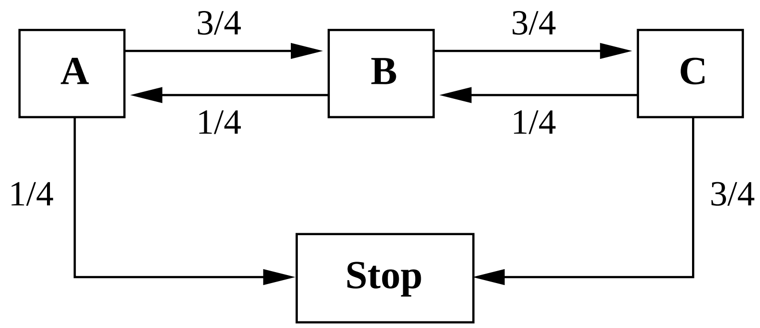
STATS 325 SC	Term Test	Semester 2, 2010

Answer **ALL QUESTIONS**. Marks are shown for each question.
Write your **name and ID number** at the top of your answer sheet.

1. A professor owns a CD with three songs on it, where the songs are A, B, and C. He always starts with song A, and plays the three songs according to the transition diagram below.

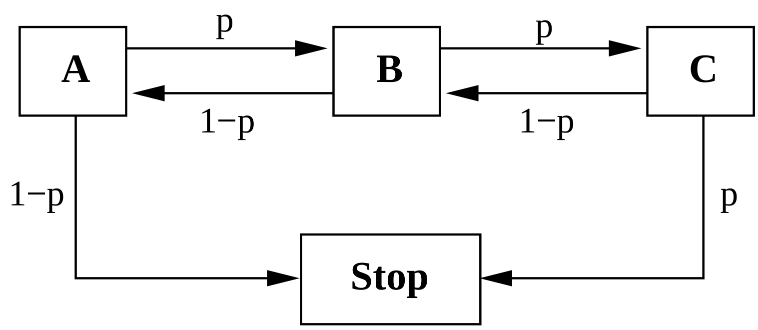


(a) We want to find the **probability** that the **last song played** is song A, starting from the beginning. Define

$$p_A = \mathbb{P}\left(\text{last song played is song } A \,|\, \text{start at song } A\right) ,$$

and write down the appropriate definitions for p_B and p_C. Hence find p_A. (4)

The professor wants a more exciting life, so he changes his transition probabilities to give the new diagram below:



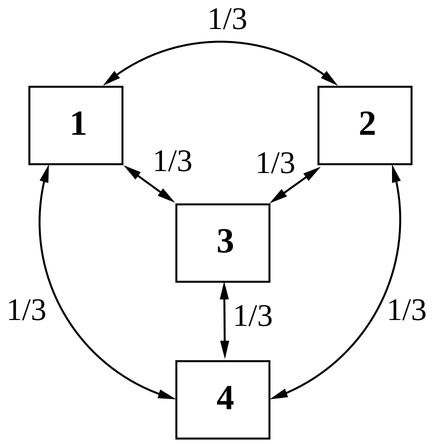
Let N be the **number of times the professor plays song** $\boldsymbol{B}$, starting from the beginning. We want to find $\mathbb{E}(N)$.

(b) Use first-step analysis to show that $\mathbb{E}(N) = \dfrac{p}{1-2p+2p^2}$.

You **must** begin your answer by defining a suitable notation. Marks are awarded for defining your notation clearly. (4)

(c) Using the answer to (b), find two values of p that will make $\mathbb{E}(N) = 1$. For which of these values of p does the random variable N have the larger variance? Briefly explain your answer. (4)

2. A policeman is chasing a robber round a set of four rooms, labelled Room 1, 2, 3, and 4. From each room, the robber will run to any of the other rooms at random with probability 1/3 each, as shown in the diagram below.



At the start, the policeman and the robber are in **different** rooms. The policeman will catch the robber when they are first in the **same** room. The policeman can choose Strategy 1 or Strategy 2 to catch the robber, where:

- In **Strategy 1**, the policeman stays still, and waits for the robber to enter his room;
- In **Strategy 2**, the policeman runs between rooms at random, following the same diagram as the robber, and moving between rooms at the same speed as the robber. Assume the policeman and the robber can not meet in transit between rooms.

(a) For **Strategy 1**, copy the transition diagram below and fill in the arrows and probabilities. Hence find the expected number of steps before the robber is caught under Strategy 1. The number of steps is equal to **the number of arrows traversed** until the state 'Same room' is reached.

(3)

(b) Repeat part (a) for **Strategy 2.** (4)

(c) Which strategy should the policeman use in order to minimize the expected time taken to catch the robber? (1)

Total: 20