STATISTICS AND FINANCE: LIVING ON THE "HEDGE"

Ricardo Gimeno
Banco de España, Spain
Ruth Mateos de Cabo
Universidad San Pablo –CEU, Spain
rgimeno@cee.upco.es

Statistics plays a leading role in finance. The explosive development of increasingly complex markets makes it more and more difficult for practitioners to correctly value financial asset. Statistical analysis has become a powerful tool for a better market valuation, taking a leading role in the development of new financial products that try to hedge the increasing amount of risks that an investor has to take. Statistics knowledge demand is steadily increasing in Hedge Funds, Investment Banking and Financial Institutions in general, where statistics students could developed a professional career. Finance can be seen as a way to motivate students on the applications of almost any statistical tool we would like to teach them, since we could always find an example where these techniques are put into practice.

SOME HISTORY ON THE EXPLOSION OF STATISTICS IN FINANCE

The major economic crisis experienced in Europe between 1875 and 1895 brought about the need for a major application of the academic statistical concepts to organize and record new statistical data such as consumer price indexes of workers, family budgets or unemployment days. This way, survey techniques and mechanical processing of information became to be widely used to solve practical problems in economics, but the development of analytical methods remained separated from the practise.

A good example on how the statistics is being used progressively into the field of finance can be shown through the derivation of the *Black-Scholes* formula (Black and Scholes, 1972 and Merton, 1973) for option pricing. The usual derivation of the Black-Scholes formula relies on the assumption that the market price of the underlying security follows a diffusion process. For the Black-Scholes model, the geometric Brownian motion is the basic model for the behaviour of stock-market prices. In 1900, Louis Bachelier, in his doctoral dissertation *Théorie de la Spéculation*, proposed that stock prices move like what it would be later known as Brownian motion. One of the most important shortcomings or Bachelier's model was that it allows the price to become negative. In 1960 the American economist Samuleson eliminated this difficulty and proposed the *geometric Brownian motion* that assumes that the logarithm of the share prices, rather than the price itself, follows a Brownian motion (Intuitively, Brownian motion is a continuous limit of a random walk) . The consistency of this modelization with Fama's popular theory of efficient markets (Fama, 1965), has made it the standard descriptor of asset prices identifying them as random variables. This theoretical framework has allowed most of actual financial analysis and development.

Actually, there are an incredible number of examples where statistics plays a key role in finance. We will describe some of them as the importance of *subjective probability* in price discovery; the efficient market hypothesis and *iid* samples; Montecarlo simulation for the definition of the efficient frontier; returns modelization; and finally, applications of risk measures. Each of the following sections will provide a brief introduction to different financial concepts at a level comprehensible to degree students with no previous knowledge of finance or economics.

ASSET VALUATION AND PROBABILITY

Finance main concern is related with the valuation of assets. An asset can be seen as the promise of receiving a set of future payments, called *cash-flows* (*CF*). The *asset value* (*P*) could be approached by the sum of all *CF* associated with that asset. But, this would mean that we do not take into account that an individual would rather prefer to receive the same amount of money today than in five years. For that reason, *CF* should be weighted inversely to the time to fulfillment. The weights are obtained from a discount function dependent on time to maturity and a parameter called *interest rate* (i) that can be seen as a price of time. The fundamental formula of asset valuation will be as shown in Equation 1.

$$P = \sum_j CF_j \cdot e^{-i \cdot t_j} \quad (1)$$

where $e^{-i \cdot t_j}$ is a continuous discount function. Once we know all the CF_j (and its correspondent t_j) and the appropriate i, P is uniquely determined. Let us suppose we have an US coupon-zero Treasury Bond that promise to pay \$100 in one year. Given an i=0.04, then P=100·$e^{-0.04 \cdot 1}$=96,08.

Unluckily, most of the *CFs* present in financial assets are not known in advance. This is the case with shares, where *CFs* are called dividends and depends on firm performance year by year. Even if your asset is a bond where the amount and time of maturity of each coupon is contractually established, you cannot be sure of the fulfillment of payments, as any borrower of Enron or Parmalat will know for sure.

In fact, any *CF* can be seen as a random variable. If we knew the probability distribution of each *CF*, then it is possible to reconstruct a mean value of an asset from the expected *CF*. But, although it is not possible to know such probabilities, everyday millions of people made such valuation (consciously or not), using their own subjective probabilities.

Although these subjective probabilities may seem feasible, we have to take into account that financial markets enclose thousands of practitioners that use its own judgment of the distribution of *CF* to valuate assets, buying and selling in consequence. The movements on prices originated by these operations, produce a so-called *price discovery process* (PDP), that allow the agents to improve their knowledge of the hidden probabilities by the joint opinion of the other investors as reflected in the operation they made.

Fama (1965) proposed the *Efficient Market Hypothesis* (EMH) that estate that if all information relevant to the pricing of an asset is known by investors, this information will be incorporated into the price via PDP, and no available information in the market can be used to improve such evaluation.

Let us now suppose we have a corporate bond that promise to pay \$100 in one year and the market price it at \$93. The difference comes from the possibility that the company does not pay it (default as it is called), compared with a Treasury Bond that is supposed to be risk-free. The \$100 *CF* can be modeled as governed by a binomial variable B(1,π), where π has been implicitly established by the market,

$$E[P] = (1-\pi) \cdot 100 \cdot e^{-0.04} = 93 \quad (2)$$

So π= 0.032 is the subjective probability given by the market to the company to default in one year. We have used a very simple asset with just one *CF* and an available risk-free asset for the same period. But the analysis could be expanded to more complex assets (Hull and White, 1999). This a very simple example for an student to see binomial distributions, just by open a newspaper, and cooperative exercises can be implemented in the classroom, as well as homework assignments, to the discovery of default probabilities (π).

In the previous exercise we have implied that the evaluator is neutral to risk, that is, it is indifferent for him to select between assets with different levels of risk. In a world where everybody is risk-neutral, all investment would be made in the asset who promises higher *CF* for fewer prices. If that where the case, everybody would invest in junk-bonds issued by troubled companies and countries, but the fact is that most people prefer assets with quite less risk like American Treasure Bonds or German Deutsche Bunds.

It was Markowitz (see Markowitz, 1991) who established that the decision on where to invest was a problem of Pareto optimal decision between risk and return. For the same level of return we would prefer assets with less risk, and for the same level of risk we would prefer more returns.

Markowitz's portfolio theory can be used as an instrument for a very simple example of the possibilities provided by Montecarlo Simulations. Our exercise consists in tracking 32 world indexes during 2003. For each one of these markets we can proxy the expected daily returns from Equation (3),

$$r_t = \log P_t - \log P_{t-1} \quad (3)$$

and risk by standard deviation of r_t, also called *volatility*. In Figure 1a, a scatter-plot of risk and expected returns for the selected basket of indexes. From the analysis of this figure it could seems that there are some stocks where it is not wise to invest but, in fact, this is not the case. Markowitz showed that sometimes it is possible to obtain the same amount of return of a single asset but with less risk via diversification with an appropriate portfolio including "not so good" assets.

To show this point we can use Montecarlo Simulation to generate a set of portfolios (linear convex combinations) until we obtain something similar to Figure 1b that illustrates how it is possible to find portfolios with better performance (measured in terms of returns and risk) than individual assets. These "efficient" portfolios include significant proportions of assets that would have been ruled out by a careless analysis of Figure 1a. These portfolios define an *efficient frontier* as it is called in Portfolio Theory.

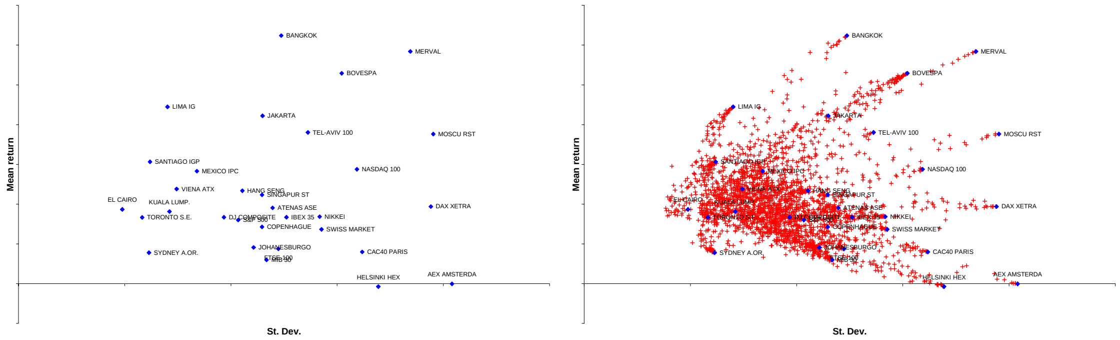

Figure 1: (a) Daily Risk and Return of 32 World Stock Indexes along 2003. (b) Daily Risk and Return of 3.000 simulated portfolios and efficient frontier.

PRICE MODELLING AND STOCHASTIC PROCESSES

Asset return definition in Equation 3 implies that asset prices evolve as some kind of random walk. This conception of prices can be tracked back to Bachelier (1900) seminal work, improved by Samuelson (1960) that defined it in continuous time as a *geometric Brownian motion*. Samuelson's improvement consisted of the appearance of *P* in the denominator of price changes, which avoid the theoretical appearance of negative prices.

$$\frac{dP}{P} = r \cdot dt + \sigma \cdot dW \tag{4}$$

A type of stochastic process where *t* represents time, *r* is the long term expected return, and σ is a measure of volatility, so price movements distribute as $N\left(r, \sigma\sqrt{t}\right)$ as consequence of the presence of the Wiener process (*dW*). Norbert Wiener showed in the early 1920s that a Brownian motion can be described directly in terms of a probability measure over a space of continuous path. This type of modeling has become standard due to its consistency with EMH. If Equation 4 stands, then the *dW* term prevents asset price forecasts, apart from its long term expected return *r*.

Efficiency will implied that returns would be independent and identically distributed (iid). The most potent test of the iid hypothesis may be the BDS test (e.g., Brock *et al*., 1996), that systematically rejects the iid hypothesis when applied to return time series.

Encouraging results have been made to overcome the "frustating" consequence of EMH. Insights in observed return distribution as pioneered by Mandelbrot (1963) lead to the discovery of three stylized facts:

- Returns diverge from Gaussian distribution by having more observations close to the mean and the extremes of the distribution (heavy tails). This tails are linked to the arrival of new information into the market what requires reevaluating Equation 1.
- Returns are skewed to the left of the distribution, as result of heavier reaction to bad news (losses) than to good news, coherent to the expected risk-aversion of economic agents.
- Extreme movements tend to cluster in some periods of time, what suggest that volatility changes over time.

Return time series are easily obtained from the Internet and can be used by students to became familiar with simple concepts of probability as the meaning of mean, variance, skewness of kurtosis. It is an easy exercise to see how divergence from normality is due to the presence of extreme values, that is, new information and reevaluation of Equation 1 (Gimeno and Gonzalez, 2004). If we take a time series of returns and evaluate its normality with a standard test of Gaussianity like Jarque-Bera test (Jarque and Bera, 1980) we will reject the Gaussian hypothesis (Figure 2).

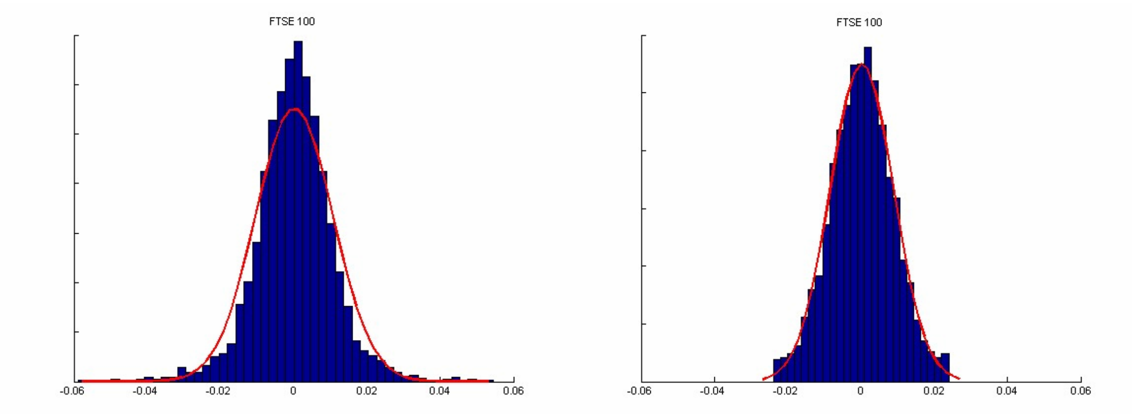

Figure 2: (a) Histogram of daily returns on British FTSE-100 against normal distribution. (b) Histogram of daily returns of FTSE-100 and normal distribution after subtracting 2.3% of largest movements.

In spite of this, if we order the sample by the magnitude of absolute returns and recursively test normality subtracting the largest movements (Figure 3), it can be seen that we just need to erase a small part of the sample to obtain a gaussian distribution, where there is no track of the leptokurtosis or skewness (Figure 4). The observations subtracted tend to cluster in some periods of time and reinforce the theory of the connection between extreme values and the arrival of new information.

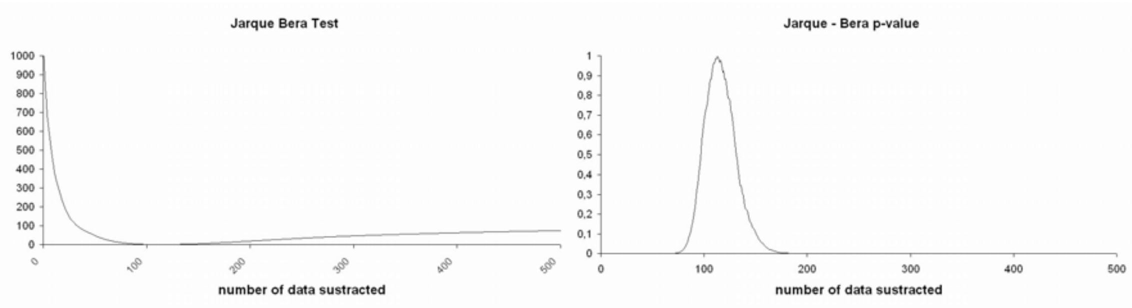

Figure 3: Recursive Jarque-Bera Tests on FTSE-100 daily returns. Test statistic is represented on the left while the test p-value is on the right.

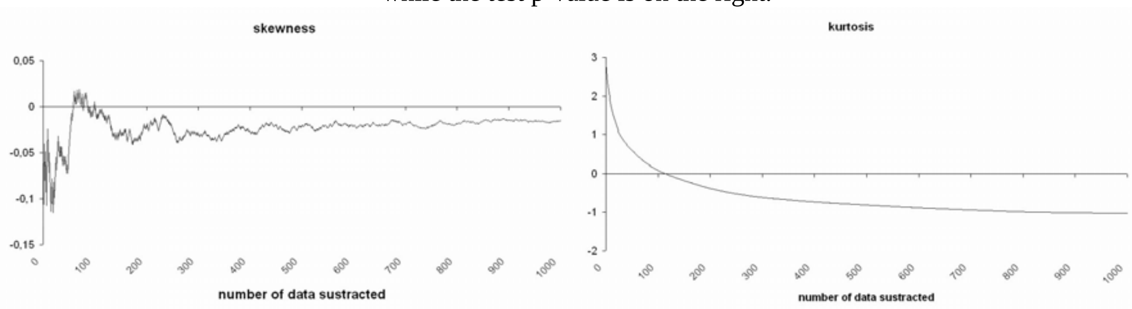

Figure 4: Skewness and Kurtosis coefficients on FTSE-100 daily returns in function of the number of extreme values subtracted

RISK MANAGEMENT

Nobel Prize winner Engle (1982) has supposed a recent revolution in finance developing new models (Generalized Autoregressive Conditional Heterokedasticity, or GARCH Models) that successfully capture these clusters of volatility allowing parameter σ of Equations 4 and 5 to evolve over time. A Typical GARCH(1,1) model would be,

$$\left.\begin{aligned} r_t &= \mu + \sigma_t \varepsilon_t \\ \sigma_t^2 &= \delta + \alpha \sigma_{t-1}^2 + \beta \varepsilon_{t-1}^2 \end{aligned}\right\} \qquad (5)$$

Promising results as the ones that have been obtained with GARCH models or its variants has put lately the focus on asset risk. Some measures allow quantifying and managing the risk of a portfolio. That is the case of *Value at Risk* (VaR) that international agreements like the one known as Basel II have established as a standard tool for financial institutions. VaR is the maximum loss we could have in a determined period of time with a given probability. A VaR(95%) of a portfolio could be obtained by simply recovering a record of past returns and using them to estimate parameter σ. Then $VaR(95\%) = 1'645 \cdot \hat{\sigma}$ if Gaussianity of returns is assumed. GARCH modeling allows for better estimation of σ_t as seen in Figure 5. The wide variety of *VaR* model that it is possible to find allows for an interesting project in which students can compete in a stochastic process contest on implementations of *VaR* models for a given portfolio.

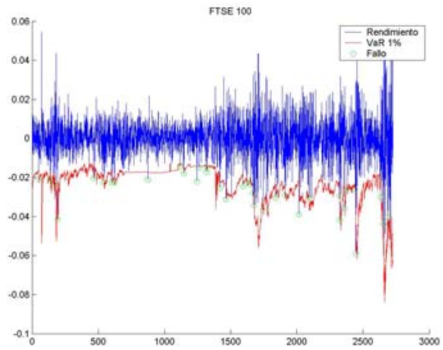

Figure 5: FTSE-100 daily returns and VaR(1%) estimated with a GARCH(1,1) model

CONCLUSION

In the new millennium the contact between statistics and economics, and especially finance, is of the major relevance. In the new economy, we are witnessing the rapid emergence of new financial products and derivates, the advances of the information technology and telecommunications produces rapid expansion of international trade and capital flows, and the globalization and economic linkages make important to track trade and investment flows among the companies operating locally and overseas. These are only a few reasons of why in this increasingly complex and globalized economy it is vital that statistics and economics work together to develop new concepts and methodologies.

These issues pose a challenge for statistical education. Although it seems evident that an understanding of basic statistical theory is useful for most university statistics students, what our students need is to develop a broader appreciation of statistics and their applications. For this reason, it is vital that students, especially in a business school, were exposed to practical problems that are likely to be encountered in practical statistical world such as those used in financial markets every day. Only in this way, we can achieve a better comprehension of the statistics and their applications in our students and competent and well-trained financial practitioners.

In this way, the use of Monte-Carlo simulations, so frequent in finance to obtain a valid pricing of assets, can serve as useful learning problems that help students to stop viewing econometrics as a collection of mechanical procedures with an accompanying set of formulas. Besides, the Monte-Carlo exercises are suitable vehicles (Kennedy, 2001) for motivating students to ensure that they understand the sampling distribution concept that captures the basic logic of statistics.

We cannot be exhaustive with the tools showed in the paper. Therefore, some other financial concepts could have been used to introduce some concepts as linear regression (*Capital Asset Pricing Model*), factor analysis (*Arbitrage Pricing Theory*), Logit/Probit (*Credit Scoring*), Neural Networks (*Price Forecast*), Stochastic calculus (*Black and Scholes optin valuation*) or nonlinear regression (*Term Structure of Interest Rates*).

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