STATS 721 Question.

(a) $E[M_{t+1} \mid X_0, \dots X_t] = X_{t+1} + \lambda(t+1)$

$= X_t + E[Y_{t+1}] + \lambda t + \lambda$

$= M_t + \left(\frac{2}{5} - \frac{3}{5}\right) + \lambda$

$= M_t + \lambda - \frac{2}{5}$.

So we need $\underline{\lambda = \frac{1}{5}}$.

(c) The event $\{T = t\}$ is

$$\{X_t = 0\} \cap \bigcap_{s=0}^{t-1} \{X_s \neq 0\}$$

which depends only on $X_0, \dots X_t$.

(b) $E[M_{500}] = E(M_0)$ by martingale property

$= E[X_0] = 100$.

(d) Optional stopping theorem: we have $E[T] < \infty$, and

$|M_{t+1} - M_t| = |Y_{t+1} + \lambda| \leq \frac{6}{5}$ bounded.

By the theorem, $E[M_T] = E[M_0] = 100$.

$E[X_T + \lambda T] = 100$

or since $X_T \equiv 0$, $E[T] = \frac{100}{\lambda} = \underline{500}$