STATS 721	Test	Semester 2, 2008

Answer **ALL QUESTIONS**. Marks are shown for each question.

Write your **name and ID number** at the top of your answer sheet.

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1. We wish to find the probability of getting a run of at least 3 consecutive 0s or 1s in 15 tosses of a fair coin.

   Set up a Markov chain similar to the one given in 325 Assignment 2, Question 3. Create a difference equation, and solve it to find the required probability. Also give a general formula for the probability of finding 3 consecutive 0s or 1s in t tosses of a fair coin. (10)

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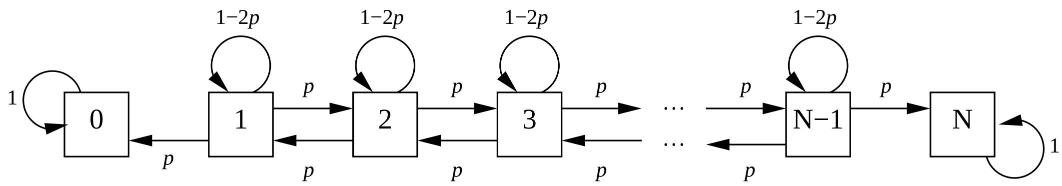
2. Consider a game following the transition diagram below. We say that the game is *won* if it finishes in state N. Let $S_0, S_1, S_2, \ldots$ be the process defined such that S_t is the state at time t, and suppose we start with $S_0 = x$. Assume that $p < 0.5$.



   (a) Choose a suitable martingale with respect to $S_0, S_1, S_2, \ldots$. Define the stopping time T to be the time at which the game finishes, either win or lose. Using the optional stopping theorem for bounded martingales, show that

   $$w = \mathbb{P}(\text{game is won}) = \frac{x}{N},$$

   where $x = S_0$ is the start state. Your answer should include a proof that you have chosen a valid martingale. You may assume that $\mathbb{P}(T < \infty) = 1$. (5)

   (b) Define a new process $M_0, M_1, M_2, \ldots$, such that

   $$M_t = S_t^2 - 2pt.$$

   Show that $M_0, M_1, M_2, \ldots$ is a martingale with respect to $S_0, S_1, S_2, \ldots$. (4)

   (c) Recall that that $\mathbb{P}(T < \infty) = 1$, and $S_0 = x$. Using the optional stopping theorem for bounded martingales, together with a suitable choice of martingale from (a) or (b), find $\mathbb{E}(T)$. You must clearly state which martingale you are using. (5)

Total: 24