A STRUCTURAL EQUATION MODEL ANALYZING THE RELATIONSHIP STUDENTS' STATISTICAL REASONING ABILITIES, THEIR ATTITUDES TOWARD STATISTICS, AND LEARNING APPROACHES

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Recent research in statistical reasoning has focused on the developmental process in students when learning statistical reasoning skills. This study investigates statistical reasoning from the perspective of individual differences. As manifestation of heterogeneity, students' prior attitudes toward statistics, measured by the extended Survey of Attitudes Toward Statistics (SATS: Schau, Stevens, Dauphinee and DeVecchio, 1995), and students' learning approaches, measured by the Inventory of Learning Styles (ILS; Vermunt, 2005) are used. Students' statistical reasoning abilities are identified by the Statistical Reasoning Assessment instrument (SRA; Garfield 1998, 2003). The aim of the study is to investigate the relationship between both attitudes and learning approaches versus reasoning abilities by estimating full structural equation models. Instructional implications of the models for the teaching of statistical reasoning are discussed.

INTRODUCTION

Recent research in statistical reasoning and the different types of it, such as reasoning about variation, distribution, and sampling distributions, has created important insights into the developmental process of a student's learning of statistical reasoning skills. Most research has focused on the identification of subsequent, hierarchically ordered stages of reasoning development by means of qualitative research methods such as thinking aloud sessions or interviews. Two recent issues of the *Statistics Education Research Journal* (2004:2; 2005:1) and an edited volume (Ben-Zvi and Garfield, 2005), contain a wealth of empirical studies into the cognitive process of developing reasoning abilities, and instructional tools that might foster these developments. The present research investigates statistical reasoning from a somewhat different perspective. It examines individual differences between students learning statistics and statistical reasoning. Students enter learning processes with different background characteristics, and different perceptions of the learning context. In as far as students' learning paths are dependent upon individual differences, diversity in learning paths has to be accounted for. As one of the manifestations of students' heterogeneity, this study uses students' prior attitudes toward statistics, measured by the extended Survey of Attitudes Toward Statistics instrument (SATS; see Schau, Stevens, Dauphinee and DeVecchio 1995; Schau, personal communication, November 30, 2003). As a second manifestation of students' heterogeneity, students' preferred learning approaches is used, measured by the Inventory of Learning Styles (ILS; see Vermunt 1996, 1998, 2005). The main aim of this contribution is to investigate the relationship between students' prior statistical reasoning abilities when entering an introductory statistics course and relevant personal background variables as attitudes toward statistics or learning approaches. We identify statistical reasoning by the Statistical Reasoning Assessment instrument (SRA; see Garfield 1998, 2003). All instruments are quantitative of nature, and generate observations that can be regarded as proxies for the underlying, but unobservable theoretical constructs. Therefore, the investigation of the relationship between attitudes, respectively learning approaches, and reasoning abilities requires the estimation of confirmatory latent factor models for each of the background factors on the one side, and for statistical reasoning on the other, and the integration of these factor models into a full structural equation model. To this model, we add two indicators of course performance: latent variables measuring the strongly cognitive based scores in the final exam, and the more effort-based scores in quizzes. Prime reason to do so is that it allows characterizing the particular position statistical reasoning takes within the spectrum of different performance indicators.

MEASURES

The Statistical Reasoning Assessment, shortly SRA, is a multiple-choice test consisting of 20 items developed by Konold and Garfield as part of a project evaluating the effectiveness of a new statistics curriculum in US high schools (Konold, 1989; Garfield, 1998, 2003). Each item in the SRA describes a statistics or probability problem and offers several choices of responses: correct and incorrect. Most responses include a statement of reasoning, explaining the rationale for a particular choice. Those responses are based on well-described classes of misconceptions and their antipodes, the learned or unlearned correct conceptions, that emerged from cognitive science research into reasoning fallacies.

Attitudes toward statistics are measured with the Survey of Attitudes Toward Statistics (SATS) developed by Schau and co-authors (Schau, Stevens, Dauphinee, and DelVecchio, 1995; Dauphinee, Schau, and Stevens, 1997; Hilton, Schau and Olsen, 2004). There exist two versions of the SATS, both consisting of seven-point Likert-type items measuring aspects of post-secondary students' statistics attitudes. The 28-item version of SATS contains four scales: Affect (measuring positive and negative feeling concerning statistics); Cognitive Competence (measuring attitudes about intellectual knowledge and skills when applied to statistics); Value (measuring attitudes about the usefulness, relevance, and worth of statistics in personal and professional life); and Difficulty (measuring attitudes about the difficulty of statistics as subject). Recently, Schau has developed a 36-item version of the SATS, containing two additional scales (Schau, personal communication, November 30, 2003): Interest (students' level of individual interest in statistics); and Effort (amount of work the student expends to learn statistics).

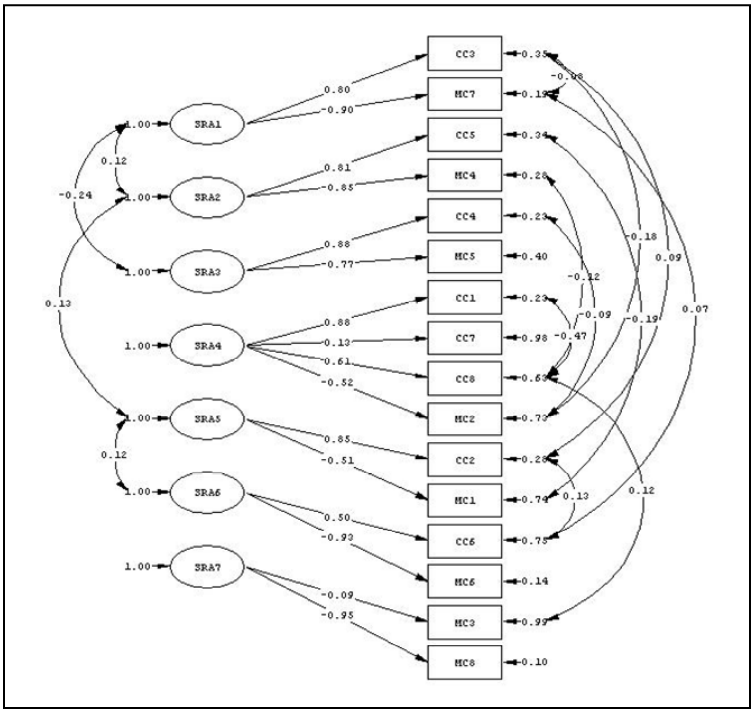
Students' learning approaches are assessed by the Inventory of Learning Styles (ILS). The ILS aims at measuring the following components of student learning: cognitive processing strategies (thinking activities that students use to process subject matter), metacognitive regulation strategies (activities that steer the cognitive and affective activities), conceptions of learning (coherent systems of knowledge and beliefs about learning and related phenomena), and learning orientations (students' personal goals, intentions, motives); see e.g., Vermunt (1996, 1998, 2005). On the basis of profiles for each of these learning components, students can be classified according their preferred or typical learning approach. Learning approaches repeatedly found in empirical factor-analytic studies of the ILS are meaning directed learning -often labeled as the deep approach to learning-, reproduction directed learning –often labeled as surface approach to learning-, application directed learning, and undirected learning.

STATISTICAL ANALYSES

This study integrates several techniques of structural equation modeling (SEM). A SEM model is distinct from a path or regression model in that it hypothesizes that crucial variables, such as attitudes in this study, are not directly observable and are better modeled as latent variables than as observable ones. In doing so, a SEM makes it possible to distinguish two different types of errors: errors in equations, as does the path model, and errors in the observation of variables. Making this distinction is especially worthwhile, when errors in important constructs have rather different sizes. Studying reliabilities of several attitude components, suggests that this argument applies to this study. In our study, SEM's were estimated with *LISREL* (version 8.54) using maximum likelihood estimation. The standard approach to estimate a SEM distinguishes two steps (Schumacker and Lomax, 2004). In the first phase of the two-step model building approach, measurement models for all latent variables in the model are estimated. Measurement models are in general factor models that allow factors, also called traits, and sometimes uniqueness, i.e., the errors in indicators, to be correlated. In our study, we need to estimate four of such correlated trait, correlated uniqueness, confirmatory factor analysis models: for the SATS data, for the SRA data, for ILS data, and for course performance data. In the second model building step, the structural part of the SEM is estimated. This structural part specifies the relationships between the independent and dependent latent variables.

SRA MEASUREMENT MODEL

Empirical studies of the SRA instrument all have used aggregated correct conceptions, and aggregated misconceptions, as scales. This would suggest a measurement model with two latent constructs and the correct reasoning and misconception scales as indicators. However, this modeling approach has important drawbacks, given the low correlations between reasoning ability scores, implying low reliabilities of aggregated scales. In this study, and alternative design of the measurement model is applied, based upon bivariate relationships between several pairs of a correct reasoning scale, and a misconception scale, as evident from the patterns of the correlation matrix. The seven-factor correlated traits, correlated uniqueness confirmatory factor model for SRA based on that design, as depicted in the figure, provides adequate fit.



FULL STRUCTURAL EQUATION MODELS

With the SRA factor model as one of the building blocks, two structural equation models are estimated: one explaining SRA factors, together with two latent course performance constructs (score in exam, and score in quizzes), by attitudinal latent constructs, and one explaining the same set of variables by latent learning approaches. Both SEMs indicate that the relationships between student background factors and reasoning abilities is very different in nature from the relationships between the same background factors and the course performance constructs. For example, the only attitude predicting statistical reasoning is planned Effort; the relationship is however negative, indicating that ‘hard-working’ students underachieve in comparison to their more ‘relaxed’ peers. In contrast, Effort is positively related to quiz score. Similarly, the learning approach SEM indicates that whereas deep learning best predicts the exam construct, and surface learning best predicts the quiz construct, the relationship with reasoning abilities is very different: surface learning is the best predictor, but again with a negative sign. Surface learning is inferior to just being inefficient: it is a real obstacle in learning to reason.

INSTRUCTIONAL IMPLICATIONS

This study adds evidence to the difficulty of educating statistical reasoning. Empirical studies as documented in *Statistics Education Research Journal* (2004:2; 2005:1) and Ben-Zvi and Garfield (2005) clarify that in order to learn reasoning and to unlearn misconceptions, the usage of specific educational tools is indispensable. The strong dependency on these instructional tools might be at odds with educational principles on which student-centered programs are based, in the sense that they delimitate students’ own responsibility to organize the learning process. The outcomes of this study might bring forward some further limitations. In most learning processes students enter the learning context with a given set of background characteristics, such as a preference for deep learning versus surface learning, and specific subject attitudes. Most of these contexts allow all students to achieve satisfactory learning outcomes, be it along different learning paths. As a concrete example: our structural equation model suggest that both surface learning oriented students and deep learning oriented students can pass the course. But our empirical analyses also suggests that statistical reasoning might be the odd man out in this context: the learning of statistical reasoning seems not to easily assimilate to variation in students' background characteristics as is the case with other cognitive goals. If this conclusion is correct, it implies we need an more focused type of educational tools than already described in the sources referred above: above content, the tools should address general learning approaches and attitudes.

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