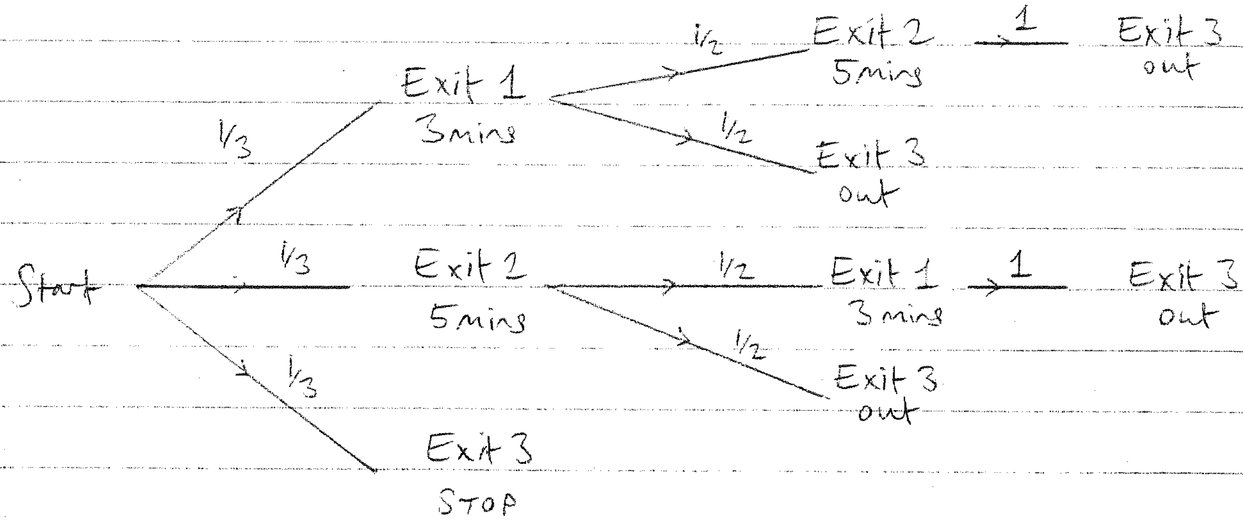
STATS 325 Mock Test Solutions

1a) The lecturer cannot take the same exit twice, so this is not a "loop" style question. Use probability tree idea.



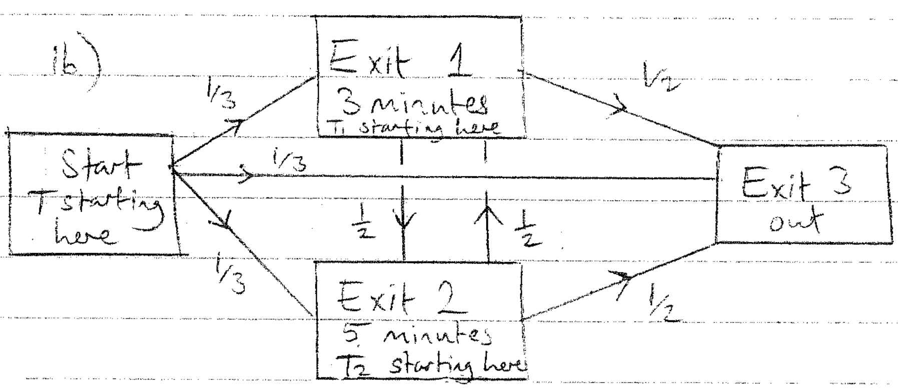
Reading off the tree, and conditioning on the first choice of exit:

$$\mathbb{E}(T) = \frac{1}{3}\left\{ \mathbb{E}(T \mid \text{Exit } 1) + \mathbb{E}(T \mid \text{Exit } 2) + \mathbb{E}(T \mid \text{Exit } 3) \right\}$$

$$= \frac{1}{3}\left\{ \left(3 + \tfrac{1}{2} * 5\right) + \left(5 + \tfrac{1}{2} * 3\right) + 0 \right\}$$

$$\underline{\underline{\mathbb{E}(T) = 4 \text{ minutes}}}$$

1b)



From the diagram, $\mathbb{E}(T_1) = 3 + \frac{1}{2}\, \mathbb{E}T_2 + \frac{1}{2}\, 0$

time lost in Exit 1   go to Exit 2   go to Exit 3

so $\underline{\underline{\mathbb{E}(T_1) = 3 + \frac{1}{2}\mathbb{E}(T_2)}}$ as stated.

Similarly

$$\underline{\underline{\mathbb{E}(T_2) = 5 + \frac{1}{2}\mathbb{E}(T_1)}}$$

1c) $\mathbb{E}(T_1) = 3 + \frac{1}{2}\mathbb{E}(T_2)$ ①
$\mathbb{E}(T_2) = 5 + \frac{1}{2}\mathbb{E}(T_1)$ ②

Substitute ② in ①: $\mathbb{E}(T_1) = 3 + \frac{1}{2}\left(5 + \frac{1}{2}\mathbb{E}(T_1)\right)$

$$\mathbb{E}(T_1)\left(1 - \frac{1}{4}\right) = 3 + \frac{5}{2}$$

$$\Rightarrow \quad \underline{\underline{\mathbb{E}(T_1) = \frac{22}{3} \quad (7.33) \text{ minutes.}}}$$

In ②, $\mathbb{E}(T_2) = 5 + \frac{1}{2}\left(\frac{22}{3}\right)$

$$\underline{\underline{\mathbb{E}(T_2) = \frac{26}{3} \quad (8.67) \text{ minutes}}}$$

d) From the diagram in (b), $\mathbb{E}(T) = \frac{1}{3}\left(\mathbb{E}(T_1) + \mathbb{E}(T_2) + 0\right)$
$\underline{\underline{\mathbb{E}(T) = \frac{16}{3} \quad (5.33) \text{ minutes.}}}$

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2) a)

y	0	1	2
$\mathbb{P}(Y=y)$	0.1	0.7	0.2

b) $G(s) = \mathbb{E}(s^Y) = s^0 * 0.1 + s^1 * 0.7 + s^2 * 0.2$

$$\underline{\underline{G(s) = 0.1 + 0.7s + 0.2s^2}}$$

c) $G_2(s) = G(G(s)) = G(0.1 + 0.7s + 0.2s^2)$

$$\Rightarrow \underline{\underline{G_2(s) = 0.1 + 0.7\{0.1 + 0.7s + 0.2s^2\} + 0.2\{0.1 + 0.7s + 0.2s^2\}^2}}$$

d) $\mathbb{P}(Z_2 = 0) = G_2(0) = 0.1 + 0.7\{0.1\} + 0.2\{0.1\}^2$
$\Rightarrow \underline{\underline{\mathbb{P}(Z_2 = 0) = 0.172}}$

e) We have $G_2'(s) = 0.7(0.7 + 0.4s) + 2*0.2\{0.1 + 0.7s + 0.2s^2\}\{0.7 + 0.4s\}$

So $\mathbb{E}(Z_2) = G_2'(1) = 0.7(0.7+0.4) + 0.4\{0.1 + 0.7 + 0.2\}\{0.7 + 0.4\}$
$\underline{\underline{\mathbb{E}(Z_2) = 1.21 \text{ ghosts.}}}$
