Answer **ALL QUESTIONS**. Marks are shown for each question.
Write your **name and ID number** at the top of your answer sheet.
If the wording of a question is not clear, **ask for advice.**

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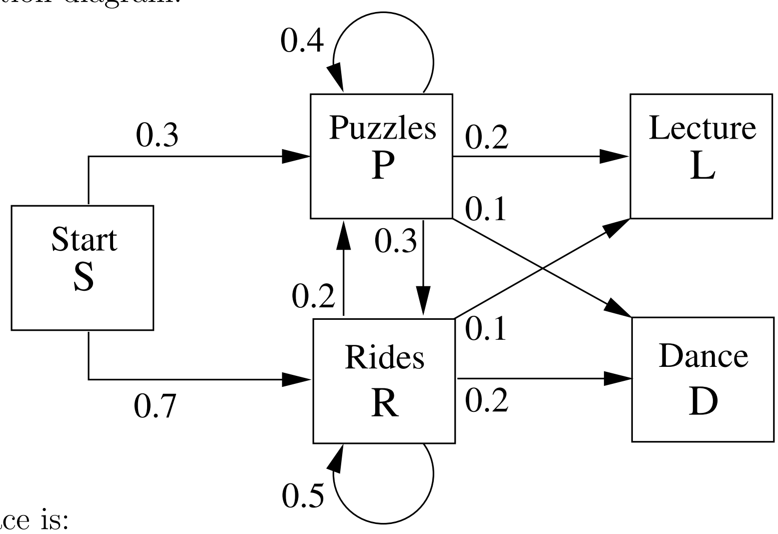
1. The Probability Fun Park is an open-air amusement park that promises a day of fun for all the family. There are two types of activity at the park: Probability Puzzles and Random Rides, and each type offers lots of crazy choices. When they are ready to leave, visitors must choose one of two attractions to attend before they go: either a **Dance display** by the dance group The Markov Mixups, or a riveting **Statistics Lecture** delivered by a noted professor.

   The sequence of activities chosen by visitors is modelled by a stochastic process with the following transition diagram:



   The sample space is:

$$\Omega = \{ \text{ all paths starting at state } S \text{ and finishing at either state } D \text{ or state } L \ \} .$$

   (a) We want to find the probability that a randomly chosen visitor attends the **Lecture** before leaving. Define

$$p_S = \mathbb{P}\,(\text{visitor eventually attends the Lecture} \,|\, \text{start at state S})\,,$$

   and write down any other definitions that you need. Then write down the equations that you need to solve, and solve them. Finish your answer by clearly stating the probability that a randomly chosen visitor attends the Lecture before leaving. (5)

   (b) We wish to find the average number of **Rides** that a visitor will go on during their visit from start to finish. Define

$$m_S = \mathbb{E}\,(\text{number of } \textbf{Rides} \,|\, \text{start at state S})\,,$$

   and write down any other definitions you need. Write down the equations you need to solve, and solve them. Finish your answer by clearly stating the average number of Rides that a visitor goes on during their visit. (4)

Continued ...

(c) It is nearly closing time, and no new visitors are being admitted into the park. There are still 12 people doing the Puzzles, and 48 people on the Rides. Out of these 60 people, what is the expected number who will attend the Lecture before leaving? Assume that all people behave independently according to the transition diagram shown overleaf, and use your answers from previous parts of the question as appropriate. (3)

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2. A fire station has a problem with **hoax callers**: people who ring up and pretend that there is a fire when there isn't one. Each hoax call must be answered as if it were a real fire, so it costs time and money. Hoax calls tend to be more common when there are a lot of real fires, so the problem tends to be worst when the fire station is busiest with real fires.

Fireman Sam proposes the following model for the numbers of real fires and hoax calls.

Define random variables X and Y such that:

Y = number of **real fires** that will occur during the next week;

X = number of **hoax calls** that will occur during the next week.

Fireman Sam's model for X and Y is:

$$\begin{aligned} Y &\sim \text{Poisson}(\lambda)\,; \\ X \,|\, Y &\sim \text{Poisson}(3+Y)\,. \end{aligned}$$

The following information is provided.

> If $Z \sim \text{Poisson}(\mu)$, then $\mathbb{E}(Z) = \text{Var}(Z) = \mu$, and $\mathbb{P}(Z = z) = \dfrac{\mu^z}{z!}e^{-\mu}$ for $z = 0, 1, 2, \ldots$.
>
> Law of total expectation: $\mathbb{E}(X) = \mathbb{E}_Y\left\{\mathbb{E}(X \,|\, Y)\right\}$.
>
> Law of total variance: $\text{Var}(X) = \mathbb{E}_Y\left\{\text{Var}(X \,|\, Y)\right\} + \text{Var}_Y\left\{\mathbb{E}(X \,|\, Y)\right\}$.

(a) Fireman Sam is concerned with $X + Y$, which is the *total* number of calls, both real and hoax, that the fire station must deal with in the week. By selecting a suitable method, find $\mathbb{E}(X + Y)$ and $\text{Var}(X + Y)$ in terms of λ. Show all working. (5)

(b) Suppose we wish to find $\mathbb{P}(X + Y = 10)$. Write down an expression for $\mathbb{P}(X + Y = 10)$ as a sum. Do *not* attempt to evaluate the sum. Your final answer should include the unknown value λ. (3)

Total: 20