A FRAMEWORK FOR EXAMINING TEACHER KNOWLEDGE AS USED IN ACTION WHILE TEACHING STATISTICS

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Research on teacher knowledge has typically examined teachers outside of the classroom in which they use their knowledge. Recognising that it is difficult to separate a teacher's knowledge from the context in which it is used, there has been a move towards studies being conducted in the classroom. Statistics presents its own challenges for teaching and learning compared with mathematics teaching and learning, especially with the growing recognition of and research around statistical thinking. Consequently there is need for an approach to examining teacher knowledge in relation to the actual work of teaching of statistics. This paper suggests a framework for examining the knowledge of primary (elementary) teachers as they engage in teaching statistics. The framework recognises that teacher knowledge is dynamic and dependent on the context of the classroom and students within it.

INTRODUCTION

In the last twenty years, there has been a significant and developing research focus on teacher knowledge. Shulman (1986) provided a major focus through his classification of teacher knowledge into various components, the most significant arguably being pedagogical content knowledge. Shulman defined this new term to include aspects of teacher knowledge such as knowledge of the most useful forms of representation of ideas within a topic, "the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject to make it comprehensible to others … [and] includes an understanding of what makes the learning of specific topics easy or difficult" (Shulman, 1986, p. 9), and consequently knowledge of how learners may be assisted in their learning of these concepts. Other components of teacher content knowledge discussed by Shulman (1986) were subject matter knowledge and curricular knowledge. Subject matter knowledge refers to the knowledge of facts and concepts, and understanding the structure of the subject, while curricular knowledge includes knowledge of the sequence of topics or concepts to be taught and the materials and resources suitable for a particular topic. Following on from Shulman's work a number of other researchers have developed and refined their own classifications of teacher knowledge, including a significant number that have focused specifically on mathematics education.

Much of the research on teacher knowledge however has been conducted away from the classroom, the main context or site in which teachers use their knowledge. In spite of the data about teacher knowledge may be obtained from a survey or an interview, or from an examination of evidence 'artefacts' such as the teachers' lesson plans, what actually happens in the classroom could be considered to be of greater importance. Teachers have to operate under the pressure of dealing with what is happening in the real time moment of teaching and consequently research needs to occur within the classroom, searching for evidence of what knowledge a teacher has and uses in the immediate act of teaching. For instance, the discourse-related practices of the classroom (such as having to provide an extra explanation for a student, or needing to respond to a student's question, or deciding on the next question to ask) cannot be separated from a teacher's mathematical content knowledge (Boaler, 2000). It has also been suggested that through the interactions that occur in the classroom, teachers can themselves learn, and that this learning is more likely to occur as a result of investigative approaches rather than in 'frontal teaching' situations (Margolinas, Coulange, and Bessot, 2005).

This paper examines teacher knowledge frameworks in general, and teacher knowledge in relation to teaching statistics and how it might differ from knowledge for the teaching of mathematics. Some of the recent literature about statistics learning is examined, leading to a framework being proposed that would enable teacher knowledge for teaching statistics to be unpacked.

TEACHER KNOWLEDGE FRAMEWORKS

Shulman (1986) provided a significant impetus to research in the field of teacher knowledge. Because his research investigated categories of teacher knowledge, Shulman's work as well as that of many subsequent researchers can be considered to fit a 'knowledge system analysis' paradigm (Sherin, Sherin, and Madanes, 2000). This type of research is characterised by descriptions of knowledge categories that contribute to successful teaching, and how knowledge changes and evolves with time. In comparison with knowledge system analysis, research that fits a 'cognitive modeling' paradigm focuses on more short-term activity of the teacher and describes aspects of teacher knowledge that can account for particular teaching behaviours. This type of research, such as that of Leinhardt and Greeno (1986) and various other researchers who have examined teachers' knowledge in regard to such things as their goals for lessons and lesson 'scripts and routines,' gives accounts for particular teaching behaviours. These two broad research paradigms (knowledge system analysis and cognitive modeling) differ because they are broadly concerned with respectively the 'content' of teacher knowledge and the 'form' of teacher knowledge (Sherin *et al*., 2000). The content of teacher knowledge identifies and describes what the knowledge is about or what it is used for, whereas the form of teacher knowledge relates to the structures of how the teacher knowledge is organised, linked, and represented in the teacher's mind. There is research that combines these two paradigms together in order consider what the teacher is doing and why. Sherin *et al*. (2000) describe how Schoenfeld's framework (Schoenfeld, 2000) encapsulates both research paradigms, in that it "specifies different kinds of knowledge and how they function as part of a cognitive mechanism" (Sherin *et al*., 2000, p. 368), but it also allows for classroom incidents changing the teacher's behaviour and how this may affect or change teacher knowledge in the longer term. Such frameworks that can address both the short-term aspects of teaching and teacher knowledge as well as changes in teacher knowledge and therefore potential for its longer term development would undoubtedly be the most useful.

The development of teacher knowledge is considered to be dynamic, rather than fixed and static. Situating research on teachers' knowledge in the classroom therefore has the potential to be able to account for knowledge growth. A teacher's knowledge continues to grow while they are teaching (Manouchehri, 1997) and some research has been conducted to investigate this knowledge growth (for example, Sherin, 2002). Sherin's research focused on the teachers' content knowledge (both subject knowledge and pedagogical content knowledge), with evidence gathered about the teachers' roles in discussion of mathematical concepts, the way the teachers presented curriculum materials and the teachers' responses to students' questions and ideas. From the analysis, it was apparent that in some cases, the teachers' content knowledge changed during the course of the lessons. The research design that enabled such conclusions to be drawn included data collected from interviews with the teachers, videotaping of lessons and observation notes from those lessons, and teacher 'video club meetings' where the teachers and the researcher watched and discussed excerpts of lessons.

Some frameworks for teacher knowledge have focused on only one component of knowledge. For instance, Kahan, Cooper and Bethea (2003) examined teachers' mathematics content knowledge using a 6×4 matrix: the 6 categories were 'elements of teaching' such as selection of tasks and representations, motivation of content, development through connectivity and sequencing, while the other dimension of the matrix split the 'processes of teaching' into four components. They analysed lesson plans to identify aspects of the teachers' mathematical content knowledge that would fit the 24 cells of the matrix, each of which was described in relation to the connection between the element of teaching and the process of teaching. Kahan *et al*. commented that for other knowledge bases such as pedagogical content knowledge, a third dimension might be needed for the framework. As with many other models of teacher knowledge, this framework has limitations with regard to the indistinct boundaries between the various categories.

Taking some of the tasks of teaching into consideration when examining teacher knowledge is important. It has been argued that teacher knowledge is organised in a content-specific way, rather than around the generic tasks of teaching such as lesson planning, etc. (Hill, Schilling, and Ball, 2004). They looked at breaking down mathematical content knowledge and pedagogical content knowledge into categories that would usefully describe the various aspects of teacher knowledge relevant to teaching particular content. Mathematical content knowledge consists of *common knowledge of content*, that which any reasonably educated adult should know and be able to do, and *specialised knowledge of content*, that which teachers but not necessarily other adults know and can do (Hill *et al*., 2004). Common knowledge of content would include the ability to identify incorrect answers or inaccurate definitions, and the ability to successfully complete the students' problems. Specialised knowledge of content may include the ability to analyse mathematically whether a student's unconventional answer or explanation is reasonable or mathematically correct, or to give a mathematical explanation for why a process (such as a particular algorithm) works. The subcategories of pedagogical content knowledge were further refined and described by Ball, Thames, and Phelps (2005). They describe *knowledge of content and students* to include the ability to anticipate student errors and misconceptions, to interpret incomplete student thinking, to predict how students will handle specific tasks, and what students will find interesting and challenging. The other component of pedagogical content knowledge is *knowledge of content and teaching*, which gives the ability to appropriately sequence the content for teaching, to recognise the instructional advantages and disadvantages of particular representations, and weigh up the mathematical issues in responding to students' unexpected approaches. These four categories of mathematical knowledge address the belief that the work of teaching entails and cannot be separated from various aspects of mathematical knowledge, skills, and habits of mind. Hill *et al*. (2004) went on to develop an assessment tool to measure these aspects of teacher knowledge in the domain of number, number operations and algebra and to look for any relationship between teacher knowledge and student achievement. Although their research was conducted with a limited range of mathematical content, these classifications of teacher knowledge are seen as potentially useful in relation to teaching statistics.

TEACHING AND LEARNING STATISTICS COMPARED WITH MATHEMATICS

Although statistics is generally a part of the school mathematics curriculum, and in New Zealand this has been the case in elementary (primary) education since 1969, it is recognised widely that there are differences between mathematics and statistics (for example, Cobb and Moore, 1997; delMas, 2004; Pereira-Mendoza, 2002). These differences arise from the deterministic nature of mathematics and the 'reasoning under uncertainty' that is the feature of statistics. Whereas in mathematics education the use of real-life contexts are advocated as a generally useful means of developing students' understanding of mathematical concepts, in statistics it is considered essential that students come to realise that data are numbers with a context and are used to address a particular issue or question (Cobb, 1999; Gal and Garfield, 1997), in statistics

The implication from this is that teaching therefore must take these differences into account and that teachers be ready and able to encourage students to think differently in statistics. Rather than focusing on statistical skills, procedures, and computations, there has been a growing call to encourage students to reason and think statistically (Ben-Zvi and Garfield, 2004). These terms (namely, statistical reasoning and thinking) although not clearly defined and delineated, present a challenge to teachers to help students develop some of the 'big ideas' of statistics. Wild and Pfannkuch's (1999) model for statistical thinking has provided some clarity to and an important way of examining what constitutes statistical thinking. The five components of statistical thinking (recognition of a need for data, ability to 'transnumerate' the data; recognition of variation; being able to reason from models; and being able to integrate statistical and contextual knowledge) need to be developed through a 'constant dialogue' with various representations of the data (Wild and Pfannkuch, 1999). To support this development, investigations are advocated as a worthwhile approach to teaching and learning. However, two studies of pre-service primary teachers found that when the teachers undertook an investigation of their own, they lost sight of the goal of the investigation and instead tended to focus on the production of a graph. (Burgess, 2002; Heaton and Mickelson, 2002). Effective teaching therefore requires an understanding and implementation of the investigative and interrogative cycles along with certain dispositions, some examples of which are imagination, scepticism, being logical, and perseverance (Wild and Pfannkuch, 1999).

Many frameworks have been developed for particular aspects of statistics skills and reasoning of students, particularly in relation to misconceptions. Some of these focused on the mean and other averages, graphing, variation, sampling, and probability. Other frameworks have looked more generally at students' understanding. Teachers have been the subject of research in various ways. Some examples of such research are: a profiling tool was developed for classifying teacher knowledge across seven knowledge domains (Watson, 2001); what teachers and students should know and be able to do with respect to understanding of graphs (Friel, Bright, Frierson, and Kader, 1997); teacher knowledge required to teach probability (Kvatinsky and Even, 2002); teacher knowledge with respect to investigations (Heaton and Mickelson, 2002); and teacher knowledge for teaching data analysis and statistics (Sorto, 2004). However, most of this research has not been conducted in the classroom in spite of some of the researchers acknowledging the importance of conducting such research in the classroom.

FRAMEWORK FOR TEACHER KNOWLEDGE IN STATISTICS

Combining ideas from the research literature on teacher knowledge, mathematics education, and statistics education has led to a proposed framework that could be used to investigate teacher knowledge for and as used in the teaching of statistics. The classifications of content knowledge for teaching as described by Hill *et al*. (2004) and Ball *et al*. (2005) are combined in a matrix with the components of statistical thinking and related aspects of empirical enquiry as described by Wild and Pfannkuch (1999) (see Figure 1 below).

		Statistical knowledge for teaching			
		Content knowledge		Pedagogical content knowledge	
		Common knowledge of content (CKC)	Specialised knowledge of content (SKC)	Knowledge of content and students (KCS)	Knowledge of content and teaching (KCT)
Thinking	Need for data				
	Transnumeration				
	Variation				
	Reasoning with models				
	Integration of statistical and contextual				
Investigative cycle					
Interrogative cycle					
Dispositions					

Figure 1: Components of teacher knowledge in relation to statistical thinking and investigating.

The cells of the matrix will enable descriptions of teachers' statistical knowledge developed from data collected from a variety of sources relevant to teaching. Video recordings of a sequence of lessons will focus on the teacher. Following the videotaped lessons, the teacher will be interviewed by the researcher, in relation to some significant incidents during the lesson. These incidents will illustrate the use of some aspect of teacher knowledge. For example: Why did the teacher respond to the student's question in that way? Did the teacher consider another response? Did you notice … with the student's work/answer/justification? In addition to the videotaped lessons and the interview, the researcher will have a copy of the teacher's written planning, and the textbook-type resources that the teacher and/or the students use or have access to. A written assessment-type task for the teacher will be used to explore some aspects of the teacher's knowledge.

Hill *et al*. (2004) and Ball *et al*. (2005) listed and described features of the four categories of knowledge in relation to mathematics education. These were:

- Common knowledge of content: ability to identify incorrect answers or inaccurate definitions, and the ability to successfully complete the students' problems;
- Specialised knowledge of content: ability to analyse mathematically whether a student's unconventional answer or explanation is reasonable or mathematically correct, or to give a mathematical explanation for why a process (such as a particular algorithm) works;
- Knowledge of content and students: ability to anticipate student errors and misconceptions, to interpret incomplete student thinking, to predict how students will handle specific tasks, and what students will find interesting and challenging;
- Knowledge of content and teaching: ability to appropriately sequence the content for teaching, to recognise the instructional advantages and disadvantages of particular representations, and weigh up the mathematical issues in responding to students' unexpected approaches.

Given the acknowledged differences between mathematics and statistics learning, some of the above descriptors for the four categories of teacher knowledge are not necessarily appropriate for the teaching and learning of statistics. For example, because of the more subjective nature of statistics with reasoning under uncertainty and understanding of the concept of variation, 'correct answers' or 'explaining particular algorithms' may not be a feature of teacher knowledge in the statistics classroom. Research using the proposed framework will aim to develop descriptors of the four knowledge categories in relation to the rows which focus on statistics for teaching.

SUMMARY

The development of research on teacher knowledge has in recent times moved toward a focus on how that knowledge is used in the classroom and how it impacts on student achievement. Because statistics education research has not had the same length of history as research in mathematics education, developments relevant to teacher knowledge for teaching statistics are needed. Research using a framework such as the one proposed in this paper will hopefully provide a clearer understanding of how teacher knowledge in statistics plays out in the classroom. The research may identify that some 'cells' of the framework are more obvious and present than others during the actual process of teaching. There may be some aspects that are conspicuously absent. Once a fuller description of statistics knowledge for teaching has been established, then ways of measuring such knowledge reliably and examining the relationship between teacher knowledge and student learning will be able to proceed with more clarity.

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