STUDENT UNDERSTANDING OF HISTOGRAMS: A STUMBLING STONE TO THE DEVELOPMENT OF INTUITIONS ABOUT VARIATION

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The findings reported in this article came from a study which took place in an introductory college-level statistics course and which adopted a nontraditional approach to statistics instruction that had variation as its central tenet. The conjecture driving the study was that poor understanding of statistical concepts might be the result of instructional neglect of variation and that instruction which puts emphasis on building student intuitions about variation and its relevance to statistics should also lead to improved comprehension of other statistical concepts. The results of the study point to a number of critical junctures and obstacles to the conceptual evolution of variation. The article discusses one of those critical junctures and obstacles, the understanding of histograms.

INTRODUCTION

We have, for a number of years, been involved in a joint research effort to understand the obstacles to the learning of statistics and use this understanding to find ways to create learning environments that facilitate deeper understanding. In a previous study by Meletiou, Lee, and Fouladi (in press), it was found that the students we interviewed, regardless of whether they came from a lecture-based classroom or from a course that made wide incorporation of technology and engaging activities, had poor intuitions about the stochastic and tended to think deterministically. This led to the conclusion that the reason behind students' difficulties might be instructional neglect of variation and gave the motivation for a new study driven by the conjecture that statistics instruction would be more successful in achieving its objectives if it were to put more emphasis on helping students build sound intuitions about variation and its relevance to statistics (Ballman, 1997). A nontraditional approach to statistics instruction that had variation as its central tenet was laid out and adopted in a college level statistics classroom.

The study allowed us to identify the kinds of intuitions students use to make sense of stochastic phenomena and the ways in which their intuitions are shaped by the learning environment. We were able to find out structures that facilitate, as well structures that inhibit, the articulation of intuitions about variation and its relation to statistics. The results of the study point to a number of critical junctures and obstacles to the conceptual evolution of the role of variation. This article discusses findings relating to one of the critical junctures and obstacles identified – understanding of histograms.

THE STUDY

Context and Participants

The site for the study was an introductory statistics course in a mid-size Midwestern university in the United States. The second author was the instructor of the course, which took place in the summer of 1999 and lasted for six weeks. There were 33 students in the class (19 males and 14 females). Most of these students were majoring in a business-related field of study. Only few had studied mathematics at the pre-calculus level or higher.

Data Collection

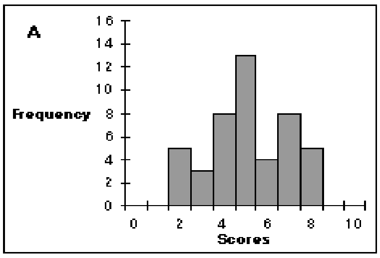
In examining students' learning progress and outcomes, a variety of both qualitative and quantitative data gathering techniques were employed throughout the course. These included open-ended questionnaires given to students on the first and last day of class in order to assess their thinking prior to instruction and at the completion of the course. In addition to collecting data from the whole class, in-depth follow-up interviews of a primary group of eight students were conducted both at the beginning and at the end of the course.

Classroom Setting

The classroom setting modeled realistic statistical investigations. The building of student intuitions about variation was seen as a dynamic process subject to development for a long period of time and through a variety of contexts and tools (Pfannkuch, 1997). The instructor was trying to increase students' awareness of variation by helping them realize that the need for statistical investigations is created due to the existence of variation. Graphical and numerical statistical tools were introduced not in isolation, but as means to describe trends and patterns and deviations from those patterns existing in the data because of the variation inherent in every process.

Histograms were among the main graphical tools employed in the classroom to assess the shape of distributions. Developing student understanding of histograms was a special emphasis of the course since having good understanding of spread when visually interpreting a distribution displayed in a histogram is necessary to be able to fully grasp the meaning of one of the most important and yet most difficult concepts encountered in the course – the concept of sampling distribution. Our past experience was suggesting that understanding of histograms is not as trivial as some people might think. For example, students in the second author's class had during the previous semester done extremely poorly in a question given at the end of the course, asking them to describe how the distribution of salaries for individuals forty years or older but not yet retired would look like. Only four out of thirty-five students had given the right response. Everybody else had argued that the distribution would be right-skewed, not because they understood how the histogram would look like but because they confused it with a scatterplot, thinking that *"the graph is skewed-to-the right because as people approach retirement, their salary gradually drops"*.

Findings from the pre-assessment given at the beginning of the course, further stressed the need for helping students improve their understanding of histograms. Several of the tasks investigated familiarity with histograms and bar graphs. Students' performance in one of those tasks (from Garfield, delMas, & Chance, 1999) asking them to decide by looking at the histogram of two distributions (Figure 1) which of the two distributions has more variability, was indicative of their limited understanding of histograms:



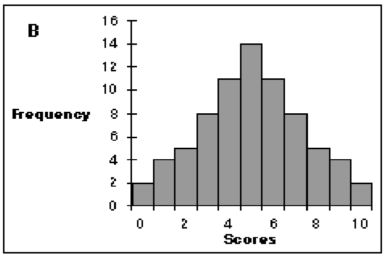


Figure 1. Examples of Histograms.

One fourth of the students taking the pre-assessment (8 students; 24) gave the wrong response to this question. They argued that *"distribution A has more variability because it's bumpier."* Obviously, these students had to improve their understanding of histograms.

Instruction put a lot of emphasis on helping students improve their ability to read and understand histograms and other graphs and to relate features of a distribution to the shape of its graph. Challenging tasks, not typically included in an introductory course, were employed throughout the course. Examples are the *"Matching Histograms to Variables"* task taken from Scheaffer, Gnanadesikan, Watkins, and Witmer (1996), which asked students to match a list of variables and a set of histograms using their knowledge of the variables, or the *"Matching Statistics to Graphs"* task also taken from Scheaffer et al. (1996), that required them to estimate the parameters of different distributions by looking at their histograms.

RESULTS

At the completion of the course, an open-ended questionnaire given to the whole class, followed by interviews of a primary group of eight students, allowed thorough investigation of student reasoning. The results of this assessment suggest that the instructional approach employed, with its simultaneous focus on variation and on the process of statistical investigation, did help students develop statistical thinking that goes beyond the superficial knowledge of terminology, rules and procedures. Instruction proved quite effective in achieving one of its main goals – helping students move away from "uni-dimensional" thinking and integrate center and variation into their analyses and predictions. Although not totally letting go of their deterministic mindset, students were much more willing to interpret situations using a combination of stochastic and deterministic reasoning. They had much better intuitions regarding the variation of population distributions and of distributions of single samples than students in previous studies we had conducted.

Despite the positive effects of instruction on students' skills and dispositions, the majority still failed to adequately develop important ideas related to inferential statistics. In particular, students had serious difficulties in grasping the concept of the sampling distribution of the mean (and other statistics). The end-of course-assessment indicated several factors contributing to poor understanding of this so important, yet so difficult concept. Naturally, the notion that the sampling distribution of the mean is based on the 'imaginary' concept of 'all possible' sample means posed a serious challenge for students. However, the assessment also showed that limited understanding of histograms is another main factor contributing to poor understanding of sampling distributions. Student difficulties in dealing with histograms were not confined to histograms of sampling distributions where indeed, it is not easy to understand the reduction in variance resulting in moving from the population distribution to the distribution of a statistic. Several students still had difficulties in constructing and interpreting simple histograms of population distributions. Even at the end-of-the course, when given again the seemingly easy question of having to decide, by looking at the histogram of two distributions of scores which one had more variability (Figure 1), five students (15%) gave the wrong response (distribution A). Even among those who chose distribution B, some might have had misunderstandings, as I found in the follow-up interview of Tim. This is how he explained why distribution B has more variability:

> *Tim: I mean, it has more variability cause like the people here…the highest frequency is here…and the highest frequency on this one is 2…12 different variables here. This has more people… Because this is 14 and this 2…a difference of 12. And this is 3 and this is 13, and so…B should have more variability.*

Instead of looking at the horizontal axes of the histograms to compare their spread, Tim was looking at their vertical axes and was comparing differences in the heights of the bars (i.e. differences in frequencies among the different categories). The explanation Lucas gave for having chosen histogram A in the previous assignment of the task, showed that he had also been looking at the wrong axis: "*I had chosen it because…the height is very… there is a lot of different heights. I was looking at the height of the graph…but now I understand it that it's here that we should be looking at…I was looking at the wrong side."* He, however, now realized that *"variability means spread…and also* [he] *would include the range of the scores",* and for this reason gave the right response at the final assessment. Tiffany also said that she had, in the pre-assessment, claimed that histogram A has more variability because she *"didn't know then the definition of variability."* She explained: *"I was confused between this…like the height and the width. I just got totally confused because…in some ways I was thinking that this has more variability and it doesn't make any sense right now…I just got confused from the height of it…"*

At the completion of the course, the same question given at the end of the previous semester, asking for a description of the histogram of the distribution of salaries for individuals 40 years or older, was given again. As already discussed, students in the previous semester had almost unanimously argued that the distribution would be skewed-to-the right, not because they understood how the histogram would look like, but because they confused histograms with scatterplots. In this study, 42% of the students realized that on a histogram describing distribution of salaries, *salary* goes on the *x-axis* and *(relative) frequency of people* on the *y-axis,* and that the distribution should be right-skewed, as *"most would make around the same but a few would make lots more."* Nonetheless, similarly to the previous semester, the rest of the students saw the graph as a scatterplot of *salary* vs. *age*. One of those students was Peter. In the follow-up interview, he confessed that he still *"struggle*[d] *with histograms".* Although with prompting he did realize that his reasoning was wrong, he was still not very optimistic about his ability to construct or interpret histograms: *"May be you give me another histogram today, I'd probably still mess it up."*

CONCLUSIONS

The close examination of how students' intuitions evolved during the course enabled us to identify not only structures that facilitated, but also structures that inhibited the articulation of intuitions about variation. In particular, findings indicated that despite the emphasis of the course on helping improve students' ability to understand and interpret histograms, a noticeable proportion still had limited understanding. The end-of-course assessment indicated several students who, when comparing the histograms of two distributions, could not figure which of the two distributions had more variation, and several others who confused histograms with scatterplots. Students' limited understanding of histograms proved to be one of the main factors contributing to students' difficulties in building the concept of sampling distribution of the mean.

The research literature supports our findings. Research has shown that even medical researchers might confuse histograms and bar graphs (Kelly, Sloane, & Whittaker, 1997). The fact that people often encounter histograms in the media and elsewhere does not mean that they understand them. Histograms – as well as bar graphs and other graphs – are a transformation from raw data into an entirely different form. Understanding of this transformation is challenging, and statistics instruction needs to find ways to support it. As the research literature tells us very little about how understanding of histograms and other graphical representations develops, a possible direction of future research is to find ways to help students recognize the different functions of the horizontal and vertical axes across different graphical representations (Friel, Bright, Frierson, & Kader, 1997). This is essential since, as findings from this study point out, understanding of histograms and its relation to variation is one of the stumbling stones in statistics instruction.

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