A STATISTICS COURSE FOR ELEMENTARY AND MIDDLE SCHOOL TEACHERS

Gary Kader and Mike Perry
Appalachian State University
USA

This paper will describe a content-pedagogy course designed to prepare elementary and middle school teachers to teach statistics in the schools. The course is organized around the newly revised content standards developed by the National Council of Teachers of Mathematics. A central objective is to encourage teachers to see statistics as a problem solving process. The course has been implemented as one component of the "Learning Math" Project. Produced by WGBH with funding from the Annenberg/Corporation for Public Broadcasting Math and Science Project, "Learning Math" is developing a series of five college-level courses designed to teach mathematics content to elementary and middle schools teachers. In the statistics course, nine video sessions follow an actual class of teachers through content classes, with footage edited to highlight critical statistical concepts. An on-line course, which parallels the nine videos, is also being developed.

INTRODUCTION

There is an evolving and growing school statistics curriculum throughout the United States. This is primarily due to the impact of the National Council of Teachers of Mathematics' (NCTM) *Curriculum and Evaluation Standards for School Mathematics* (1989) and their newly revised *Principles and Standards for School Mathematics* (2000). In its report, *A Call for Change: Recommendations For the Preparation of Teachers of Mathematics* (1991), the Mathematical Association of America (MAA) maintains that implementing a school mathematics curriculum like that envisioned by the Standards will require significant changes in the preparation of future mathematics teachers. In order for teachers to promote "mathematical power" in their students, teachers themselves must experience first hand the "doing" of mathematics in their college courses. The MAA proposes that teachers learn mathematics through exploration, conjecture, logical reasoning, and problem solving using a variety of mathematical methods, and recommends that college courses designed for teachers should consider the nature of the school curriculum. The MAA identifies several strands of "standards" which teachers should encounter in their preparation. Experiences in probability and statistical problem solving are recommended for teachers at all instructional levels (Elementary, Middle, and Secondary).

An emerging consensus among mathematics educators is that mathematics in the schools should be taught for understanding, and not simply for establishing proficiency. Teachers are necessarily the change agents required to enhance the learning environment of their students. However, teachers are products of a system, which has historically emphasized memorization and computational routine. As students, future teachers have had few opportunities to see examples of learning experiences that promote conjecture and understanding. According to Ma (1999), teacher preparation must link the study of the mathematics with the study of how to teach mathematics. If teachers are expected to teach probability and statistics for understanding and to become advocates of change, they must become constructive learners and experience learning probability and statistics through a process similar to that of their students. Through such experiences, teachers come to realize that active exploration of statistical concepts provides the necessary structures for building understanding.

PREPARING K-8 TEACHERS TO TEACH STATISTICS

What statistics should be taught in order to prepare teachers to teach statistics in grades K-8? Statistical problem solving begins with the notion that we use data to answer questions. A statistical investigation includes a set of four interrelated components:

- *Formulate a question* - identify a specific question or questions to explore and decide what variables to measure in order to address the question(s).
- *Collect suitable data* – determine an appropriate data collection design to obtain suitable data as well as actually collect the data.

- *Analyze the data* – organize, summarize and describe the variation present in the data.
- *Interpret the results* - use the results from the analyses of the data to provide the best possible answer(s) the original question.

The main purpose of this model is to give structure and direction to the type of reasoning used in statistical problem solving. The individual components in the model are not necessarily self-contained and the process is not necessarily sequential.

Formulate a question. It is essential that teachers have a clear understanding of the relationship between the problem under study, the specific types of questions that can be answered with statistics, and the variables identified to address these questions. The processes involved in developing question formulation skills are best acquired through experience.

Collect suitable data. Data is the critical ingredient of statistics. Data are not simply numbers – they are measurements. How to manage data collection depends on the nature of the question(s) and the variables identified in formulating the question. The types of conclusions that can be made from the data depend on the data collection design. For many investigations a census of an entire group may be appropriate. Other questions may require that a sample be selected. Cause and effect type questions must be examined through data from controlled/comparative experiments. Notions of fairness and the role randomness must be addressed in both sampling and experimentation.

Recording and measurement errors are also important issues. Attention must also be given to identifying various sources of variation in data.

Analyze the data. Data analysis focuses on ways to represent data for the purpose of seeking understanding of variation in data and identifying patterns present in the variation. Opportunities should be provided for teachers to create their own data representations. Often such representations provide insights into their understanding of the question at hand and how the data relates to that question.

There are of course many standard statistical representations that must be developed. For a given problem, there are often multiple representations that may be appropriate for the data. It is important that teachers be aware of the connections between the various representations, as well as the strengths and weaknesses of each representation.

Statistical representations of data include graphical displays and numerical summaries. Graphical displays provide visual descriptions of variability in data. Statistical concepts such as representative value, spread/dispersion/variability, and shape develop naturally through graphical representations. Numerical representations of data provide a means for summarizing the conceptual ideas conveyed through graphical representations.

Teachers must also understand that appropriate representations for data depend upon the nature of the question(s) and the type of data collected. That is, *how* the data are represented depends on *why* the data have been collected and *what* type of data has been collected.

Interpret the results. An analysis focuses attention on the variation in the data. Interpreting results requires making sense of the analysis as it relates to the question under study. The results from the analysis must now be translated from graphical descriptions and numerical summaries into meaningful statements that address the question at hand. Developing skills that incorporate relevant information concerning the variation present in data into statistical answers is critical in teacher preparation.

The statistical education of teachers should focus on data, the above four-step problem solving process, and statistical reasoning rather than mostly on techniques. With the emphasis on statistics as problem solving, the development of statistical reasoning is quite natural because it is integral to the problem solving process. It is important that the inductive nature of statistical reasoning be continually nurtured in teacher preparation.

A CONTENT-PEDAGOGY COURSE FOR TEACHER PREPARATION

A major goal of teacher preparation is to encourage active learning through problem solving in the teaching of statistics in the schools. Consequently, statistics should be presented to teachers in the same style considered appropriate for their students. This Content-Pedagogy approach to teacher preparation allows teachers to experience an activity-based approach to

instruction while at the same time learning appropriate content for teaching statistics in the elementary and middle grades.

The basic principle is to "teach the teachers in the same style that they should teach their students." This requires an approach that has three key characteristics:

- It is *Activity* based,
- It employs *Integration of Content and Pedagogy,*
- It employs *Sequences* of activity units.

Activities. The activities used in teacher education should be the same, as the ones the teachers will eventually use in their classrooms. There are two types of learning activities: *Problem-Solving* activities based on the complete problem solving process (Question, Data, Analysis, and Interpretation) and *Concept* activities designed for learning a concept through active experience, and may or may not be based on the complete problem solving process.

Integration of Content and Pedagogy. Lessons for integrating the study of statistics with the study of how to teach statistics should include three parts. Part 1 is "Participation" where teachers complete an activity in a similar fashion that it is designed for school use. The learning objectives for the teachers include the same learning objectives intended for the students, plus the objective of experiencing the learning activity so that they might use it in their future teaching. Part 2 is "Reflection" where teachers examine more closely and in more depth the concepts and principles underlying the activity and pursue any questions that might have arisen from the activity phase. Part 3 is "Readings" where materials, which discuss the concepts and ideas and techniques in the activity, are examined in parallel with the activity and reflection phases.

Sequences of Units. The sequencing of activity units should be designed is such a way that the underlying concepts might evolve in an appropriate order and be related to other statistical and mathematical ideas.

THE LEARNING MATH PROJECT

The Learning Math Project, produced by WGBH-TV (Boston) with funding from the Annenberg/CPB Math and Science Project, is a series of five college-level courses designed to teach mathematics content to elementary and middle school teachers. These multimedia courses emphasize the content that underlies the elementary and middle school curriculum and are organized around the content standards developed by the NCTM *Principles and Standards* (2000). The five courses are (1) Number and Operation; (2) Patterns, Functions, and Algebra; (3) Geometry and Spatial Sense; (4) Measurement; and (5) Data Analysis, Statistics, and Probability. They are video based and are available through a web site so that they may be used for distance learning.

The primary goal of the courses is to help teachers develop a robust understanding of mathematics through a series of carefully sequenced multimedia activities and support materials. Related goals include having teachers look at how they learn and what they think mathematics is; helping teachers re-examine their beliefs about learning and teaching mathematics; and helping them make sense of mathematics and take ownership of this process.

Learning Math courses are college-level courses which can be used in a variety of ways. They may be offered individually to pre-service or in-service teachers, or as a package by a college or university or by a professional development unit of a school system.

THE DATA ANALYSIS, STATISTICS, AND PROBABILITY COURSE

The goal of this course is to help teachers better understand statistics itself, pushing beyond surface familiarity to promote a deeper knowledge of the content that underlies elementary and middle school statistics curricula. The course is composed of the following nine lessons:

#1 Statistics as Problem Solving
   a. Statistics as a Problem-Solving Process
   b. Measurement, Variation in Data, Sources of Variation

#2 Data Organization and Representation
   a. Distribution

b. Representing the Distribution for Categorical Data
c. Representing the Distribution for Numerical Data

#3 Describing Distributions
a. Grouping Data and Histograms

#4 Location and Spread
a. Dividing data in half
b. Dividing data into fourths
c. The Five-Number Summary and the Box Plot

#5 Variation about the Mean
a. What is the mean?
b. Quantifying variation about the mean

#6 Experiments
a. Controlling sources of variation
b. The role of randomization in data collection

#7 Linear Relationships in Data

#8 Probability

#9 Random Sampling and Estimation

Videos. Nine thirty-minute videos follow an actual class of teachers through each of the nine class sessions utilizing the Content-Pedagogy approach described earlier. Footage has been edited to highlight critical statistical concepts and to expose and address common misconceptions. Each video presents comments from the teachers about the ideas in the class, and also includes an interview with a professional statistician who discusses how ideas in the class session are being used in a real world application. A tenth video provides a classroom application setting focusing on each of the three grade bands (K-2; 3-5; 6-8).

A central objective of the class sessions is to illustrate the use of engaging explorations of statistics that encourage participants to see statistics as a problem solving process or allow them to experience statistical concepts. The classes also show how appropriate activities can stimulate the development of statistical thinking skills. The underlying concepts and techniques are presented through the activities and explorations.

Session 6, for instance, begins with a problem – *Is it easier to remember strings of characters which spell familiar words than it is to remember arbitrary strings of characters which do not spell words?* The class considers all aspects of the problem. Working in small groups the class begins with the issue of how to measure memory recall so that the desired comparison can be made. This leads to group discussion and the class discussion of each group's ideas. A consensus is reached on a method of measurement. The class then considers five alternative designs for the experiment. Working again in small groups the discussion eventually focuses on issues such as confounding, randomization and pairing. The class as a whole discusses the optional designs and then agrees on a design for the experiment. The class then conducts the experiment. The data collection design is implemented, the data is analyzed and an interpretation of results is discussed. The activity encourages statistical thinking in the small group discussions and allows opportunities for the students to develop their own ideas. There is no lecture to describe how things are done; instead the students are directed toward the ideas by a carefully designed investigation of the entire problem solving process.

Website. The web site provides nine sessions, which parallel the video sessions. These present ideas in the same exploratory, activity based style employed in the video class sessions. The video sessions are intended to convey the general nature of the activity based approach and the resulting classroom atmosphere whereas the web sessions allow a closer look at the underlying concepts and techniques. In some cases the web sessions explore ideas through the same activities used in the videos and in other cases present additional activities, which are not illustrated in the videos.

In addition to text presentations, the web sessions contain portions of the video to illustrate ideas and each session has at least one interactive activity. These interactive activities allow students to actively participate. Several of these are described below.

Session 1 has two interactive activities. The first allows the user to make personal judgments about the lengths of line segments to illustrate how perception bias might result in

measurement bias. The second uses an activity based on sampling from a population of circles to estimate the size of the circles; this is a popular activity, which illustrates how personal selection of a sample often results in estimation bias. Results for the most frequent 100 users are accumulated and used later in Session 9 to compare the distribution of estimates resulting from personally selected samples with the distribution of 100 samples resulting from random selection.

Session 4 uses a physical representation of measurements (noodles) to introduce the idea of quartiles. The interactive activity allows the user examine and manipulate the noodles to investigate quartiles. Session 5 uses a physical representation (stacks of coins) and the dot plot to introduce the idea of the mean in terms of deviation from the mean. Then the idea of Mean Average Deviation (M.A.D.) is explored in the interactivity activity. The user is allowed to arrange values on a dot plot to create distributions with specified mean and M.A.D.

Session 6 investigates the design of experiments. The interactive activity allows the user to participate in an experiment concerning memory recall. Students in a class can be tested for memory recall of word and non-words to obtain the measurements needed for a paired difference design. Session 7 develops the concept of the least squares regression line. The user is presented with a scatter-plot and is allowed to subjectively choose a line and then compare his choice with the least squares line.

Session 8 investigates the binomial random variable and then uses this model to determine if a person has developed skill playing the game of "Push-Penny" by comparing the player's results to the results for a random player. The interactive activity gives a graphical display of random plays of the game and relates to results to a tree diagram to demonstrate that the binomial random variable describes a random player.

Session 9 develops the concept of a sampling distribution. The population is represented spatially as black dots on a white background. The population is divided into sub-regions and samples of sub-regions are selected to estimate the size of the population. The interactive activity allows the user to select random samples, visualize the sample and then examine the resulting estimate. Repetition provides a picture of the differences in samples and the resulting sample to sample variation of estimates.

DISCUSSION

This discussion addresses a content-pedagogy course designed to prepare elementary and middle school teachers to teach statistics in the schools. The proposed course presents statistics as a problem solving process involving the four components: (1) formulate a question; (2) collect suitable data; (3) analyze the data; (4) interpret the results. The course advances that notion that statistical content and issues related to the teaching statistics in the schools must be developed simultaneously. The integration of content and pedagogy is achieved through a proper sequencing of experiential activities designed to promote either statistical problem solving or statistical concepts.

A video series illustrating a group of teachers participating in a content-pedagogy statistic course has been developed through the "Learning Math" Project. These materials, together with on-line resources, may be used to prepare pre-service or in-service teachers to teach statistics in the elementary and middle grades.

REFERENCES

Ma, Liping (1999). *Knowing and teaching elementary mathematics*. Mahwah, New Jersey: Lawrence Erlbaum Associates, Publishers.

Mathematical Association of America (1991). *A call for change: recommendations for the preparation of teachers of mathematics.* In R.C. James (Ed). Washington, D.C.

National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics*. Reston, Virginia.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, Virginia.