STATS 325 SC **Term Test** **Semester 2, 2009**

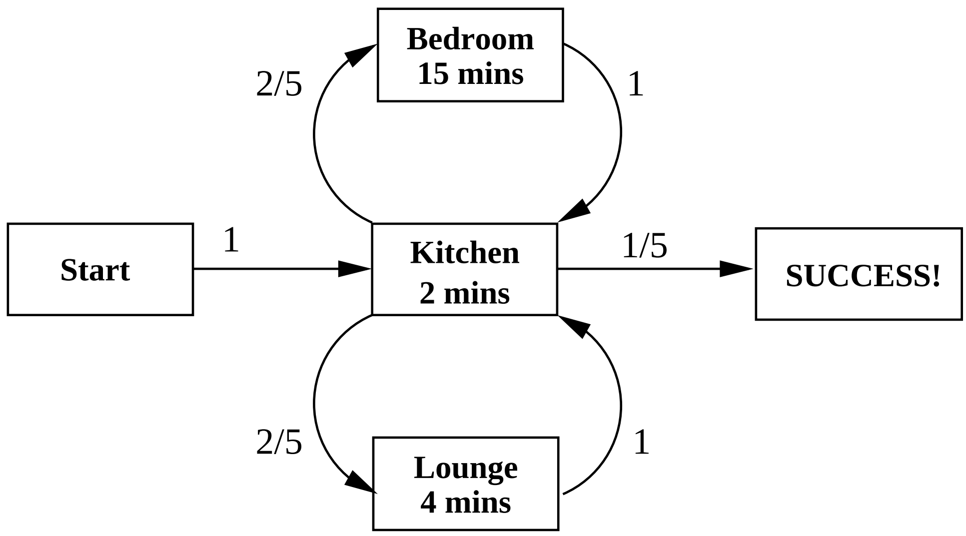
Answer **ALL QUESTIONS**. Marks are shown for each question.

Write your **name and ID number** at the top of your answer sheet.

A **Formula Sheet** with information about probability generating functions is overleaf.

Mrs Figg needs to give some medicine to her cat each day. Unfortunately, the cat doesn't know what's good for him, and he always tries to escape.

They start in the kitchen. After 2 minutes of struggling, either Mrs Figg will be successful, or the cat will escape to the bedroom or the lounge with equal probability. If he escapes to the bedroom, it will take Mrs Figg 15 minutes to coax him out from under the bed and return him to the kitchen to try again. If he escapes to the lounge, it will take her 4 minutes to return him to the kitchen to try again. The transition diagram for Mrs Figg's daily drama is below.



(a) We want to find the expected time taken for Mrs Figg to successfully give the medicine to the cat, starting from the beginning. Define the following expectation:

$$m_K = \mathbb{E}\,(\text{time to finish, starting upon entry to the kitchen})\ .$$

Use first-step analysis to find m_K. (4)

(b) Using first-step analysis, find the **expected number of times** that Mrs Figg and the cat will visit the **bedroom**, starting from the beginning. You must begin your answer by writing down a suitable notation for the expectation(s) you need. Marks will be awarded for defining your notation appropriately. (4)

(c) The cat makes an **escape** every time he enters the bedroom or lounge. What is the **expected number of escapes** that the cat makes, starting from the beginning? (Hint: you do not need to do much calculation.) (2)

(d) Define the random variable N to be the number of times that Mrs Figg and the cat will **visit the bedroom** before the process ends, starting from the kitchen. Let $G(s) = \mathbb{E}(s^N)$ be the probability generating function of N. Show that

$$G(s) = \frac{1}{3 - 2s}\,.$$

(3)

(e) Using the expression for $G(s)$ given in part (d), name the distribution of N, giving the values of any parameters. Show your working. Hence find the probability that Mrs Figg and the cat **never** visit the bedroom before the process ends. (3)

(f) Mr Figg always wakes up exactly 10 minutes after the process begins. Find the probability that Mrs Figg and the cat are in the bedroom when Mr Figg wakes up. Similarly, find the probability that they are in the lounge, and the probability that the process has already finished, when Mr Figg wakes up. You may use any method that you like. (4)

Total: 20

Formula Sheet: Discrete probability distributions and PGFs:

Distribution	$\mathbb{P}(X = x)$	PGF, $\mathbb{E}(s^X)$
Geometric(p)	pq^x (where $q = 1 - p$), for $x = 0, 1, 2, \ldots$	$\dfrac{p}{1 - qs}$
Binomial(n, p)	$\binom{n}{x} p^x q^{n-x}$ (where $q = 1 - p$), for $x = 0, 1, 2, \ldots, n$.	$(ps + q)^n$
Poisson(λ)	$\dfrac{\lambda^x}{x!} e^{-\lambda}$ for $x = 0, 1, 2, \ldots$	$e^{\lambda(s-1)}$