Answer **ALL QUESTIONS**. Marks are shown for each question.
Write your **name and ID number** at the top of your answer sheet.
If the wording of a question is not clear, **ask for advice.**

The Beckham family are Mum, Dad, and Baby Beckham. When they go to the park, Baby Beckham organises them into a line with Baby in the middle and Mum and Dad at either end, and they play a game of passing a ball. Baby passes to Mum and Dad with equal probability. Mum and Dad can pass either to Baby or to each other. Mum is more likely to stop the game than Dad, but Baby will never stop it. The ball always starts with Baby.

The Beckham family transition diagram is below. Each arrow between states M, B, and D corresponds to the ball being passed between Mum, Baby, and Dad. (The two arrows pointing into state Stop do not constitute a pass of the ball.)
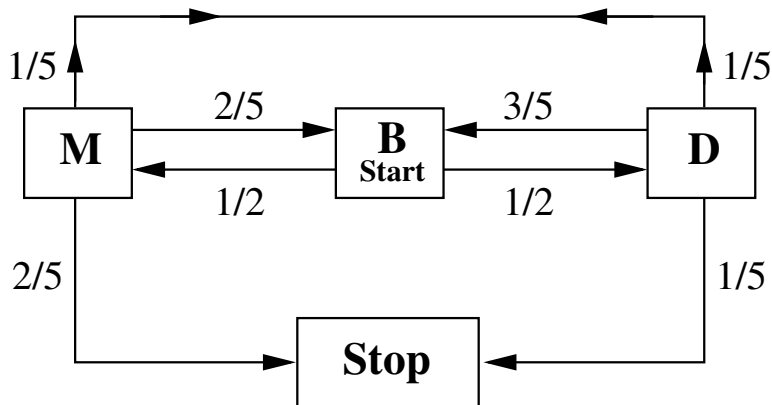
(a) We want to find the **expected number of times that Baby passes the ball** during the game, from start to finish. This corresponds to counting the expected number of arrows **out of the state 'Baby'.**

Define

$$m_B = \mathbb{E}\,(\text{number of times Baby passes the ball} \mid \text{start at state B})\,,$$

and write down any other definitions that you need.

Then write down the equations that you need to solve, and solve them. Finish your answer by stating the expected number of times that Baby passes the ball during the game. (6)

(b) Starting from the beginning, what is the probability of the trajectory B, D, M, Stop? (2)

(c) What is the probability that the game consists of **only one pass**? (2)

Continued . . .

The diagram is printed again here to help you:
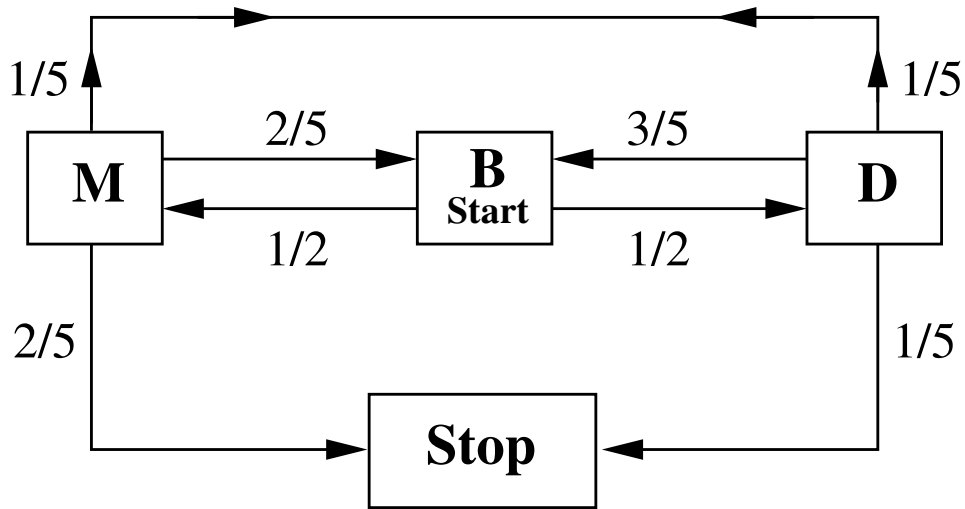
(d) We want to find the **probability that the game is stopped by Mum**. That is, we want the probability that Mum is the last person to hold the ball before the game stops.

  (i) Sketch a new diagram suitable for answering this question.

 (ii) Define a suitable notation for answering this question.

(iii) Write down the equations that you need to answer the question.

    Do not solve the equations. Marks are awarded for formulating your answer, defining your notation, and setting up your equations. (5)

(e) Find the probability that a game **ever includes a pass from Mum to Dad**. That is, find the probability that Mum **ever** passes the ball to Dad before the game stops. Marks are awarded for clearly defining your notation. (5)

Total: **20**