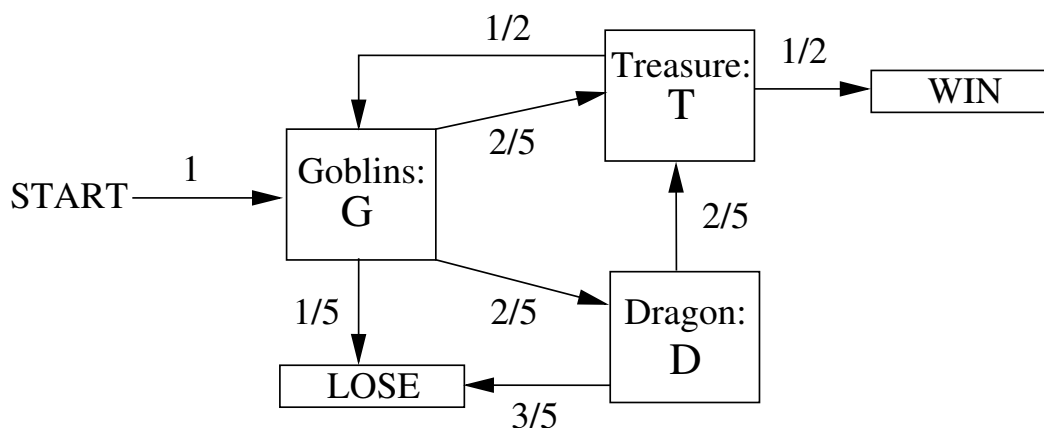


Answer **ALL QUESTIONS**. Marks are shown for each question.
Write your **name and ID number** at the top of your answer sheet.
If the wording of a question is not clear, **ask for advice**.

The Hobbit plans a short adventure following the transition diagram below. His adventure ends in one of two states: **win** or **lose**.



Throughout, we will be working in the following sample space:

$$\Omega = \{ \text{all paths starting at state } G \text{ and finishing at Win or Lose} \}.$$

- (a) We want to find the probability that the Hobbit **wins** the adventure, starting from the beginning. Define

$$p_G = \mathbb{P}(\text{Hobbit wins} \mid \text{start at state } G),$$

and write down any other definitions that you need.

Then write down the equations that you need to solve, and solve them. Finish your answer by stating the probability that the Hobbit's adventure ends in state **Win**. (8)

- (b) Define events D and W as follows:

$$D = \{ \text{Hobbit ever meets the Dragon} \}; \quad W = \{ \text{Hobbit wins eventually} \}.$$

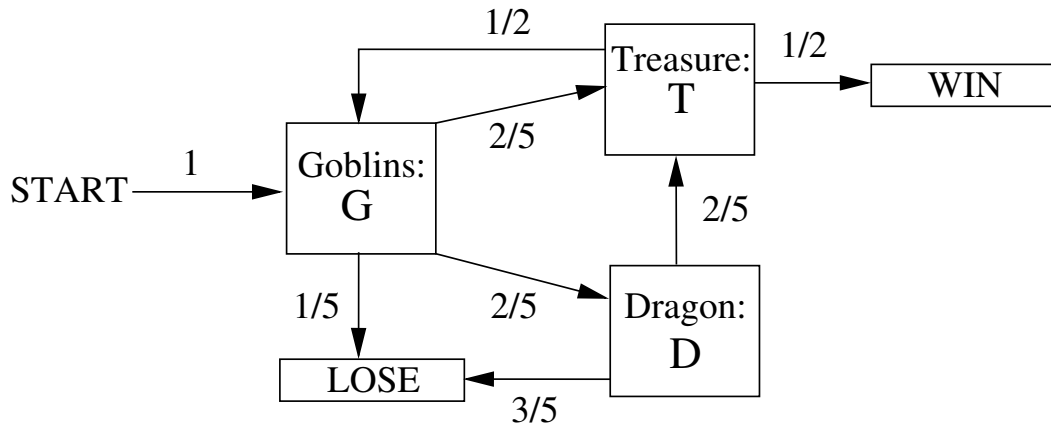
Find $\mathbb{P}(D)$. You will need to define notation h_G and h_T .

Note: marks are awarded for clearly defining your notation, for setting out any relevant equations, and for clearly stating your final answer. (6)

- (c) Find $\mathbb{P}(D|W)$, the probability that the Hobbit ever has to meet the Dragon, given that he wins the adventure eventually. (6)

Continued ...

The diagram is printed again here to help you:



- (d) Define the random variable N to be the **number of times the Hobbit visits the state Treasure**, starting from the beginning. Find $\mathbb{E}(N)$. You will need to define notation m_G , m_D , and m_T .

Note: marks are awarded for clearly defining your notation, for setting out any relevant equations, and for clearly stating your final answer. (6)

- (e) The sample space we have been working in is

$$\Omega = \{ \text{all paths starting at state } G \text{ and finishing at Win or Lose} \}.$$

Elements of the sample space are called **sample points**. When we **condition** on an event A , we restrict the sample space to the set of sample points that are members of the event A .

Recall that the random variable N is the number of times the Hobbit visits the state Treasure, and W is the event that the Hobbit wins eventually. Write down two examples of sample points that are members of the event $\{N = 2\}$.

By looking at your example sample points, consider how you would calculate $\mathbb{P}(W | N = 2)$. Hence state or calculate $\mathbb{P}(W | N = 2)$, using your results from earlier parts of the test as appropriate, and justifying your answer. Would your answer change if you were calculating $\mathbb{P}(W | N = n)$ for any other integer $n \geq 1$? (6)

- (f) The Hobbit recognises that if he Loses the adventure, it won't matter how much treasure he has, because he will be dead. Therefore, he is interested in $\mathbb{E}(N | W)$.

Write down the definition of $\mathbb{E}(N | W)$ as a sum. Hence find the value of $\mathbb{E}(N | W)$, fully justifying all your working.

[Hint: several previous parts of the question are useful for this question. Very little calculation is required: you do *not* need any long calculations. Marks are awarded for correctly formulating and justifying your answer.] (8)

Total: 40