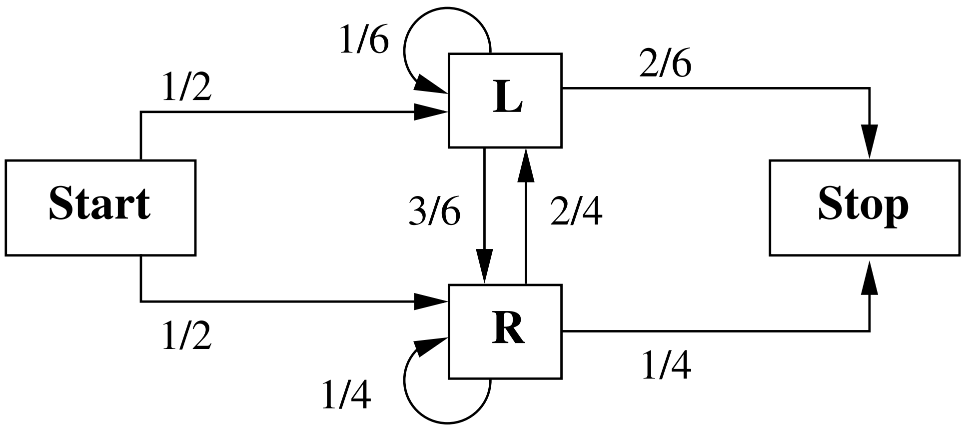
STATS 325 SC **Term Test** **Semester 2, 2011**

Answer **ALL QUESTIONS**. Marks are shown for each question.
Write your **name and ID number** at the top of your answer sheet.
If the wording of a question is not clear, **ask for advice.**

Tennis players use ball-machines to practise against. The machines throw out balls for the players to hit, and they can be programmed to shoot balls to random directions.

Venus has programmed her ball-machine to shoot balls to the left (L) or the right (R) according to the transition diagram below.



The **boxes L and R** correspond to a ball being fired. The arrows correspond to decisions made by the machine. We want to find the **expected number of balls** that the machine fires, starting from the beginning. This corresponds to the **expected number of times that we enter boxes L and R**.

(a) Define

$$m_L = \mathbb{E}\,(\text{number of balls before the end}\,|\,\text{start just before entering box L})\ ,$$

and write down the appropriate definition for m_R. Hence find m_L and m_R. (5)

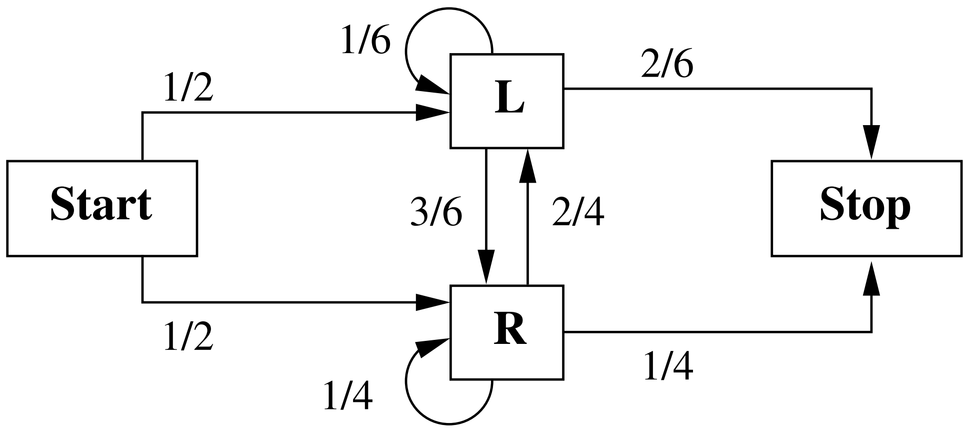
(b) Let N be a random variable giving the total number of balls that the machine fires, starting from state **Start** and finishing at state **Stop**. Using your answer to (a), find $\mathbb{E}(N)$. (1)

(c) Starting from state **Start**, what is the probability of the trajectory L, R, R, Stop? (2)

(d) What is the **probability of getting no Lefts** from the beginning to the end? You must begin your answer by defining a suitable notation. (3)

Continued . . .

The diagram is printed again here to help you:



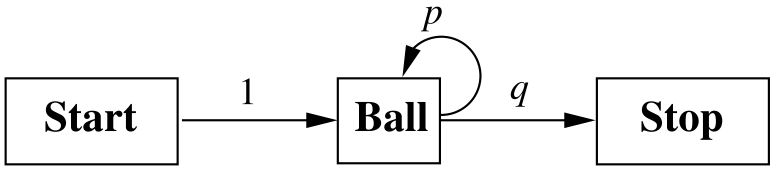
(e) We now want to find the **probability of ever getting two Lefts in a row**, starting from the beginning. To find this probability, you will need to draw a new transition diagram, and define a suitable notation. Marks are awarded for defining your notation clearly.

Draw a transition diagram suitable for this calculation, and find the probability of ever getting two Lefts in a row, starting from state **Start** and finishing at state **Stop**. (6)

(f) Recall that parts (a) and (b) asked for the expected number of balls that the machine fires before the end. We weren't interested in whether the ball was fired to the Left or to the Right; we were just interested in counting the number of balls fired.

Recall that the random variable N is the number of balls fired from the beginning to the end. In part (b) we found $\mathbb{E}(N)$.

Consider the new transition diagram below. It does not distinguish between Left and Right balls: it just has the state **Ball** which means that a ball has been fired in some direction.



We want to know whether it is possible for the new diagram to replace the old diagram. Specifically, we want to know whether it is possible to choose values of p and q such that the random variable N (the number of balls fired) has exactly the same distribution in the new diagram as it does in the old. Say whether it is **true** or **false** that the new diagram can give the same distribution for N as the old diagram.

If you think it is **true**, find the appropriate values of p and q.

If you think it is **false**, give a brief explanation why. (3)

Total: 20