WHERE DO STUDENTS GET LOST: THE CONCEPT OF VARIATION? ®

Carl Lee, Central Michigan University, USA
Aklilu Zeleke, Alma College, USA
Howard Wachtel, Delaware Valley College, USA

Many college students have difficulties in understanding and making connections among the main concepts of statistics. Compounding the difficulties is the misconception of a variety of statistical concepts that students hold even before taking any statistics course. It is, thus, crucial to investigate how the understanding of statistical concepts is constructed and at which stage students start to lose making connections among various concepts. This article reports some findings from our study of investigating the path of learning statistical concepts, specifically on how students learn the concept of variation. We focus on investigating the missing connections about their understanding of variation. The framework of statistical thinking, PPDAC investigative cycle, is used as our guideline for analyzing our interview data.

INTRODUCTION

Many college students have difficulties in understanding and making connections among the main concepts of statistics. Frustrations, boredom, dislike and, at times, fears of the course are some of the common attitudes we observe in students (Yilmaz, 1996). Compounding the difficulties is the misconception of a variety of statistical concepts that students hold even before taking any statistics course (Hawkins, 1997). It is, thus, crucial for educators to investigate how the understanding of statistical concepts is constructed and at which stage students start to lose making connections among various concepts. Garfield (1995) discussed some useful principles to help student learning. Rossman and Chance (2001) proposed four different sequencing topics for introductory statistics with an argument for each sequencing. The path that students follow to make connection among different concepts is certainly closely related to the issue of sequencing topics in introductory statistics.

The learning theory of constructivism is widely accepted among educators as a guide for instructional design. This theory emphasizes learning by constructing knowledge and active involvement. In the learning of statistics concepts, it translates to the instructional designs such as 'activity-based learning' (Scheaffer, Gnanadesikan, Watkins, & Witmer, 1996), 'PACE learning strategy' (Lee, 1999), and 'Workshop Statistics' (Rossman & Chance, 2000). These pedagogical strategies have gained a wide support among statistics educators. However, there has been little research on assessing the effectiveness of these strategies. Lee (1998) conducted an assessment of students taught by using the PACE strategy and indicated that students taught by the PACE method applied very different problem solving strategies when compared with students from a traditional class. To solve a problem, the class taught by the traditional method followed a mechanical approach: look for a formula that can be used, plug numbers into the formula and get the right numerical output. In contrast, the group that was taught by the PACE method demonstrated a conceptual approach: identify strategies used to solve similar problems in the class, modify the strategy or the model so that one is appropriate for the other and interpret the numerical results by making connections to the original problem. The different problem solving strategies suggest that the path of learning statistical concepts is crucial. Moreover, along the path of understanding subtle statistical concepts, if research could identify the stage where students begin to get lost, it will help instructors develop adequate instructional strategies and help students in learning statistical concepts.

This article reports some findings from our study of investigating the path of learning statistical concepts, specifically on how students learn the concept of variation. Our focus is on investigating the lost connections about their understanding of variation. The framework of statistical thinking, the investigative cycle (PPDAC) (Pfannkuch & Wild, 1999), is used to analyze the data. The preliminary analysis showed some surprising findings. For most students, the concept of variation from samples to samples was no trouble. However, the issue of valid measurement to be used when making comparison was difficult. In this paper, we will discuss the implications of these findings and suggest some strategies to help students with (1) retention of the learnt material and (2) using standard deviation in real world applications.

BACKGROUND

Students from three different institutions participated in this study. The institutions involved are Central Michigan University (a comprehensive university), Alma College (a liberal arts institution) and New York State University at York (a liberal arts institution). The students were taught elementary statistics by two different methods: one group was taught by a traditional lecture method and the other group was taught using the PACE model developed by Lee (1999). PACE stands for Projects, hands on Activities, Cooperative learning and Exercises. Students who were taught by the PACE model were much more involved in active learning by collecting their own data, through group work and classroom discussions. In contrast, the traditional method emphasized lectures and note-taking.

THE INTERVIEW DESIGN

The interview was a semi-experimental design. Nine students from the traditional lecture format class were selected at random from each institution and nine students from the PACE model class were selected at random from Central Michigan University and Alma College. We made sure that in each group we had three students representing those students who received the grade A, the grade B and the grade C or lower in the course. The students were contacted about two months after taking their first introductory statistics course for an in-depth interview. The interview protocol remained the same. The first half-hour focused on motivations, learning styles, attitudes and beliefs. The rest of the time focused on the process of statistical problem solving skills and the applications of a variety of essential concepts. A project scenario was described to students. The scenario was*: "Recently, there has been an impression that students coming to this institution have weaker quantitative backgrounds compared with students 20 years ago. You are assigned the project to find out if this is indeed the case."* A sequence of interview questions surrounding this project was designed to investigate how well students solve each question and at what stage students begin to lose the connection. Most of the participants replied to the above question by stating *"We do hypothesis testing"*.. We proceeded with the interview by raising specific questions that eventually lead to hypothesis testing. Here are some examples of questions presented to the students*: "How do you compare the two groups?" "What do you do first?" "What is the target population?" "How do you describe the center and/or variation numerically, graphically?"* etc. We are aware that this is an interview study with a small sample size. Therefore, we focus on students' thought processes. It is not suggested to making a general inference to the entire college student population.

RESULTS

Our main goal was to find out students' understanding of the concept of variation and how it relates to other important concepts in statistics. One can view variation from different perspectives. We focus in this paper on two aspects: numerical and geometric understanding of variation. A typical question we posed to investigate their numerical understanding was *"How do we differentiate between data sets that are widely scattered or are cuddled together?"* Not all our interviewees were comfortable with these kinds of questions. In fact our interview data suggests that about 85% of the class taught by the traditional method was not able to remember any numerical quantity that can be used to describe variation. In contrast 63% of the PACE class mentioned standard deviation and another 10% mentioned range as possible answers. After the term standard deviation was mentioned, the formula was given to the students and they were asked to describe its meaning. From the traditional group about 25% were able to explain the meaning of standard deviation verbally, and 37.5% were able to correctly explain the meaning of the formula. From the PACE group, 50% of the interviewees were able to explain verbally what standard deviation is, and 60% were able to explain the meaning of the formula. Table 1 summarizes some of the responses to the questions about standard deviation.

An important point we want to make here is the following: Once students were told that standard deviation is one measure used to describe variation, they were able to connect in general terms low and high standard deviations to low and high variation respectively. We were curious to find out if this response was due to real understanding of the concept of standard deviation or an instinct response. We will come back to investigate this issue later when we address the issue of "making connections" at the end of this section.

Table 1
Responses to Questions about Standard Deviation

Traditional	*PACE*
Standard deviation describes the difference between each score.	*Standard deviation is a number that tells you how scattered the data is around the mean*
Standard deviation means that it's like average. I don't want to say difference, but the average of how much it might vary on average	*Standard deviation describes how far your numbers are from the center of your data*
The meaning of the standard deviation? See, with this class, I just took it, basically, as learning formula and applying it to the test and not so much.	*Standard deviation tells you how far your data is from the mean, the further away it is the more spread you have in your data.*

The geometric approach of describing variation was the other point we wanted to investigate. In this regard typical questions include: *"How do you describe variation or the spread graphically?" "Can you draw different graphs and describe variation?"* etc. In response to the first question, 25% of students from the traditional class and 90% from the PACE group mentioned histograms. Once histograms were mentioned, 13% of the students from the traditional group were able to explain how histograms are constructed and 38% were able to remember different kinds of histograms (symmetric versus skewed) and correctly identify the types of distributions from the histograms. The percentages to similar questions in the PACE group are 80% and 70% respectively. Similar to the numerical description of variation, we see evidence that students from the PACE model class did better in the conceptual understanding of variation. However, when it comes to differentiating between "skewed to the left" and "skewed to the right" histograms we see confusion in both groups. We address this problem in the last section of this paper. Table 2 summarizes some responses we received from the two groups in regard to questions about the construction of histograms and identifying the different types.

Table 2
Responses about Construction of Histograms

Traditional	*PACE*
Well you would.oh my God I can't really ah I can picture it in my head, but I can't remember exactly how it's made. .	*I guess I take scores between 100 and 200 and count how many scores there are in that range and so on. Then I draw a bar graph that represents the data.*
You are asking me what this curve is called if you give a very easy test? Well that will be skewed to the higher end because more people would probably do better.	*This distribution is skewed to the left. Why I say this? Well I think because there is more data on the left.*
A very difficult test? Well obviously it is not gonna be symmetrical. You have a lot of failures but you may have few high score. Your average tends to one end, but to which end? It depends I am confused wait wait wait.	*A very easy exam? Let em see. I have many high scores and few low scores. The tail in on the left side. I think it is skewed to the left, but I am not sure.*

Finally, we were interested in finding out if students can use various concepts to make connections and be able to explain variation. The following hypothetical problem was created to analyze their responses: "We are comparing SAT scores of two institutions. For the first institution the mean SAT score is 540 with standard deviation 40 and the median is 535. For the second institution the mean SAT score is 540 with standard deviation 1 and the median is 535 Please explain the distribution of scores for these two institutions".

Table 3
Responses about Making Connections among Various Concepts and Variation

Traditional	*PACE*
I think they are both symmetric the difference of five units in hundreds of scores is not much.	*With standard deviation 40, the distance between the mean and the median is very small.*
The one with standard deviation 1 is symmetric. Isn't there something like small standard deviation means less variation?	*If the standard deviation is 1, then the mean is 5 standard deviation units away from the mean. That is far and it is not symmetric.*
I think it is very hard to get standard deviation 1. There must be some mistake. We can't compare the two groups	*In the first case the mean is within one standard deviation away from the median. In the second case it is away about 5 standard deviation unit. That indicates to me an outlier pulled the mean and hence skewed distribution*
Both are roughly symmetric because to be perfectly symmetric the mean and the median have to be equal	*Even though the difference between the mean and the median is 5 units I think the standard deviation makes a difference here. I guess the second seems skewed.*

Here is a summary of their responses. 73% from the traditional group and 52% from the PACE group of the students said the distributions are symmetric because the mean and the median are close to each other (a difference of five), a major factor for identifying symmetric distributions. However, from the PACE group nearly 50% of the students analyzed the problem by including the effect of the difference in standard deviation. They were also able to connect the difference in standard deviation with the 68-95-99.7 percent empirical rule to show how symmetric or skewed a distribution can be by finding the number of students within one or two or three standard deviation units. These students have also explained that it is very important to consider the standard deviation when it comes to deciding whether the mean and the median are "close" or "far". Table 3 summarizes some responses we received from the two groups in regard to questions about making connections among various concepts and variation.

CONCLUSION

Students in the PACE model were more complex thinkers and more articulate in their explanation of concepts related to variation. In fact, comments from students taught by the traditional method indirectly support this statement. As mentioned in the introduction, a large hurdle for both groups was the ability to make connections among different concepts of variation. One example we found in this direction is the inability to connect the concept of "skewed to the right" and "skewed to the left" to the presence of the outliers in the tails. It seems that in student's minds there is an association between "skewness" and "higher frequency." Again and again we have witnessed responses like *" this is skewed to the left because the majority of the scores are on the left side."* We think that we need to emphasize in our teachings that "skewness" is related to "outliers" and not to "more observations." In our interviews we were able to accomplish this by bringing an example that compares the salary distribution of college graduates. The example was *"After you graduate from college, you are hired by a professional sports club for ten million dollars while your classmates were hired by different companies for fifty thousand dollars. What will the salary distribution look like in this case?"* This example helped almost all of the interviewees in the PACE group to make connections between "skewness" and "outlier."

In summary, an activity based with group activities, that engages students in their learning process has shown a much more promising outcome on students' understanding of variation and the applications of this concept in real world situations. Putting this in the framework of the PPDAC Investigative Cycle, the activity-based instruction such as the PACE model creates a more engaging learning environment for students to develop and encapsulate the investigative process. Through a sequence of hands-on activities in which students are required to actively participate in the process of problem solving, they develop a better understanding of the 'big' picture and are able to relate and apply the statistical concepts in diverse situations. However, there are many other factors associated with learning statistics. The dimensions of attitude and motivation are important factors, which have often been overlooked due to the difficulty of quantifying their impact. Further research of this dimension will be valuable.

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