ON AVERAGE AND OPEN-ENDED QUESTIONS

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This paper reports on our ongoing research on the teaching and learning of averages in secondary mathematics education based on a questionnaire that combine open-ended and multiple-choice questions. Analysis has led us to note that many students who choose the correct answers in multiple-choice questions were completely unable to demonstrate any reasonable method of solving related open questions.

INTRODUCTION

In the last decades some studies have shown that the teaching and learning of averages, although apparently easy turn to be a difficult subject giving rise to tremendous difficulties when students of a different ages are confronted with this concept within different domains, i.e. teaching, learning and everyday situations. For example, Pollatsek, Lima and Well (1981) have shown that most students seem to know the average calculation rule or algorithm. However, if these students have merely an instrumental knowledge of averages, they will make foreseeable kinds of mistakes in all questions, except the most obvious. According to Watson and Moritz (2000), for a large number of children, the average is simply a value in the centre of distribution. A study undertaken by Cai (1995) has shown that most students know the mechanism of “adding all together and dividing” which constitutes the simple average calculation algorithm. However, only some of them were able to find an unknown value in a series of data where the average is known. Mokros and Russell (1995) demonstrate various difficulties faced by students in their understanding of averages. This study identified and analysed five different constructions of representation used by students: the average as mode, the average as algorithm, the average as something reasonable, the average as mean point and the average as mathematical point of equilibrium.

The debate on how to assess students’ conceptions has given rise to different points of view about the suitability of each form of assessment. Garfield (2003) describes a questionnaire for assessing statistical reasoning, consisting of twenty multiple-choice questions involving concepts of Probability and Statistics. Garfield (2003) believes that most assessment instruments are centred more on the abilities for calculation or problem solving than on reasoning and understanding. Cobo and Batanero (2004) and Cai (1995) underline the importance of open questions for assessment, and suggest that this type of question be used to examine students’ ideas about the concept of arithmetical average and the processes of solving problems. Gal (1995) states that it is difficult to judge fully what a person knows about averages as an instrument for solving problems based on data unless a context is given that would motivate the use of that instrument.

In this work we set out to analyse how students act when faced with open-ended questions that are closely related to multiple-choice questions. We wanted to find out if those students who choose the correct options in multiple-choice questions have done so using clear criteria, basing our observations on the students’ actions during the process of solving open-ended questions. We tried to find answers to questions like: Do those students who correctly answer the multiple-choice questions carry out reasoned actions in open problems? Are their actions a real, convincing indication of having had a solid basis when answering the multiple-choice questions correctly?

Our hypothesis is that most students who choose the correct answers in multiple-choice questions do not appear to do so on a solid basis. Our review of current related research has shown no clear evidence supporting our hypothesis and that is why we have conducted our research in that direction.

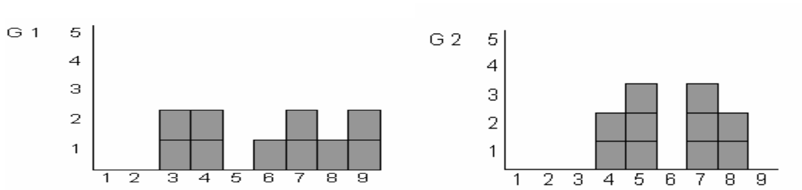
Our analysis could possibly highlight elements for reference to clear up various positions that are constantly being assumed in the debate on which forms of assessment are the most appropriate when examining the understanding of statistical concepts. In this respect we will cross reference results of pairs of interrelated problems. We also want to remark that this is part of our ongoing research on averages which is not yet finished**.**

METHOD

Our study was undertaken with 94 students in their final year of secondary education, average age being 17 years old. Throughout their schooling they had received specific instruction in arithmetical averages and other topics concerning descriptive and inferential statistics and also elementary probability.

We are developing a broad questionnaire on averages as part of our ongoing research; we want to design a teaching and learning unit for secondary education as well. Due to space reasons we will concentrate here on data gathered from one question drawn from the whole questionnaire which is made up of both multiple-choice and open-end items.

Question “Marks Graph”: *Twenty high-school students take part in a mathematics competition. Ten of the students form Group 1 and the other ten Group 2. The marks they achieve in the competition are shown in the graphs below:*



Each rectangle in the graph represents the mark achieved by an individual student. For example, in Group 1 the two rectangles appearing above Number 9 show that two students in this group achieved a score of 9.

5.A *Group 1 has an average mark of 6.*
- *a* *Check that the average mark for Group 2 is also 6.*
- *b* *Which group seems better to you? Justify your choice.*

5.B *Which of the following statements is true?*
- *a* *Group 1 is better than Group 2 because the students who got higher marks are in this group.*
- *b* *Group 2 is better because there are no students with marks below 4.*
- *c* *There is no difference between the two groups because the average is the same.*
- *d* *Although the averages are the same for both groups, Group 2 is more homogeneous.*

This question is of our own devising, although there is some similarity to one described by Garfield (2003). The aim of this question is to see how students interpret distributions shown in the form of a graph, find out if they know how to manipulate data graphically in order to calculate and examine what criteria they use when checking two samples based on their visual appearance.

RESULTS

We shall basically analyse the actions in open questions of those students who chose the right answer in the multiple-choice questions. Table 1 shows the cross referencing of results for the items “Marks Graph, 5.A.b)” (rows) and “Marks Graph 5.B” (columns).

As we can see 36 (38%) students marked the correct justification for the multiple-choice question (MHOMO), where this option referred to homogeneity and indicated the need to take into account the dispersion of data when comparing groups with the same averages. With respect to the open question, we could only find 7 (7%) students who gave the correct justification (MHOMO) using the same criteria. Also, of the 36 students who chose the correct option in the multiple-choice question, 30 (83%) had used incorrect arguments for the open item, that is, only 6 (17%) students gave acceptable arguments. The baseless justification most in evidence was the one that took for its criteria of comparison the fact that there were more students passing (NAPROB), given by 39% (14) of these students. We also found that 6 (17%) students who failed

	IGUMEDIA	MHOMO	MNOTA	N<4	NC	SC	Total
IGUMEDIA	13	1	0	0	0	0	14
MHOMO	0	6	0	0	0	1	7
MNOTA	2	1	4	0	0	0	7
N<4	0	1	0	1	0	0	2
NAPROB	5	14	1	14	3	6	43
NC	1	6	0	0	0	1	8
NJUST	0	1	1	0	0	0	2
SC	2	6	2	1	0	0	11
Total	23	36	8	16	3	8	94

Table 1: Cross referencing of results for the *open-ended question* "Marks Graph, 5.A b)" (rows) and *multiple-choice question* "Marks Graph 5.B" (columns). The categories we have set out to codify students answers are shown at the end of this paper.

to answer (NC), another 6 (17%) who put forward confused justifications (SC), 1 who said that there were no differences between the groups because the averages were the same (IGUMEDIA), and 2 who only looked at the maximums or minimums of the distribution (MNOTA and N<4). With regard to those students who correctly justified the open question, the data show that nearly all of the students also correctly got the answer to the multiple-choice question, except 1 (SC). These results lead us to suspect that those students who chose the correct answer for the multiple-choice question did so without taking into account a formal basis, since, after choosing the answer, they did not bother to rectify the wrong justifications they had put forward in the open question. This also shows that the students are not consistent in their affirmations. In the same situation they use completely different criteria! The difficulties arising when comparing samples in which students merely analyse only one part of the distribution of the maximum or minimum values were also found by Godino and Batanero (1997), and Estepa and Sánchez (1996). As interpreted by Estepa and Sánchez (1996), these difficulties are due to the fact that students have a local concept of association of variables and believe that this is the analysis which explains the differences between the two samples.

CONCLUSIONS

Our study has allowed us to see that many students who choose the correct answer for the multiple-choice questions are not able to demonstrate reasonable methods for solving open-ended questions. The above example is one among others we have from our ongoing research but the limited space in this article does not allow us to show more data to support our arguments. The actions in open-ended questions by those students who choose the correct answer in the multiple-choice questions, suggest that they choose these answers without any rational criteria. The results also show incoherence in students' actions when they correctly mark the answer to the multiple-choice question and are unable to solve a related open-ended question or vice versa. Furthermore, the results show that students are not consistent in their affirmations, given that in the same situation they use completely different criteria. Finally we want to highlight the fact that the combined use of open-ended questions and multiple-choice questions has allowed us to detect some difficulties and incoherence in students' actions, which are hard to detect with the use of only one type of question. We think that this is mainly due to the use of open-ended questions, underlining once more the importance of this type of question when assessing the concepts held by students.

CATEGORIES USED TO CODIFY STUDENTS ANSWERS:

IGUMEDIA - Justifies that the groups have the same averages or chooses the statement that indicates that there is no difference between the groups because the averages are the same.

MHOMO - Puts forwards a justification based on homogeneity or chooses the statement referring to this criteria.

MNOTA - Uses as criteria for justification the greater mark factor or marks the option referring to this criteria.

N<4 - Uses as an argument the fact that there are no marks less than 4.

NAPROB - Uses as justification the fact that there are more students who pass, achieved mark greater than 4, or marks the option referring to this factor.

NJUST - Indicates that one group is better, but without justifying this idea.

NC - No answer.

SC - Undertakes incoherent transformations or puts forward confused justifications or chooses more than one option.

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