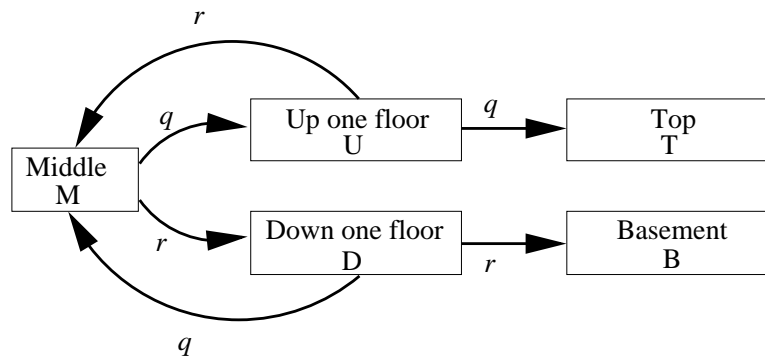


Answer **ALL QUESTIONS**. Marks are shown for each question.

Write your **name and ID number** at the top of your answer sheet.

1. The University of Brokeland is situated in a 5-level building with a faulty lift. Visitors enter the lift at the Middle floor (state M). At each step, the lift will take them up one floor with probability q , and down one floor with probability $r = 1 - q$, completely at random. When the lift reaches the Top floor (T) or the Basement (B), it gets stuck and cannot move any further.



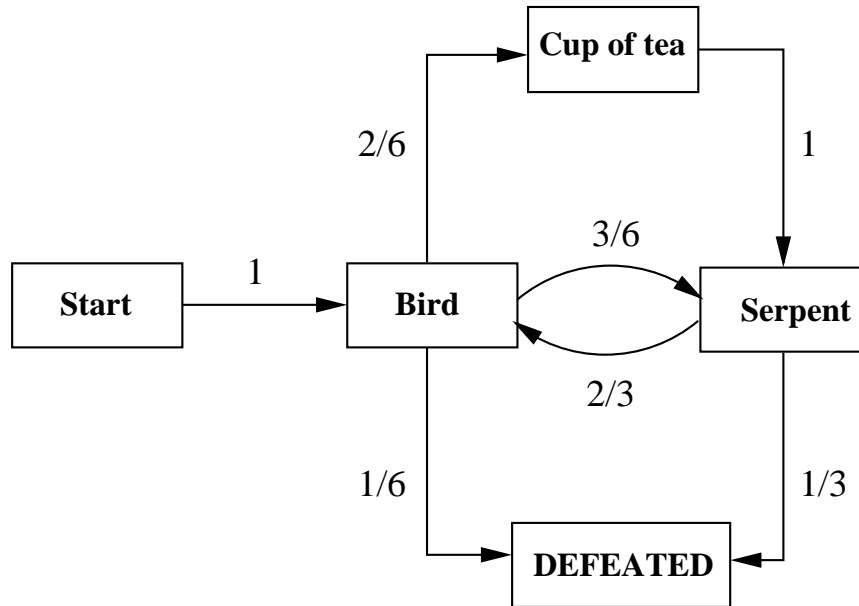
We want to find the probability that the lift eventually gets stuck at the top (state T). Define the following probabilities:

$$\begin{aligned}
 p_M &= \mathbb{P}(\text{lift finishes in state } T \text{ starting from state } M) \\
 p_U &= \mathbb{P}(\text{lift finishes in state } T \text{ starting from state } U) \\
 p_D &= \mathbb{P}(\text{lift finishes in state } T \text{ starting from state } D).
 \end{aligned}$$

Using first-step analysis, show that

$$p_M = \frac{q^2}{1 - 2qr}. \tag{4}$$

2. Remember Hou Yi, the Chinese adventurer from Assignment 1. Hou Yi sets out to conquer the Wind Bird and the Water Serpent again, this time with the transition diagram below. Occasionally he has time for a cup of tea in between battles with the Bird and the Serpent. Unfortunately, Hou Yi is certain to get defeated eventually (state **Defeated**). Nonetheless, his wife wonders how many cups of tea he will need before his defeat.



- (a) Using first-step analysis, find the **expected number of cups of tea** that Hou Yi will drink before he gets defeated, starting from the beginning. You **must** begin your answer by writing down a suitable notation for the expectations you will need. Marks will be awarded for defining your notation correctly. (6)
- (b) Using first-step analysis, find the probability that Hou Yi has **no** cups of tea before he is defeated, starting from the beginning. (4)
- (c) Hou Yi has a battle every time he goes through state Bird or state Serpent on the diagram above. Define random variables N and M as follows:
- N is the number of battles Hou Yi will have before defeat, starting upon entry into state Bird.
 - M is the number of battles Hou Yi will have before defeat, starting upon entry into state Serpent.

Define the probability generating functions of N and M to be:

$$G(s) = \mathbb{E}(s^N)$$

$$H(s) = \mathbb{E}(s^M)$$

Show that

$$G(s) = \frac{3s + 5s^2}{18 - 10s^2},$$

and find a similar expression for $H(s)$. (6)

Total: 20