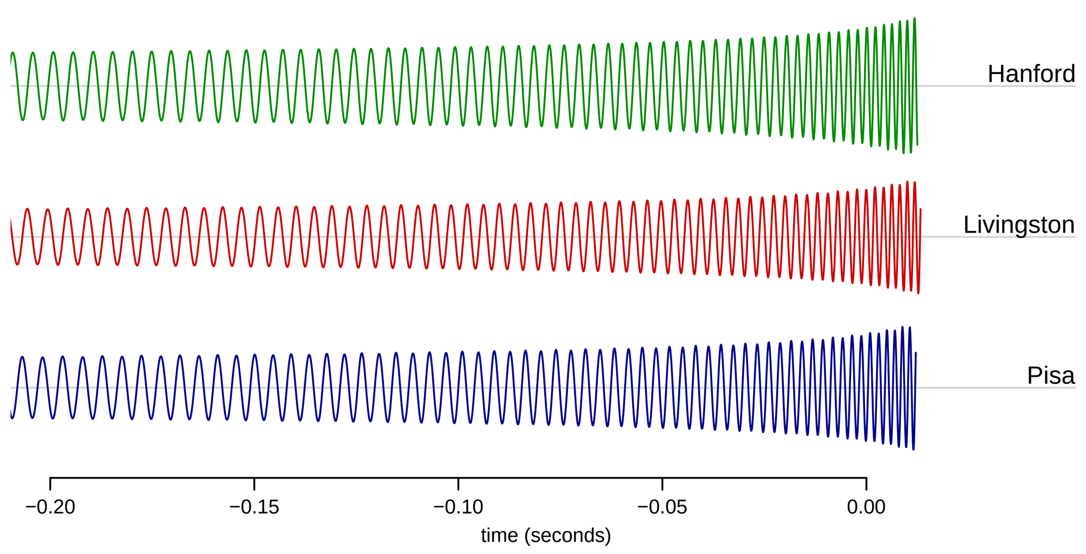
▶ Gravitational waves

General Relativity Theory predicts the existence of gravitational radiation—distortions in space-time that propagate through space at the speed of light. Around the world, laser interferometers are being built and put into operation in order to measure the effect of gravitational waves. Direct measurement of gravitational radiation would not only confirm the General Relativity Theory, but also complement 'traditional' observations in the electromagnetic spectrum [1].
We propose a Bayesian framework to estimate event parameters from gravitational wave measurements.

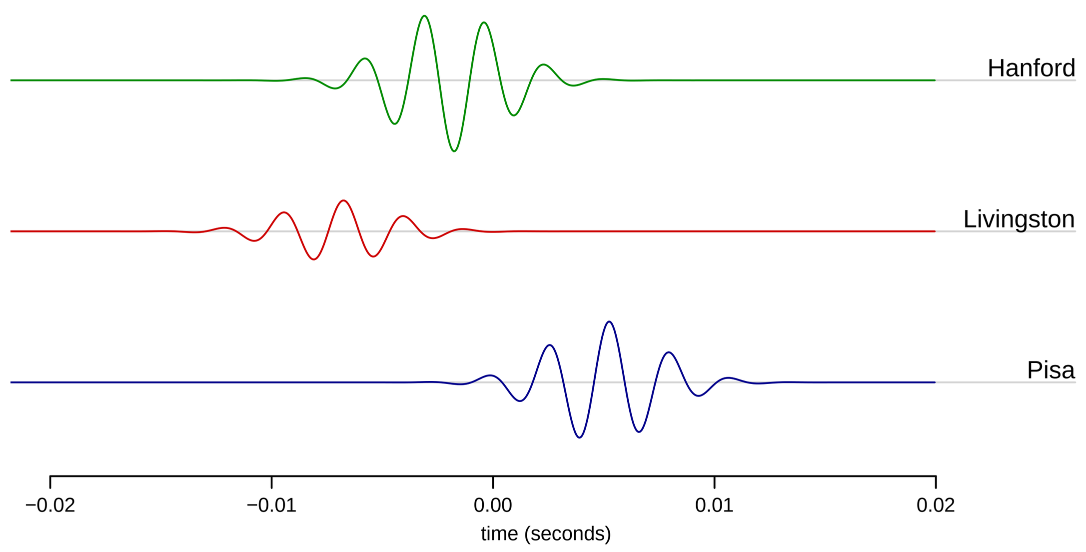
▶ Measuring chirps & bursts

Among the event candidates that are expected to be detected first are *binary inspirals* and *bursts*. Inspiralling binary star systems emit sustained '*chirp*' signals of increasing frequency and amplitude, whose evolution is predicted with great precision [2].



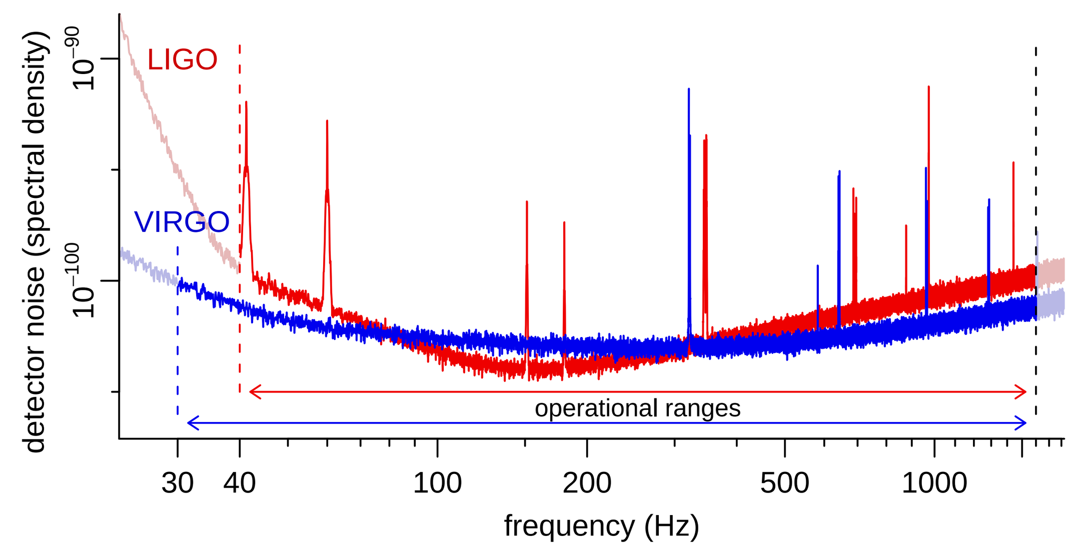
A binary system's last moments before coalescence, measured at different interferometer sites as 'chirps'.

Burst signals are short-duration signals that are expected from different sources, e.g. from supernova events. Their waveforms are not exactly known. For now we use simple (e.g. sine-gaussian) waveforms.



A burst signal, arriving at different interferometers with different time delays and amplitudes.

The gravitational wave measurements are superimposed with interferometer-specific coloured noise. Combining data from several interferometers improves the signal-to-noise ratio, and also makes the estimation of certain parameters (e.g. sky location) possible.



Noise spectra of two different interferometers.

▶ Parameters

The measured detector response depends on the signal waveform as well as mutual orientations of detectors and source (sky location (δ, α) and polarisation angle (ψ)).
A binary inspiral's chirp waveform is determined by the masses of involved objects (m_1, m_2), coalescence time (t_c), distance (d_L), phase (ϕ_0) and inclination (ι); not all of these are of primary concern, the latter two as well as the polarisation are rather nuisance parameters.
Burst waveforms can have a range of parametrisations; the *sine-gaussian* bursts we use are defined by their center (μ), width (σ), amplitude (a), frequency (f) and phase (ϕ_μ).

▶ The coherent likelihood

The individual likelihoods (for measurements from one interferometer I) are computed based on Fourier transforms of data ($\tilde{z}$) and waveform ($\tilde{s}(\vartheta)$), and the detector's noise spectrum:

$$p^{(I)}(z|\vartheta) \propto \exp\left(-\frac{2}{\delta_t}\sum_{i=i_L}^{i_U}\frac{\big|\overbrace{\tilde{z}^{(I)}(i\times\Delta_f)}^{\text{data}}-\overbrace{\tilde{s}^{(I)}(i\times\Delta_f,\vartheta)}^{\text{waveform template}}\big|^2}{\underbrace{S_n^{(I)}(i\times\Delta_f)}_{\text{noise spectrum}}}\right)$$

[3]. Waveform templates $s(\vartheta)$ are generated in the time domain, and then (numerically) Fourier-transformed for each likelihood evaluation.
Assuming that the noise is independent across different interferometers, the coherent network likelihood is the product of the individual detector likelihoods:

$$p(z|\vartheta) = \prod_I p^{(I)}(z|\vartheta).$$

▶ Priors

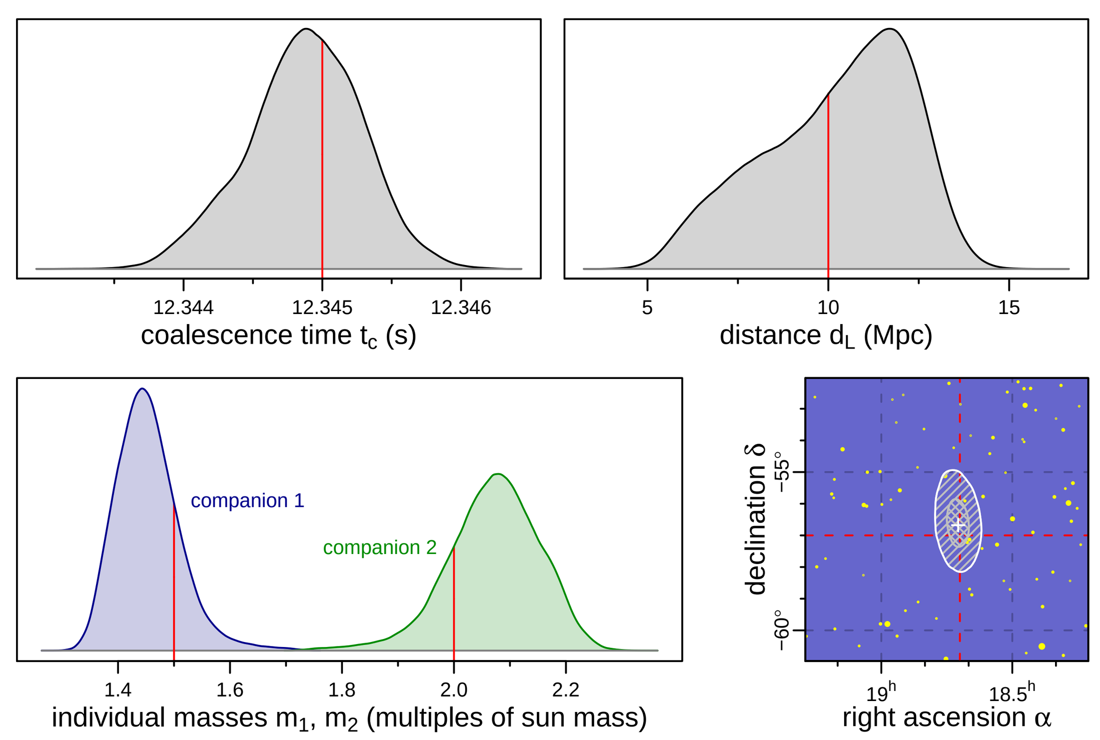
The prior information about parameters is specified (in terms of 'proper' priors) assuming for example that different orientations are equally likely, potential events are evenly spread across time and space, neutron star masses comply with predicted frequencies, and also taking into account the interferometers' sensitivities [4].

▶ MCMCMC Posterior simulation

Integration of the posterior distribution is solved using MCMC simulation. The posterior distribution exhibits many local modes amongst which a 'regular' Metropolis-sampler would have trouble converging towards the global optimum. A Metropolis-Coupled MCMC (MCMCMC) scheme (also known as 'parallel tempering') was implemented. This algorithm runs several chains sampling from 'tempered' likelihoods $p(z|\vartheta)^{\frac{1}{T}}$ with increasing temperatures T in parallel, so only the first chain ($T = 1$) samples from the actual likelihood. Additional steps then allow to randomly switch between chains, improving sampling and convergence [5].

▶ Chirp analysis

In the example below, data was generated to simulate a binary neutron star inspiral event that is measured at three different interferometers. The amount of data was 23, 23 and 44 seconds for the different sites, downsampled to 4–5 kHz (1.6 MB in total).

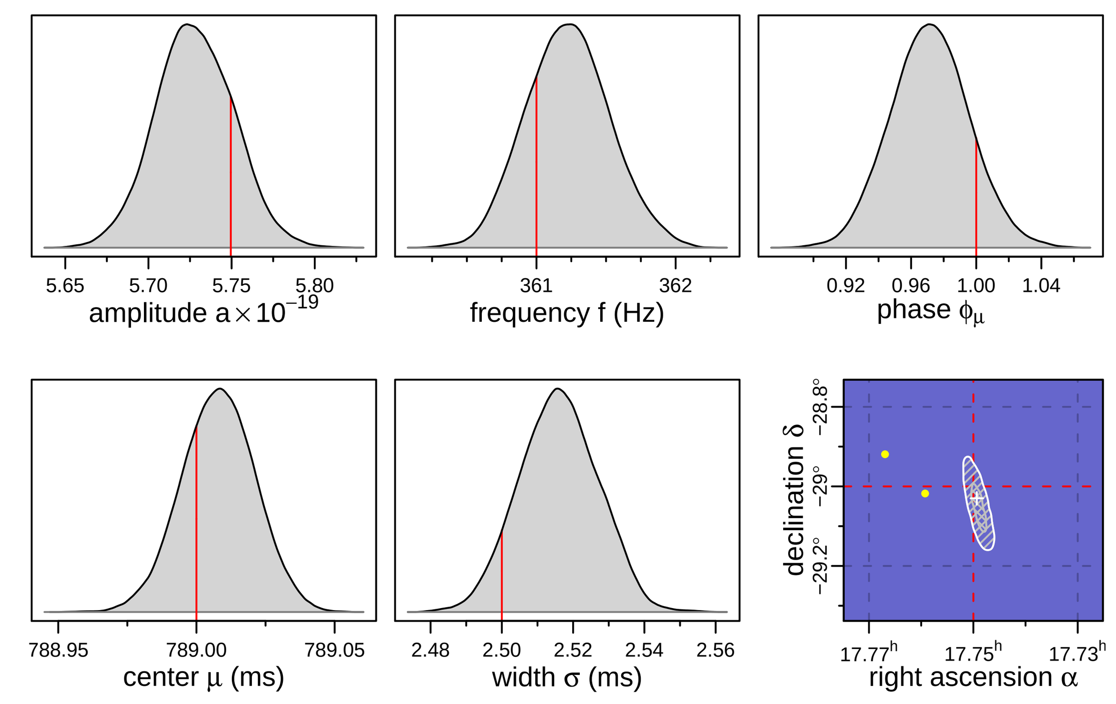


Posterior densities for some of the inspiral parameters, and a 95% posterior region for the sky location. True parameter values are indicated in red.

The parallel tempering algorithm ran 6 parallel chains, generating 80 draws per minute on a 3.2 GHz desktop PC.

▶ Burst analysis

Far less data needs to be considered when analysing burst signals; only 2 seconds for each of 3 interferometers (0.1 MB) were processed in the example below.



Posterior densities for the burst parameters, and a 95% posterior region for the sky location. True parameter values are indicated in red.

Due to the burst posterior's different characteristics, more parallel chains need to be run in order to find the posterior's global mode; here 15 chains were used.

▶ Outlook

We are planning to extend our research to more complex signal waveforms, like e.g. inspirals involving greater masses (black holes), or taking into account the effects of spins the two companions may have.
Eventually, the developed methodology is supposed to be installed at the end of a 'detection pipeline' that monitors the interferometer measurements and triggers the analysis when the presence of a signal is detected.

References

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[5] W. R. Gilks, S. Richardson, and D. J. Spiegelhalter. *Markov chain Monte Carlo in practice*. Chapman & Hall / CRC, Boca Raton, 1996.