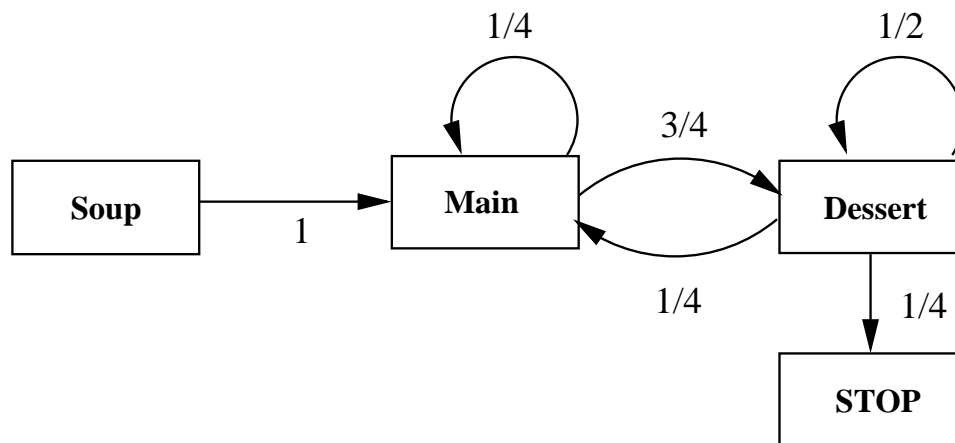


Answer **ALL QUESTIONS**. Marks are shown for each question.

Write your **name and ID number** at the top of your answer sheet.

1. The most remarkable thing about graduate students is their eating habits. To prepare for this test, I took my graduate student to an All-You-Can-Eat buffet and observed his behaviour closely. After paying the \$20 for the buffet meal, customers can help themselves to as many courses as they like.

The transition diagram for my grad student's buffet meal is shown below. Probabilities are shown on the arrows. The student always begins his meal in state **Soup**.



- (a) We are interested in the expected number of courses eaten by the student according to the diagram above. Each box (Soup, Main, or Dessert) counts as one course.

Define the following expectations:

$$\begin{aligned}
 E_S &= \mathbb{E} \{ \# \text{courses starting at entry into state Soup} \} \\
 E_M &= \mathbb{E} \{ \# \text{courses starting at entry into state Main} \} \\
 E_D &= \mathbb{E} \{ \# \text{courses starting at entry into state Dessert} \}.
 \end{aligned}$$

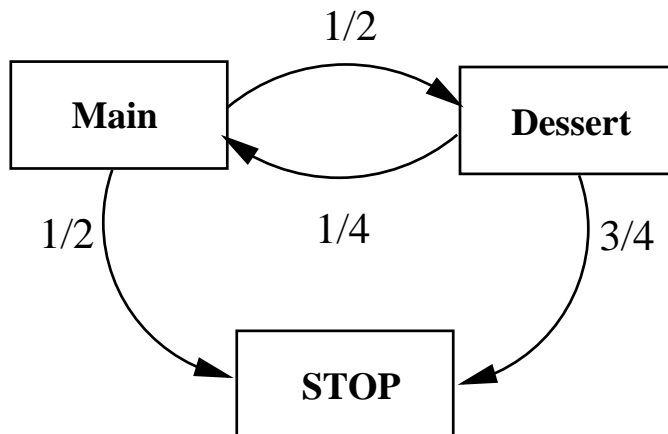
Show that  $E_D = \frac{16}{3} = 5.33$  courses, and find  $E_M$  and  $E_S$ . (8)

- (b) On average, it takes the student 3 minutes to eat a Soup course, and 5 minutes to eat a Main course or a Dessert course. **Without** re-doing the simultaneous equations in part (a), find the expected time, in minutes, required for the student to eat his meal. You may use any quantities already calculated in part (a).

(Hint: the answer requires a little thought but very little working.) (3)

- (c) Using first-step analysis, find the **expected number of Desserts** that the student eats in his meal. You **must** begin your answer by defining a suitable notation and writing it down, in the same format as the notation defined in part (a). (4)

2. After completing graduate studies, people tend to lose interest in Desserts. Below is the lecturer's transition diagram for the same buffet meal. The lecturer always starts the meal in state **Main**.



- (a) Using first-step analysis, find the **probability** that the lecturer's **last course eaten** before finishing is a Main. You must begin your answer by defining a suitable notation and writing it down, in the same format as the notation defined in Question 1(a), but for probabilities rather than expectations.

**Hint:** it might help to re-draw the transition diagram on your answer sheet with two separate 'Stop' states: state  $LM$  in which the last course eaten is a Main, and state  $LD$  in which the last course eaten is a Dessert. (6)

- (b) Using any method you like, find the probability that the lecturer eats more than one dessert:  $\mathbb{P}(\#desserts > 1)$ . (3)

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**Total: 24**