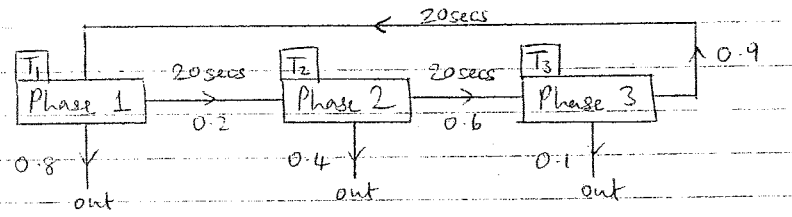
Stats 325 Term Test Solutions Semester 2, 2003

1)



a) $\mathbb{E}(T_1) = 0.8 * 0 + 0.2\,(20 + \mathbb{E}(T_2))$

$\Rightarrow \underline{\mathbb{E}(T_1) = 4 + 0.2\,\mathbb{E}(T_2)}$ as stated.

Similarly, $\mathbb{E}(T_2) = 0.4 * 0 + 0.6\,(20 + \mathbb{E}(T_3))$

$\Rightarrow \underline{\mathbb{E}(T_2) = 12 + 0.6\,\mathbb{E}(T_3)}$

and $\mathbb{E}(T_3) = 0.1 * 0 + 0.9 * (20 + \mathbb{E}(T_1))$

$\underline{\mathbb{E}(T_3) = 18 + 0.9\,\mathbb{E}(T_1)}$

b) $\mathbb{E}(T_1) = 4 + 0.2\,\mathbb{E}(T_2)$ ①

$\mathbb{E}(T_2) = 12 + 0.6\,\mathbb{E}(T_3)$ ②

$\mathbb{E}(T_3) = 18 + 0.9\,\mathbb{E}(T_1)$ ③

Substitute ③ in ②: $\mathbb{E}(T_2) = 12 + 0.6\{18 + 0.9\,\mathbb{E}(T_1)\}$

$\Rightarrow \mathbb{E}(T_2) = 22.8 + 0.54\,\mathbb{E}(T_1)$ ⊛

Substitute in ①: $\mathbb{E}(T_1) = 4 + 0.2\{22.8 + 0.54\,\mathbb{E}(T_1)\}$

$\mathbb{E}(T_1)\,(1 - 0.2 * 0.54) = 8.56$

$\underline{\mathbb{E}(T_1) = \frac{8.56}{0.892} = 9.6 \text{ seconds}}$

as stated.

1b cont.) Returning to ⊛:

$\mathbb{E}(T_2) = 22.8 + 0.54\,\mathbb{E}(T_1)$

$= 22.8 + 0.54 * 9.6$

$= 27.98$ seconds

$\underline{\mathbb{E}(T_2) = 28.0 \text{ seconds}}$ (3sf).

and from ③, $\mathbb{E}(T_3) = 18 + 0.9\,\mathbb{E}(T_1)$

$= 18 + 0.9 * 9.6$

$\underline{\mathbb{E}(T_3) = 26.6 \text{ seconds}}$.

1c) $W \sim \text{Uniform}(0, 20)$, so $\underline{\mathbb{E}(W) = \frac{20+0}{2} = 10}$

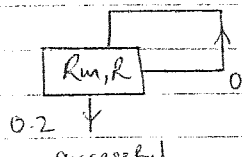
So $T = W +$ { T_1 with prob $1/3$; T_2 " " $1/3$; T_3 " " $1/3$

$\Rightarrow \mathbb{E}(T) = \mathbb{E}(W) + \frac{1}{3}\{\mathbb{E}(T_1) + \mathbb{E}(T_2) + \mathbb{E}(T_3)\}$

$= 10 + \frac{1}{3}(9.6 + 28.0 + 26.6)$

$\underline{\mathbb{E}(T) = 31.4 \text{ seconds}}$

2)



$R \sim \text{Exponential}(0.1)$

$\Rightarrow \mathbb{E}(R) = 1/0.1 = 10$ sec

$\mathbb{E}(T) = \mathbb{E}(R) + 0.8\,\mathbb{E}(T)$

$= 10 + 0.8\,\mathbb{E}(T)$

$0.2\,\mathbb{E}(T) = 10$

$\Rightarrow \underline{\mathbb{E}(T) = \frac{10}{0.2} = 50 \text{ seconds}}$.

Q3) a)

y	0	1	2	3
$\mathbb{P}(Y=y)$	0·3	0	0	0·7

b) $G_1(s) = \mathbb{E}(s^Y) = 0.3\,s^0 + 0.7\,s^3$

$\Rightarrow$ $\underline{\underline{G_1(s) = 0.3 + 0.7s^3}}$

c) $G_2(s) = G_1(G_1(s))$

$= 0.3 + 0.7\,G_1(s)^3$

$\underline{\underline{G_2(s) = 0.3 + 0.7\left(0.3 + 0.7s^3\right)^3}}$

d) $\mathbb{P}(Z_2 = 0) = G_2(0) = 0.3 + 0.7\,(0.3)^3$

$= 0.3189$

$\underline{\underline{\mathbb{P}(Z_2 = 0) = 0.32}}$ (2sf).

e) $\mathbb{E}(Z_2) = G_2'(1)$

Now $G_2(s) = 0.3 + 0.7\left(0.3 + 0.7s^3\right)^3$

$G_2'(s) = 0.7 \cdot 3\left(0.3 + 0.7s^3\right)^2 \cdot (3 * 0.7s^2)$

So $G_2'(1) = 0.7 * 3\left(0.3 + 0.7\right)^2 (3 * 0.7)$

$\underline{\underline{\mathbb{E}(Z_2) = G_2'(1) = 4.41}}$